UNSTABLE RELATIONSHIPS*

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Abstract

We analyze models where agents search for partners to form relationships (employment, marriage, coauthor, etc.), and may choose to continue searching for better partners while in relationships. Matched agents are less inclined to search if their match generates more instantaneous utility, and also if it is more stable. If one partner searches the relationship will be less stable, and so the other is more inclined to search. Thus, instability can be a self-fulfilling prophecy. This new source of multiplicity can lead to a continuum of steady state equilibria. We also show that it tends to lead to too much search, and also to too much unemployment and too much inequality in equilibrium.

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"To be faithful to one would be cruel to the rest." Don Giovanni

1 Introduction

We analyze models where agents search for partners to form relationships (e.g., employment, marriage, or coauthor relationships). While in a relationship, an agent may or may not choose to continue to search, at a cost, for a different partner. Matched agents are less inclined to search – or, more inclined to be "faithful" to their current partner – if the match is *better* in the sense of the instantaneous utility derived from the relationship, and also if the match is more *stable*. What lends stability to a relationship? If one partner is searching then the relationship is less secure for the other, since he is more likely to be abandoned, and hence he is more inclined to search himself. In this way, endogenous instability can be a self-fulfilling prophecy. We show these considerations can lead to multiple equilibria, and indeed to a continuum of steady state equilibria, in some cases. We also show that they tend to lead to too much search, too much unemployment, and too much inequality in equilibrium.

The multiplicity in the current model is new, and has nothing to do with the standard thick-market effects that have been understood in equilibrium search theory since Diamond (1982). The standard thick-market effect works as follows: assuming increasing returns in the matching technology, if there is more search then it is easier to meet people, which makes people more inclined to search. This is *not* what is going on here. To emphasize the distinction, we assume constant returns to scale in the meeting technology, so that your probability of meeting a potential partner is independent of the aggregate amount of search, and also make some other assumptions that reduce the strategic interaction between market activity and the private gains from search. This allows us to focus clearly on the strategic interactions within relationships, and to show that there can be more unstable relationships simply because people think relationships will be more unstable.[1]

The rest of the paper can be summarized as follows. Section 2 specifies the basic framework, and in particular our assumption that when any two people meet they draw a random variable x that gives the instantaneous utility each will receive if they form a partnership; thus, agents are homogeneous ex ante, although relationships are heterogeneous ex post due to match-specific idiosyncracies. Section 3 considers a relatively simple version where relationships come in two varieties, x_1 and $x_2 > x_1$. We characterize the set of equilibria in terms of when agents enter into relationships and when they search. For some parameter values there is a unique equilibrium, while for others there are multiple equilibria. The leading example is the following: agents will accept x_1 matches, and may either be: "unfaithful" and continue to search for a better match; or "faithful" and stop searching because they are satisfied with x_1, this satisfaction arising because these matches are relatively stable, and this stability arising because agents in x_1 matches do not search.

Indeed, we show that there can even exist a "perverse" equilibrium where agents prefer x_1 over x_2 matches, even though $x_2 > x_1$, because they believe

[1] In the labor market contect, searching while matched is referred to as *on-the-job search*, and has been previously studied by Burdett (1977) in a single-agent framework, by Mortensen (1978) in the context of two agents, and by Pissarides (1994) and Webb (1998) in simple general equilibrium models, for example. See Mortensen and Pissarides (1998) for other related references. None of the existing papers looks at the main issue considered here, however, which is the possibility of endogenous instability.

the former are more stable, and this belief is correct because agents search in x_2 matches and not in x_1 matches (a special case occurs when $x_1 = x_2$, so that there is really no fundamental difference between matches, but people simply believe that certain relationships will be less stable than others). We also analyze welfare, and derive the result that there tends to be too much search in equilibrium.[2] We also show that this leads to excessive unemployment and excessive inequality: compared to the solution to the social planner's problem, as the equilibrium tends to have too much search, it also has too many people unmatched, too many in x_2 matches, and too few in x_1 matches.

We go on in Section 4 to consider the case where the match value x is drawn from a general, rather than a two-point, probability distribution. We consider the class of equilibria where agents choose a reservation match value R, such that they enter relationships iff $x \geq R$, and a critical match value $Q > R$, such that they search while matched iff $x \leq Q$. Even within this potentially limited class, it turns out that under reasonable parameteric assumptions we can generate a continuum of equilibria: any value of Q in some interval is consistent with equilibrium. We also derive the distribution of agents across states and distribution of match values across existing relationships, and show how these vary across equilibria. In general, we show that equilibria with more search (a higher value of Q) generate more unemployment and greater inequality. We interpret our findings, that there can

[2]To be more precise, we show that endogenous instability tends to generate too much search, although there is another effect that tends to generate too little search, according to an ex ante welfare criterion, that arises here as in any typical search models where agents are impatient. If the discount rate is not too big, however, this standard impatience effect is dominated by the new endogenous instability effect, and there will be either too much search or an efficient amount of search depending on other parameter values.

exist a great multiplicity of equilibria, with different search behavior and different implications for unemployment, inequality and welfare, to mean that at least theoretically endogenous instability can be a powerful force.

2 The Basic Framework

Time is continuous. There is a $[0, 1]$ continuum of infinitely-lived agents who are interested in forming (bilateral) partnerships. While unmatched, agents search, and while doing so they receive an instantaneous payoff b, which can be thought of as the utility from being single net of search costs. While searching, they meet other agents according to a Poisson process with arrival rate α. All agents are homogeneous ex ante but matches are heterogeneous ex post: when any pair meet, they draw a random variable x giving the instantaneous utility that each will receive if they enter into a relationship. Agents are always free to reject a potential partner in favor of continued search – as they presumably would do if they draw a low value of x. The distribution of potential match qualities is given by $F(\bar{x}) = pr(x \leq \bar{x})$ and is assumed to be exogenous; however, the distribution of match qualities across existing relationships, call it $G(x)$, is endogenous, given that agents accept some values of x and reject others.[3]

[3]One can assume that in each meeting there is a realization of total surplus X, and that agents bargain in such a way that each gets $x = X/2$ (as would follow from at least some versions of standard bargaining theory). Alternatively, one can assume nontrannsferrable utility. A closely related model with nontrannsferrable utility (but no on-the-job search) is contained in Burdett and Wright (1998), although there it is assumed that when two agents i and j meet they draw a pair (x_i, x_j) representing the utlity each would receive from the match. In that model, x_i and x_j are independent. The present model can be thought of as a version of that general structure with perfect dependence: $x_i = x_j = x$. In any case, nothing we do in this paper depends on whether one interprets utilty as nontrasferrable, or as trasferrable with bargaining dividing the surplus evenly.

While in a relationship, agents enjoy the match value x, and also decide whether to continue searching. If they search they pay cost $d > 0$ and continue to meet other agents at the same rate α; if they do not search they pay no cost and meet no one new.[4] When a matched agent meets someone new, we assume that he first leaves his current partner and then draws the new value of x associated with the other person. He may then form a relationship with the new person at that value of x or reject the new person and become unmatched, but he *cannot* go back to his previous partner. One could also analyze the model where agents are allowed to go back to their previous partner after checking out a potential alternative; obviously, in this case they will choose whichever relationship yields the greater x. However, we think that our assumption not only seems realistic, at least in some contexts, it is also the appropriate assumption if one wants to focus on the strategic interactions *within* relationships.

To explain this, consider the opportunities available when you search. First, your arrival rate of meetings α could depend on the number of other agents that are searching. As this effect has been studied extensively in the past, here we assume constant returns to scale in the meeting technology, which implies that α is a fixed constant.[5] However, even give a constant α, if we allow a matched agent who meets someone new to choose between

[4] If $d = 0$, all agents will search, as in some previous analyses of related models, such as Webb (1998) or Burdett and Coles (1999); hence, those models cannot address the key aspect on which we focus – the decision whether to search while matched.

[5] Let $M(N)$ be the total number of meetings in the market per unit time when the number (measure) of agents who are searching is $N > 0$. The arrival rate for a representative individual is given by $\alpha = M(N)/N$, and so as long as M exhibits constant returns, α does not depend on N. Intuitively, if N is bigger there are more people you might meet, but you are also competing with more people to meet them, and constant returns implies that these effects exactly offset.

the new person and his current partner, your *effective* arrival rate will still be endogenous. To see this, observe that when you meet matched agents, if they are allowed to choose, they will only enter a relationship with you if you beat their current value of x, and therefore to know your chances you need to know the endogenous distribution of x within existing matches, $G(x)$. Since our assumption is that a matched agent can either go with you or become unmatched, but cannot stay with his current partner, meeting a matched agent is the same as meeting an unmatched agent.

Summarizing, in our model your effective arrival rate (if not your payoff) is unaffected by what other agents are doing: you have a constant probability of meeting someone, and every meeting is a draw from $F(x)$ which can either be accepted or rejected in favor of unmatched search. Our assumptions may somewhat discourage search while matched, compared to alternatives such as allowing matched agents the option of staying with their current partners, but in any case we will show below that there is too much search in equilibrium, and if anything this result is more striking give these assumptions. Moreover, they simplify the analysis. One thing that keeps the model relatively tractable is the following: given that you have decided to search while matched, when you meet someone new, there is no additional decision to leave or stay with your current partner (i.e., to stay without observing the new x; once you observe the new x our assumptions do not allow you to stay). This is because all new people look identical ex ante, so if you would not leave with one new person then you would not leave with any, and therefore you would not be engaged in costly search in the first place.

Hence, matched agents engaged in search are always willing to leave their current partners as soon as they meet someone else. For generality, we also assume that all matches are terminated exogenously according to an independent Poisson process with arrival rate σ. Whenever you are thrown out of a relationship, either because of an exogenous separation or because your partner meets someone new, you return to unmatched search. You also return to unmatched search when you are in a relationship, meet someone new, and find them lacking. Thus, in the context of labor markets, the model has well-defined notions of quits and layoffs – voluntary and involuntary separations, if you will – concepts that are often hard to distinguish in theory. However, we will not belabor this interpretation in the analysis that follows.

3 A Simple Model

Suppose that $x = x_2$ with probability π and $x = x_1 < x_2$ with probability $1-\pi$. Agents can be in one of three states: unmatched, in an x_1 relationship, or and in an x_2 relationship. The fraction in each state is denoted N_0, N_1, and N_2, respectively, where $N_0 + N_1 + N_2 = 1$. Let the payoff, or value, function of an agent in each state be denoted V_0, V_1, and V_2. Agents need to choose strategies for deciding when to accept a match and when to search while matched. Let A_i be the probability that a representative agent agrees to enter into an x_i match and let S_i be the probability that he searches while in an x_i match, $i = 1, 2$.

The value functions satisfy the following continuous time dynamic programming equations:

$$rV_0 = b + \alpha\pi A_2(V_2 - V_0) + \alpha(1-\pi)A_1(V_1 - V_0)$$

$$
\begin{aligned}
rV_1 &= x_1 + (\sigma + S_1\alpha)(V_0 - V_1) + S_1\Sigma_1 \\
rV_2 &= x_2 + (\sigma + S_2\alpha)(V_0 - V_2) + S_2\Sigma_2,
\end{aligned}
\tag{1}
$$

where Σ_i is the net gain from searching while in an x_i match:

$$
\begin{aligned}
\Sigma_1 &= \alpha\pi[A_2V_2 + (1 - A_2)V_0 - V_1] + \alpha(1 - \pi)(1 - A_1)(V_0 - V_1) - d \\
\Sigma_2 &= \alpha\pi(1 - A_2)(V_0 - V_2) + \alpha(1 - \pi)[A_1V_1 + (1 - A_1)V_0 - V_2] - d.
\end{aligned}
$$

For example, the third equation in (1) equates the flow value rV_2 to the sum of the instantaneous utility x_2, plus the probability you become unmatched, either because of an exogenous separation or because your partner meets someone new, $\sigma + S_2\alpha$, times $V_0 - V_2$, plus S_2 times your net gain from search.[6]

Given (A_1, A_2, S_1, S_2) one can determine (N_0, N_1, N_2). We will do so below, but it is important to note that we do not need to know (N_0, N_1, N_2) in order to analyze equilibrium strategies, since (N_0, N_1, N_2) does not enter (1). This is a consequence of two assumptions. Due to constant returns in the meeting technology, the arrival rate α does depend on the number of agents searching, $N_0 + N_1S_1 + N_2S_2$. Moreover, even if everyone is searching, so that α is determined, suppose you are unmatched and meet a matched agent. Under our assumption that he cannot return to his previous partner, this is equivalent to meeting an unmatched agent. If we alternatively assumed that he could return to his previous partner, he will match with you only if the new match is better, and you would have to know (N_0, N_1, N_2) to calculate the probability of this event.

[6] One could be more explicit at the cost of additional notation by saying that you choose s_j taking as given others choose S_j. Then one would write $rV_2 = x_2 + (\sigma + S_2\alpha)(V_0 - V_2) + s_2\Sigma_2$, for example. In equilibrium, of course, $s_j = S_j$.

A (steady state) equilibrium can now be defined as a list including value functions (V_0, V_1, V_2) satisfying (1), the distribution across states (N_0, N_1, N_2) satisfying conditions given below, and strategies (A_1, A_2, S_1, S_2) satisfying the following best response conditions:

$$A_j = \begin{cases} 1 & if \quad \Delta_j > 0 \\ [0,1] & if \quad \Delta_j = 0 \\ 0 & if \quad \Delta_j < 0 \end{cases}, \qquad S_j = \begin{cases} 1 & if \quad \Sigma_j > 0 \\ [0,1] & if \quad \Sigma_j = 0 \\ 0 & if \quad \Sigma_j < 0 \end{cases} \tag{2}$$

where $\Delta_j = V_1 - V_0$ is the net gain from accepting an x_j match, while Σ_j is the net gain to searching while in an x_j match, defined above.

Consider for now pure strategy equilibria, given by setting each of the elements of (A_1, A_2, S_1, S_2) to either 0 or 1. There are sixteen such strategy profiles; however, only five potentially constitute equilibria. First, there is a "degenerate" or *type D* equilibrium where agents reject all matches: $A_1 = A_2 = 0$. Second, there is a "choosy" or *type C* equilibrium where agents accept x_2 but reject x_1, and – naturally, since they would not accept x_1 – they do not search in x_2 matches: $A_1 = 0$, $A_2 = 1$ and $S_2 = 0$. Third, there is a "faithful" or *type F* equilibrium where agents accept x_1 as well as x_2 and do not search in either case: $A_1 = A_2 = 1$ and $S_1 = S_2 = 0$. Fourth, there is an "unfaithful" or *type U* equilibrium, in which agents accept both matches but now continue to search in x_1 matches: $A_1 = A_2 = 1$, $S_1 = 1$ and $S_2 = 0$. Finally, there is a "perverse" or *type P* equilibrium where agents accept both and, perhaps counter to all intuition (but see below), search in x_2 but not x_1 matches: $A_1 = A_2 = 1$, $S_1 = 0$ and $S_2 = 1$.

These are all the potential equilibria, at least given the normalization $x_2 > x_1$ (if we reverse the inequality then other cases arise, but they are merely relabellings of what has already been considered). Any other candidate equilibrium can easily be ruled out. For example, suppose $A_1 = 0$, $A_2 = 1$, and $S_2 = 1$. These strategies imply $\Sigma_2 = \alpha(1-\pi)(V_0 - V_2) - dS_1$ and $\Delta_2 = V_2 - V_0$; but the best response conditions require $\Sigma_2 > 0$ and $\Delta_2 > 0$, which is a contradiction. Intuitively, it cannot be an equilibrium to accept and then spend resources to search in x_2 matches since there is no possible gain if you are going to reject x_1 matches. The other cases are similar. The next step is to derive parameter values for which each of our five candidate equilibria exist.

Consider first *type D* equilibrium, where $A_1 = A_2 = 0$, and therefore $rV_0 = b$. We use the *unimprovability principle*: to check whether strategies constitute an equilibrium, it suffices to show the payoffs from using these strategies cannot be improved by deviating, in any possible contingency, one time and then reverting to the candidate strategies. Consider the payoff to deviating from $A_2 = 0$ by accepting an x_2 match. It cannot be optimal to accept the match and then search, given that you revert to the candidate strategy $A_1 = A_2 = 0$ (it would be better to quit immediately). So we can set $S_2 = 0$, which means $rV_2 = x_2 + \sigma(V_0 - V_2)$, and this implies Δ_2 is proportional to $x_2 - b$. Hence, $\Delta_2 \leq 0$, and $A_2 = 1$ does not improve your payoff, iff $x_2 \leq b$. Similarly, $A_1 = 1$ does not improve your payoff iff $x_1 \leq b$, although this is not binding. Hence, type D equilibrium exists iff $x_2 \leq b$, shown as region D in Figure 1, which also shows the regions where each of the equilibria exist in (x_1, x_2) space (note that only the area above the 45^o line is relevant since $x_2 > x_1$).

Now consider *type F* equilibrium, where $A_1 = A_2 = 1$ and $S_1 = S_2 = 0$.

This implies

$$\begin{aligned} rV_0 &= b + \alpha\pi(V_2 - V_0) + \alpha(1-\pi)(V_1 - V_0) \\ rV_1 &= x_1 + \sigma(V_0 - V_1) \\ rV_2 &= x_2 + \sigma(V_0 - V_2). \end{aligned}$$

For this to be an equilibrium, we require $\Delta_1 \geq 0$, which holds iff

$$x_2 \leq y_1 = \frac{(r+\sigma+\alpha\pi)x_1 - (r+\sigma)b}{\alpha\pi},$$

and $\Delta_2 \geq 0$, which is not binding. We also require $\Sigma_1 \leq 0$, which holds iff

$$x_2 \leq y_2 = x_1 + \frac{(r+\sigma)d}{\alpha\pi},$$

and $\Sigma_2 \leq 0$, which is not binding. Hence, *type F* equilibrium exists in the region F, to the southeast of both lines y_1 and y_2, in Figure 1. Intuitively, for this equilibrium to exist, we need x_1 to be big enough that agents accept it and x_2 to be small enough that agents stop searching once they accept x_1.

Consider *type U* equilibrium, where $A_1 = A_2 = 1$, $S_1 = 1$ and $S_2 = 0$. This implies

$$\begin{aligned} rV_0 &= b + \alpha\pi(V_2 - V_0) + \alpha(1-\pi)(V_1 - V_0) \\ rV_1 &= x_1 + (\sigma+\alpha)(V_0 - V_1) + \alpha\pi(V_2 - V_1) - d \\ rV_2 &= x_2 + \sigma(V_0 - V_2). \end{aligned}$$

We require $\Delta_1 \geq 0$, which holds iff $x_1 \geq b + d$, and $\Delta_2 \geq 0$, which is not binding. We also require $\Sigma_1 \geq 0$, which holds iff

$$x_2 \geq y_3 = \frac{r+\sigma+\alpha}{r+\sigma+2\alpha}x_1 + \frac{\alpha}{r+\sigma+2\alpha}b + \left(\frac{r+\sigma}{\alpha\pi} + \frac{\alpha}{r+\sigma+2\alpha}\right)d,$$

and $\Sigma_2 \leq 0$, which is not binding. The region where *type U* equilibrium exists is shown as region U in the Figure. Notice that we need x_2 to be big enough that agents want to search in x_1 matches, and also $x_1 \geq b + d$, so that agents prefer to search while matched than to search while unmatched.

Consider now *type P* equilibrium, where $A_1 = A_2 = 1$, $S_1 = 0$ and $S_2 = 1$. This implies

$$\begin{aligned} rV_0 &= b + \alpha\pi(V_2 - V_0) + \alpha(1-\pi)(V_1 - V_0) \\ rV_1 &= x_1 + \sigma(V_0 - V_1) \\ rV_2 &= x_2 + (\sigma + \alpha)(V_0 - V_2) + \alpha(1-\pi)(V_1 - V_2) - d. \end{aligned}$$

We require $\Delta_2 \geq 0$, which holds iff $x_2 \geq b + d$, and $\Delta_1 \geq 0$, which is not binding. Also, we require $\Sigma_2 \geq 0$, which holds iff

$$x_2 \leq y_4 = \frac{r+\sigma+2\alpha}{r+\sigma+\alpha}x_1 - \frac{\alpha}{r+\sigma+\alpha}b - \frac{(r+\sigma)(r+\sigma+2\alpha)+\alpha^2(1-\pi)}{\alpha(1-\pi)(r+\sigma+\alpha)}d,$$

and $\Sigma_1 \leq 0$, which is not binding. See region P in the Figure. In this strange case, agents prefer x_1 over x_2 matches even though $x_1 < x_2$, because these strategies make x_1 matches are more secure. This is only possible if x_1 is not too much less than x_2, since agents will only sacrifice so much instantaneous utility for security, and also x_1 and x_2 are large relative to $b + d$, since it is a high cost of becoming unmatched that makes security important.

Finally, consider *type C* equilibria, where $A_1 = 0$, $A_2 = 1$ and $S_2 = 0$. This implies

$$\begin{aligned} rV_0 &= b + \alpha\pi(V_2 - V_0) \\ rV_1 &= x_1 + (\sigma + S_1\alpha)(V_0 - V_1) \\ rV_2 &= x_2 + \sigma(V_0 - V_2). \end{aligned}$$

Notice we have left S_1 in the expression for rV_1. It turns out that, even though no one accepts an x_1 match, it matters what they believe about S_1 (i.e., it matters what agents believe would happen off the equilibrium path). It can be shown that a *type* C equilibrium exists iff $x_1 \leq b + d$ and $x_2 \geq y_1$, although the beliefs concerning S_1 have to be different, depending on parameter values.[7] See region C in the Figure.

This completes our characterization of existence, with the results in terms of pure strategy equilibria summarized in Figure 1.[8] The figure is drawn making no assumptions about parameter values, other than obvious things such as $r > 0$, $\pi \in (0, 1)$, etc. – everything one needs to know for all of the

[7]Explicitly, given $x_1 \leq b + d$ and $x_2 \geq y_1$, there is an equilibium with $A_1 = 0$, $A_2 = 1$, $S_2 = 0$, and: $S_1 = 1$ if $x_2 \geq y_5$; $S_1 = 0$ if $x_2 \leq y_6$; and $S_1 \in (0, 1)$ if $x_2 \in (y_6, y_5)$, where

$$\begin{aligned} y_5 &= \frac{(r+\sigma+\alpha\pi)}{\pi(2\alpha+\sigma+r)}x_1 + \frac{\alpha\pi-(1-\pi)(r+\sigma)}{\alpha\pi(2\alpha+\sigma+r)}b + \frac{(r+\sigma+\alpha)(r+\sigma+\alpha\pi)}{\alpha\pi(2\alpha+\sigma+r)}d \\ y_6 &= \frac{(r+\sigma+\alpha\pi)}{\pi(\alpha+\sigma+r)}x_1 - \frac{(1-\pi)(r+\sigma)}{\pi(\alpha+\sigma+r)}b + \frac{(r+\sigma)(r+\sigma+\alpha\pi)}{\alpha\pi(\alpha+\sigma+r)}d. \end{aligned}$$

Notice that when $x_2 \in (y_6, y_5)$ the equilibrium requires agents use mixed strategies. In any case, all of the *type* C equilibria are observationally equivalent, since no one ever accepts an x_1 match in equilibrium.

[8]Although we focus mainly on pure strategy equilibria, for completeness, we show that when the *type* F and *type* U equilibria coexist one can also construct a mixed-strategy equilibrium as follows. Suppose agents accept x_1 matches and are indifferent between searching and not searching, and respond by searching with probability $S_1 \in (0, 1)$ (your partner does not know if you are searching, only the probability S_1). The payoff to searching and not searching in an x_1 match must be the same; hence

$$x_1 + (\sigma + S_1\alpha)(V_0 - V_1) + \alpha\pi(V_2 - V_1) - d = x_1 + (\sigma + S_1\alpha)(V_0 - V_1).$$

Solving for S_1, we have

$$S_1 = \frac{[\alpha\pi(x_2 - x_1) - (r+\sigma)d]\,(\sigma + r + \alpha)}{\alpha[\alpha\pi(b + d - y) + (r+\sigma)d]}.$$

It is easy to show that $S_1 \in (0, 1)$, and hence the mixed strategy equilibrium exists, iff $x_2 \in (y_3, y_2)$, as claimed.

qualitative properties (e.g., relative slopes and intercepts) holds generally. As mentioned above, *type F* and *type U* equilibria coexist when $x_2 \in (y_3, y_2)$. In this case, agents in x_1 matches may either be "faithful" or "unfaithful" depending on what other agents are doing. If other agents are searching in x_1 matches then you prefer to search in x_1 matches because these matches are unstable, and therefore, speaking heuristically, you are less inclined to be satisfied with your x_1 match, and more inclined to desire an x_2 match, which is relatively secure as well as generating higher instantaneous utility. Indeed, this effect is sufficiently powerful that one can produce a *type P* equilibrium where agents "perversely" prefer x_1 matches, even though $x_1 < x_2$, because they believe these matches are more stable, and hence they search for an x_1 match while in x_2 matches, thus rationalizing their beliefs.

A special case of the *type P* equilibrium occurs when $x_1 = x_2$ (on the 45^o line in the Figure), which means that there is really no fundamental difference between matches, but people simply believe that certain relationships will be unstable. This has an interpretation in terms of *discrimination.* For example, suppose all agents are distinguished by one of two identifiable characteristics, say black and white. It is logically possible for individuals to believe that black-white relationships will be less stable that black-black or white-white relationships, and for this belief to be true in equilibrium, even if color has zero impact on payoffs. As we said earlier, however, notice that for the *type P* equilibrium to exist we need x_1 and x_2 similar and sufficiently bigger than $b + d$. Also notice that whenever the *type P* equilibrium does exist there also exists another equilibrium, either *type U* or both *type F* and *type U*, depending on x_1 and x_2.

As mentioned, given strategies one can compute (N_0, N_1, N_2). Taking account of the flows between states, one can see that, in general:

$$\begin{aligned}
\dot{N}_1 &= N_0\alpha(1-\pi)A_1 + N_2S_2\alpha(1-\pi)A_1 \\
&\quad -N_1[\sigma + S_1\alpha + S_1\alpha(1-\pi)(1-A_1) + S_1\alpha\pi] \\
\dot{N}_2 &= N_0\alpha\pi A_2 + N_1S_1\alpha\pi A_2 \\
&\quad -N_2[\sigma + S_2\alpha + S_2\alpha(1-\pi) + S_2\alpha\pi(1-A_2)].
\end{aligned}$$

For example, the flow into x_1 matches consists of those who are unmatched, get an x_1 draw and accept, it plus those in x_2 matches who are searching, get an x_1 draw and accept it; the flow out consists of those in x_1 matches whose relationships break up exogenously, those who are abandoned by their partners, those who are searching and get an x_1 draw and reject it, plus those who are searching and get an x_2 draw (whether they accept or reject it).

Steady state solves $\dot{N}_1 = \dot{N}_2 = 0$, as well as $N_0 = 1 - N_1 - N_2$. Letting N_i^T denote the fraction of agents in state i in a *type* T equilibrium, we have:

$$\begin{array}{lll}
N_0^D = 1 & N_1^D = 0 & N_2^D = 0 \\
N_0^C = \dfrac{\sigma}{\alpha\pi+\sigma} & N_1^C = 0 & N_2^C = \dfrac{\alpha\pi}{\alpha\pi+\sigma} \\
N_0^F = \dfrac{\sigma}{\alpha+\sigma} & N_1^F = \dfrac{\alpha(1-\pi)}{\alpha+\sigma} & N_2^F = \dfrac{\alpha\pi}{\alpha+\sigma} \\
N_0^U = \dfrac{\sigma(\sigma+\alpha+\alpha\pi)}{\kappa_U} & N_1^U = \dfrac{\sigma\alpha(1-\pi)}{\kappa_U} & N_2^U = \dfrac{\alpha\pi(\sigma+2\alpha)}{\kappa_U} \\
N_0^P = \dfrac{\sigma(\sigma+2\alpha-\alpha\pi)}{\kappa_P} & N_1^P = \dfrac{\alpha(1-\pi)(\sigma+2\alpha)}{\kappa_P} & N_2^P = \dfrac{\sigma\alpha\pi}{\kappa_P}
\end{array}$$

where to save space we have written $\kappa_U = (\sigma + 2\alpha)(\alpha\pi + \sigma)$ and $\kappa_P = (\sigma + 2\alpha)[\alpha(1-\pi) + \sigma]$.

One can easily show the following. In terms if the number of unmatched agents, we have $N_0^C > N_0^P, N_0^U > N_0^F$, with $N_0^P > N_0^S$ iff $\pi > 1/2$. Intuitively, the *type* C equilibrium where agents are "choosy" and reject x_1 matches maximizes N_0, while the *type* F equilibrium where agents all matches and never voluntarily leave their partners (since they are "faithful") minimizes N_0. In particular, in the labor market context, one should interpret N_0 as unemployment, then the conclusion is that the unemployment rate is lowest when there is no on-the-job-search. In terms if the number of agents in x_1 matches, we have $N_1^P > N_1^F > N_1^U > N_1^C$. Thus, the *type* P equilibrium, where agents "perversely" prefer x_1 over x_2 matches, maximizes N_1. In terms of the number of agents in x_2 matches, we have $N_2^C = N_2^U > N_2^F > N_2^P$. Intuitively, equilibria where agents either accept only x_2 matches or accept x_1 matches but continue to search will maximize N_2, while naturally the *type* P equilibrium where agents in x_2 matches search for x_1 matches minimizes N_2.

We now proceed to a discussion of efficiency, defined in terms of the standard social planner's welfare criterion:

$$W = N_0V_0 + N_1V_1 + N_2V_2.$$

As $x_2 < b$ implies the efficient outcome is obviously $A_1 = A_2 = 0$, let us proceed to the more interesting case of $x_2 > b$. In this case, any efficient outcome clearly entails $A_2 = 1$ and $S_2 = 0$ (and so the *type* P equilibrium cannot be efficient); it is not so clear, however, what the planner will choose for A_1 and S_1 – i.e., whether he prefers the *type* C, *type* F or *type* U strategies. To determine this we need to derive the closed form for W as a function of

A_1 and S_1. Inserting V_i and N_i, after some algebra, one can derive:[9]

$$rW = \frac{\left\{\sigma\alpha S_1[2-(1-\pi)A_1]+\sigma^2\right\}b+\sigma\alpha(1-\pi)A_1(x_1-S_1 d)+\alpha\pi(\sigma+2\alpha S_1)x_2}{\sigma\alpha(1-\pi)(1-S_1)A_1+(\sigma+\alpha\pi)(\sigma+2\alpha S_1)}. \tag{3}$$

There are three relevant possibilities for the planner: he can pick the *type C* strategy, $A_1 = 0$, which yields

$$rW = \frac{\alpha\pi x_2 + \sigma b}{\alpha\pi + \sigma};$$

he can pick the *type F* strategy, $A_1 = 1$ and $S_1 = 0$, which yields

$$rW = \frac{\alpha(1-\pi)x_1 + \alpha\pi x_2 + \sigma b}{\alpha + \sigma};$$

or he can pick the *type U* strategy, $A_1 = 1$ and $S_1 = 1$, which yields

$$rW = \frac{\alpha\sigma(1-\pi)[x_1 - (b+d)] + (\sigma + 2\alpha)(\alpha\pi x_2 + \sigma b)}{(\sigma + 2\alpha)(\alpha\pi + \sigma)}.$$

It is easy to verify that the planner's strategy is $A_1 = 0$ if $x_1 \leq b + d$ and $x_2 \geq y_A$, where

$$y_A = \frac{\alpha\pi + \sigma}{\alpha\pi}x_1 - \frac{b}{\alpha\pi};$$

it is $A_x = 1$ and $S_x = 0$ if $x_2 \leq y_A$ and $x_2 \leq y_S$, where

$$y_S = \frac{\pi(\sigma + 2\alpha) + \sigma}{\pi(\sigma + 2\alpha)}x_1 - \frac{\sigma b}{\pi(\sigma + 2\alpha)} + \frac{\sigma(\sigma + \alpha)d}{\alpha\pi(\sigma + 2\alpha)};$$

and it is $A_x = 1$ and $S_x = 1$ if $x_1 \geq b + d$ and $x_2 \geq y_S$. In words, the planner wants agents to accept x_1 matches iff x_1 is sufficiently big, and to search in these matches iff x_2 is sufficiently big.

We want to compare efficient and equilibrium outcomes. For this purpose, we ignore the *type D* equilibrium, which is always efficient if it exists, and

[9] In this derviation we have already inserted $A_2 = 1$ and $S_2 = 0$. Notice that the reduced form for W is a linear combination of the instantaneous utilities in each of the states: b, $x_1 - S_1 d$, and x_2.

the *type P* equilibrium, which is always inefficient, and concentrate on the efficiency properties of *type F*, *type U* and *type C* equilibria. It facilitates the exposition to begin with the limiting case $r \to 0$, since in this case the line y_1 that divides equilibrium regions C and N coincides with the line y_A that divides the planner's choice between C and N – i.e., when $r \to 0$, the equilibrium choice of A_1 is always efficient, and we can concentrate for now on the efficiency of S_1.

Figure 2 shows a version of Figure 1 drawn with $r = 0$, and highlights two regions in which the equilibrium differs from the planner's choice. In the region labeled 1, a planner would choose the *type F* strategy but the unique equilibrium is *type U*. In this case, there is too much search, in the sense that agents always search in x_1 matches in equilibrium even though this is inefficient. In the region labeled 2, the a planner would again choose the *type F* strategy but there are multiple equilibria, including *type F* but also including *type U*. In this case, the may be too much search in the sense that there is an equilibrium where agents search in type x_1 matches even though this is inefficient, although there is also an efficient equilibrium. In all other regions the equilibrium coincides with the planner's choice. The general conclusion is that, in the limiting case when $r \to 0$, there is a tendency towards too much search. The reason is simple: when you search while matched, you take into account your own benefits and costs, but neglect the fact that when you meet someone new you abandon your current partner, and moreover, if the new partner is was already matched, they also abandon their current partner.

We can use the steady state results derived above to discuss how the distribution of agents across states differs between the equilibrium and efficient outcomes. When these outcomes do in fact differ, in regions 1 and 2 of Figure 1, the inefficiency aries when we are in a *type U* equilibrium but the efficient out come entails *type F* strategies. As has been established, $N_0^U > N_0^F$, $N_2^U > N_2^F$, and $N_1^U < N_1^F$. Hence, the planner prefers fewer unmatched agents, fewer x_2 matches, and more x_1 matches. In other words, the efficient outcome entails less inequality than the equilibrium outcome. It seems to be a general property that when there is too much search there will be too much inequality – after all, what agents are searching for is to move up in the income distribution (that is, the x distribution here). It is not hard to understand how excessive search leads to too many people in the upper tail (that is, at x_2); what is perhaps slightly less obvious is that it also leads to too many people in the lower tail (unmatched), since their excessive search generates excessive separations.[10]

The effect that generates too much search (you neglect the impact on other agents when they are abandoned) is always there; however, there is another effect that arises when $r > 0$ that goes the other way. This other effect generates too little search, according to criteria W, because impatient agents are less inclined than the planner to, in the first place, reject an x_1 match, and in the second place, to keep searching while in an x_1 match. This is because the gains to finding an x_2 match accrue only in the future. Figure 3 generalizes Figure 2 to the case of $r > 0$, and shows that, in addition to the regions 1 and 2 with too much search, there are three new regions with too little.

[10]This logic does not rely on a two-point x distribution; as we shall see, something similar occurs in the generalization of the model in the next section.

In the region labeled 3, the planner chooses the *type C* strategy but the unique equilibrium is *type F*: impatient agents accept x_1 matches and stop searching, while the planner wants them to reject these matches. In the region labeled 4, the planner chooses the *type U* strategy but the unique equilibrium is *type F*: impatient agents accept x_1 matches and stop searching, while the planner wants them to accept these matches but continue to search. Finally, in the region labeled 5, the planner chooses the *type U* strategy but there are multiple equilibria, including *type U* but also including *type F.* Note that, for a given r, these regions occur only in the range where x_1 and x_2 are relatively small – for x_1 or x_2 sufficiently big, the equilibrium must either entail too much search or efficient search.[11]

We close this section by reiterating that the source of the multiplicity here has nothing to do with the standard thick-market externality, but results exclusively from the strategic interaction between the partners: if other agents are searching then matches are less secure and hence less valuable, and so you are more inclined to search. This can lead to multiple equilibria even without thick-market effects. Moreover, our endogenous instability effect tends to generate too much search, while standard thick-market effects tends to generate too little search. While there are other effects discussed in the literature that can also lead to excessive search, the endogenous instability effect seems new and different.

[11] Equivalently, given any x_1 and x_2, we can pick r small enough that there will be either too much search or efficient search, but not too little search.

4 The General Model

We now present an extension of the model in the previous section, which is exactly the same, except that we now allow match quality $x \geq 0$ to have a general distribution described by the cumulative distribution function $F(x)$. If F is differentiable we denote the density by $f(x)$, but we do not require differentiability for anything important. We will restrict attention to equilibria with the following property: unmatched agents enter into relationships iff they draw a value of x above the *reservation match quality* R; and matched agents stop searching iff x is above the *critical match quality* Q. Moreover, we are mainly interested in equilibria with $Q > R$ (since otherwise there is no search while matched). Even given this restrictions to a certain class of outcomes, we will show that there can exist a continuum of equilibria.

Generalizing the discussion in the previous section, an equilibrium here is defined as a list including the value functions for agents who are unmatched and agents who are in a relationship with match value x, $[V_0, V(x)]$, a steady state distribution that can be characterized by the number of unmatched agents and the distribution of match values across existing relationships, $[N_0, G(x)]$, and strategies (R, Q) satisfying conditions to be given below. Note that for ease of presentation we assume as a tie-breaking rule that agents stop searching while matched iff $x > Q$ (i.e., they search when they are indifferent); this is not particularly important for anything, but it allows us to write the fraction of matched agents enaged in search as $G(Q)$.

To begin the analysis, observe that if you are unmatched your value function satisfies

$$rV_0 = b + \alpha \int_0^\infty \max[V(z) - V_0, 0]dF(z) = b + \alpha \int_R^\infty [V(z) - V_0]dF(z). \quad (4)$$

where R is the reservation match quality, given by $V(R) = V_0$. Also, if you are in a relationship with match quality x your value function satisfies

$$rV(x) = x + (\sigma + S\alpha)[V_0 - V(x)] + s\Sigma(x) \quad (5)$$

where $S = 1$ if your partner is searching and 0 otherwise, and

$$\Sigma(x) = \alpha[V_0 - V(x)] + \alpha \int_R^\infty [V(z) - V_0]dF(z) - d$$

is your expected gain from search.[12] In any equilibrium where $Q > R$, we know that $x = R$ implies $s = 1$ (agents always search in relationships at the reservation match value), which means

$$rV(R) = R + \alpha \int_R^\infty [V(z) - V_0]dF(z) - d. \quad (6)$$

Comparing (6) and (4), one sees immediately that $R = b + d$. Thus, unmatched agents will agree to form a relationship iff the instantaneous return net of the cost of continued search, $x - d$, exceeds b.[13]

[12]Equivalently,

$$\Sigma(x) = \alpha \int_0^R [V_0 - V(x)]dF(z) + \alpha \int_R^\infty [V(z) - V(x)]dF(z) - d.$$

Notice that we are maintaining the assumption that, if you search while matched, you must leave your current partner when you meet someone new. Hence, when you meet someone new and draw an x below R you will become unmatched. This assumption is made here, as in the previous section, so that one does not have to know N_0 or $G(x)$ to solve the individual agent's decision problem.

[13]Recall that in the previous section we could have an equilibrium of *type F* where agents accept matches with $x < b + d$. This does not contradict the current result that $R = b + d$, since the current result is predicated on being in an equilibria with $Q > R$, and in the case in the previous section has $Q < R$ (agents do not search while in realtionships at the reservation match quality).

To proceed, we now solve for $V(x)$. First, rewrite (5) as

$$rV(x) = x - sd + (\sigma + S\alpha + s\alpha)[V_0 - V(x)] + s\alpha I,$$

where $I = \int_{b+d}^{\infty}[V(z) - V_0]dF(z)$ does not depend on x. Then insert $V_0 = (b + \alpha I)/r$ into $V(x)$ and rearrange to yield

$$V(x) = \frac{r(x - sd) + (\sigma + S\alpha + s\alpha)b + (\sigma + S\alpha + s\alpha + sr)\alpha I}{r(r + \sigma + S\alpha + s\alpha)}. \quad (7)$$

Moreover, for future reference we note that I can be simplified as

$$I = \int_{b+d}^{\infty}[1 - F(z)]V'(z)dz = \int_{b+d}^{Q}\frac{[1 - F(z)]dz}{r + \sigma + 2\alpha} + \int_{Q}^{\infty}\frac{[1 - F(z)]dz}{r + \sigma}dz$$

by integrating by parts and inserting $V'(x)$ from (7). This expression gives $I = I(Q)$ as a function of Q, but otherwise it depends only on exogenous variable. Notice that I is a decreasing funcion of Q.

It is now useful to express a matched agent's payoff explicitly as a function of x, his search decision s, the search decision of his partner S, and the rule being used by all other agents, which is to search while matched iff $x \leq Q$. Letting this payoff be denoted $v_{sS}(x, Q)$, we have from (7)

$$\begin{aligned}
v_{11}(x, Q) &= \frac{r(x - d) + (\sigma + 2\alpha)b + (\sigma + 2\alpha + r)\alpha I(Q)}{r(r + \sigma + 2\alpha)} \\
v_{01}(x, Q) &= \frac{rx + (\sigma + \alpha)b + (\sigma + \alpha)\alpha I(Q)}{r(r + \sigma + \alpha)} \\
v_{10}(x, Q) &= \frac{r(x - d) + (\sigma + \alpha)b + (\sigma + \alpha + r)\alpha I(Q)}{r(r + \sigma + \alpha)} \\
v_{00}(x, Q) &= \frac{rx + \sigma b + \sigma\alpha I(Q)}{r(r + \sigma)}.
\end{aligned}$$

Figure 4 shows the four v_{ij} functions. Notice that they are linear in x, with $\partial v_{00}/\partial x > \partial v_{01}/\partial x = \partial v_{10}/\partial x > \partial v_{11}/\partial x$. Also, as shown in the figure, we have $v_{10} = v_{11} > v_{01}, v_{00}$ at $x = b + d$, which must be true as long as $Q > R = b+d$.[14] Given all agents search iff $x \leq Q$, your value function $V(x)$ is given by max$\{v_{01},v_{11}\}$ when your partner is searching and max$\{v_{00},v_{10}\}$ when your partner is not searching – as shown by the thick line in the figure.

Given any $Q > b + d$, it is clear from Figure 4 that there is a unique $q_0(Q) > b + d$ such that $v_{00} = v_{10}$ (that is, your best response switches from searching to not searching, given that your partner is not searching), and a unique $q_1(Q) > b+d$ such that $v_{01} = v_{11}$ (that is, your best response switches from searching to not searching, given that your partner is searching). So, given Q, your best response to your partner's behavior is to search iff $x \leq q_0$ when $S = 0$, and to search iff $x \leq q_1$ when $S = 1$. Figure 4 depicts a situation with $b+d < q_0(Q) < Q < q_1(Q)$, in which case it should be clear that it is an equilibrium for all agents to search while matched iff $x \leq Q$, and moreover that we have $Q > R$, as we have been assuming throughout the argument. To see why this constitutes an equilibrium, suppose that everyone searches iff $x \leq Q$: if you are in a match with $x \leq Q$, then $S = 1$ and you want to search because $x < q_1(Q)$; while if you are in a match with $x > Q$, then $S = 0$ and you do not want to search because $x > q_0(Q)$.

The remaining difficulty is to show that the situation depicted in the figure, $b + d < q_0(Q) < Q < q_1(Q)$, can actually arise (which is not obvious, because as Q varies all of the curves shift, and hence so do q_0 and q_1). To

[14]This last result has a simple intuitive interpretation: given that you are indifferent between accepting $x = R = b+d$ and rejecting it to remain unmatched, it does not matter to you if your partner searches, since all their search does is increase your probability of returning to the unmatched state. Since $Q > b + d$, the relevant value functions at $b + d$ are v_{10} and v_{11}, and so we conclude $v_{10} = v_{11}$ at $b + d$. The result $v_{1j} = v_{0j}$ also follows from the fact that searching is better than not searching at $b + d$, given $Q > b + d$.

this end, we first equate $v_{0j} = v_{1j}$ and solve explicitly for $q_j(Q)$:

$$\begin{aligned} \alpha q_0(Q) &= \alpha b - (\sigma + \alpha + r)d + (\sigma + 2\alpha + r)\alpha I(Q) \\ \alpha q_1(Q) &= \alpha b - (\sigma + r)d + (\sigma + \alpha + r)\alpha I(Q). \end{aligned}$$

As seen in Figure 5, the functions $q_0(Q)$ and $q_1(Q)$ are decreasing – because $I(Q)$ is – and therefore each has at most one fixed point.

Consider the following mild parameter restriction, which always holds if d is small (as one naturally needs to construct an equilibrium where agents accept matches and continue to search):

$$\int_{b+d}^{\infty}[1 - F(z)]dz > \frac{(r+\sigma)^2 d}{\alpha (r + \sigma + 2\alpha)}. \tag{8}$$

It is easy to verify that (8) guarantees $q_0(b + d) > b + d$, and so the fixed point of $q_0(Q)$, call it $\underline{q}$, satisfies $\underline{q} > b+d$. Also, one can verify that $\underline{q} > b+d$ guarantees $q_1(\underline{q}) > \underline{q}$, and so the fixed point of $q_1(Q)$, call it $\overline{q}$, satisfies $\overline{q} > \underline{q}$. Summarizing, (8) implies $b+d < \underline{q} < \overline{q}$, and so if we choose any $Q \in [\underline{q}, \overline{q}]$ we have $b + d < q_0(Q) \leq Q \leq q_1(Q)$, exactly the situation depicted in Figure 4.

In other words, there exists a nondegenerate interval $[\underline{q}, \overline{q}]$ with the property that any $Q \in [\underline{q}, \overline{q}]$ constitutes an equilibrium in the class under consideration. What lies behind this continuum of steady state equilibria? The answer is the discontinuity at $x = Q$, as seen in Figure 4. As x crosses the critical value Q there is a discrete jump upward in $V(x)$ because other people stop searching; moreover, you strictly prefer to search when $x < Q$ and strictly prefer not to search when $x > Q$ (because the relevant branches of v_{ij} change from v_{i1} to v_{i0} when other agents stop searching). If Q_1 is an equilibrium and we change Q_1 to Q_2 in the neighborhood of Q_1, it will be the case that you strictly prefer to search for $x < Q_1$ and strictly prefer not to search for $x > Q_1$. Hence, if Q_1 is a best response to itself, any Q in the neighborhood of Q_1 is also a best response to itself.

Given Q and $F(R) = \varphi$, to complete the characterization of an equilibrium we describe the distribution of agents across states (which we need this to discuss inequality). To begin, as a preliminary step, we take $G(x)$ as given, and observe that the number of unmatched agents evolves according to:[15]

$$\dot{N}_0 = (1 - N_0)\sigma + (1 - N_0)G(Q)(\alpha + \alpha\varphi) - N_0\alpha(1 - \varphi).$$

Setting $\dot{N}_0 = 0$, we can solve for the steady state value of N_0 as a function of $G(Q)$,

$$N_0 = \frac{\sigma + G(Q)\alpha(1 + \varphi)}{\sigma + G(Q)\alpha(1 + \varphi) + \alpha(1 - \varphi)}. \tag{9}$$

To derive the distribution of match quality, first note that $G(b + d) = 0$. Next, denote the measure of set of agents who are matched with match quality $x \leq \bar{x}$ by $\mu(\bar{x}) = (1 - N_0)G(\bar{x})$. For $x \in [b + d, Q]$, this evolves according to

$$\begin{aligned} \dot{\mu}(x) &= N_0\alpha[F(x) - \varphi] + (1 - N_0)[G(Q) - G(x)]\alpha[F(x) - \varphi] \\ &\quad - G(x)(1 - N_0)\{\sigma + \alpha + \alpha\varphi + \alpha[1 - F(x)]\}. \end{aligned}$$

For $x > Q$, $\mu(x)$ evolves according to

$$\begin{aligned} \dot{\mu}(x) &= N_0\alpha[F(x) - \varphi] - (1 - N_0)[G(x) - G(Q)]\sigma \\ &\quad - G(Q)(1 - N_0)\{\sigma + \alpha + \alpha\varphi + \alpha[1 - F(x)]\}. \end{aligned}$$

[15] In words, the flow into the set of unmatched agents is the number of matched agents who suffer an exogenous separation, plus the number of matched and searching agents who are abandoned by their partner or meet someone with match value below $b + d$, while the flow out is the number of unmatched agents who meet someone with match value above $b + d$.

As another preliminary step, we can insert (9) into $\dot{\mu}(x) = 0$ and solve for the steady state $G(x)$, for any $x \geq b + d$, as a function of $G(Q)$:

$$G(x) = \begin{cases} \dfrac{[F(x) - \varphi][\sigma + 2\alpha G(Q)]}{(1-\varphi)(\sigma + 2\alpha)} & \text{if } x \in [b+d, Q] \\[2ex] \dfrac{\sigma[F(x) - \varphi] + 2\alpha G(Q)[1 - F(x)]}{\sigma(1-\varphi)} & \text{if } x > Q \end{cases} \tag{10}$$

Now we can solve for $G(Q)$ by setting $x = Q$ in (10) and rearranging:

$$G(Q) = \frac{\sigma F(Q) - \sigma\varphi}{\sigma(1-\varphi) + 2\alpha - 2\alpha F(Q)}. \tag{11}$$

Then we can substitute (11) into (10) to arrive at the final expression for the distribution of match values above $b + d$:

$$G(x) = \begin{cases} \dfrac{\sigma F(x) - \sigma\varphi}{\sigma(1-\varphi) + 2\alpha - 2\alpha F(Q)} & \text{if } x \in [b+d, Q] \\[2ex] \dfrac{(\sigma + 2\alpha)F(x) - \sigma\varphi - 2\alpha F(Q)}{\sigma(1-\varphi) + 2\alpha - 2\alpha F(Q)} & \text{if } x > Q \end{cases} \tag{12}$$

Similarly, we can substitute (11) into (9) to arrive at the final expression for the number of unmatched agents:

$$N_0 = \frac{\sigma[\sigma + 2\alpha - \alpha F(Q) + \alpha\varphi]}{(\sigma + 2\alpha)[\sigma + \alpha - \alpha F(Q)]} \tag{13}$$

This yields the closed form for the distribution of agents across states, as a function of exogenous variables and Q (which is endogenous but not pinned down by the model, due to the multiplicity of equilibrium).

Several remarks are in order concerning this distribution. First, G transforms F by truncating it below $b+d$, scaling it linearly between $b+d$ and Q, and again scaling it linearly (but differently) above Q. It is easy to verify that $G(x) \leq F(x)$ for all x, with strict inequality on the interior of the support of

F (i.e., G first order stochastically dominates F). Also, G is continuous at $x = Q$ as long as F is, but in any case G has a kink at Q. Thus, if F has a density f, then G has a density g, but it is discontinuous at $x = Q$:

$$g(x) = \begin{cases} \dfrac{\sigma f(x)}{\sigma(1-\varphi) + 2\alpha - 2\alpha F(Q)} & \text{if } x < Q \\[2ex] \dfrac{(\sigma + 2\alpha) f(x)}{\sigma(1-\varphi) + 2\alpha - 2\alpha F(Q)} & \text{if } x > Q \end{cases} \tag{14}$$

Figure 6 shows the density f, and the transformed density g for two different values of the critical match value, Q_1 and $Q_2 > Q_1$; Figure 7 shows the cumulative distribution functions F, G_1 and G_2 (these were drawn assuming x was distributed log normally). Concentrating on the densities, one can see from (14) that increasing Q has the following effects: g shifts up for $x < Q_1$ and $x > Q_2$, and shifts down for $x \in (Q_1, Q_2)$. Thus, we can say the following about multiple equilibria in the model: an equilibrium with a higher value of Q leads to more weight in both tails and less weight in the middle of the distribution of x values across existing matches. Additionally, (13) implies $\partial N_0/\partial Q > 0$, and so an equilibrium with a higher value of Q also displays a greater number of unmatched agents. Summing up, across different equilibria, ones with higher values of Q entail more unemployment and more inequality.

5 Conclusion

The paper has analyzed a model where agents choose whether to search while in relationships. One key finding is that there can be multiple equilibria, and indeed a continuum of steady state equilibria, generated by endogenous instability. The intuition is basic: when other people are more inclined to search, relationships will be less stable and hence less attractive, and so you will be more inclined to search. We even showed that for certain parameters there is an equilibrium where agents prefer matches that are fundamentally inferior because they believe these matches are more secure, which is true in equilibrium simply because they believe it. We analyzed welfare, and showed that there was a tendency for too much search – intuitively, because agents neglect the cost they impose on their current partners when they meet someone new, plus the cost they impose on the current partners of the matched people they meet. We analyzed the endogenous distribution of agents across states and the endogenous distribution of match values across relationships. A general conclusion is that more instability leads to more unemployment and more inequality.[16]

How robust is the notion of endogenous instability? Why can't two married people, for example, simply sit down, talk it over, and agree to be "faithful" to each other? There are several issues here. First, recall that there are two distinct ways in which there can be too much search, corresponding to regions 1 and 2 in Figure 2. The first case is a prisoner's dilemma: your best response is to search regardless of your partner's behavior, and there is no way to credibly promise not to despite the fact that search is inefficient. The second case is a coordination failure: when both partners say they will not search, not searching is a best response, *if* they believe each other – but this can be a big *if* in some relationships. In any event, it is true by definition

[16]One is tempted to interpret this logic in terms of American versus European labor markets, as the former are conventionally regarded as having more turnover and more inequality. Pursuing this idea seems interesting, but beyond the scope of the current project.

that we can improve on a bad outcome when we have a coordination failure as long as agents just cooperate. However, although we only explicitly analyzed bilateral relationships, in principle the message is meant to apply more generally to n-person organizations, and when n is big, it may not be easy to get the entire coalition together to sit down, talk it over, and agree to cooperate.

There are many directions in which to extend this line of research. One branch involves thinking about generalizations of the technical assumptions. For example, if agents are heterogeneous ex ante – some people are more generally desirable than others – or they do not generally agree on match values – when two agents i and j meet they draw a pair (x_i, x_j) where x_i need not equal x_j – then if utilty is transferrable some interesting bargaining issues may arise. In particular, one may need to think about counteroffers when one's partner meets someone new, or one may need to think about offering one's partner a big enough share of the surplus to keep them from searching in the first place. Also, it would be useful to examine how the results are affected if we allow a continuous choice of search intensity, rather than $s = 0$ or 1. Another branch of future work would be to assess the empirical relevance of the idea. Just how susceptible are various relationships to endogenous instability? It might be interesting to examine this in the context of particular labor markets, such as those in professional sports or in academics, and also in the marriage market. We leave this to future research.

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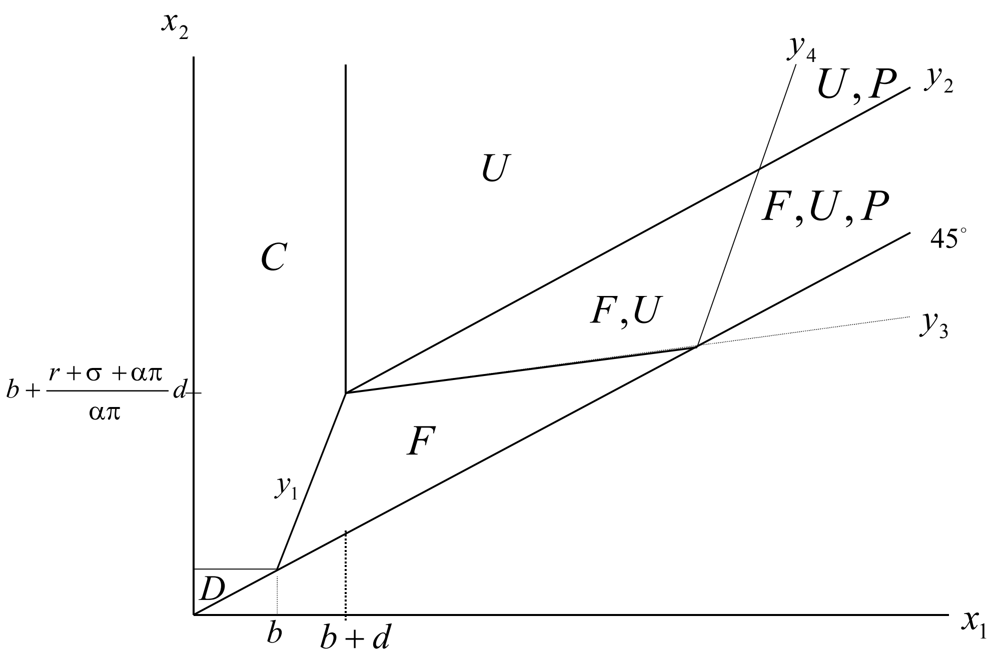
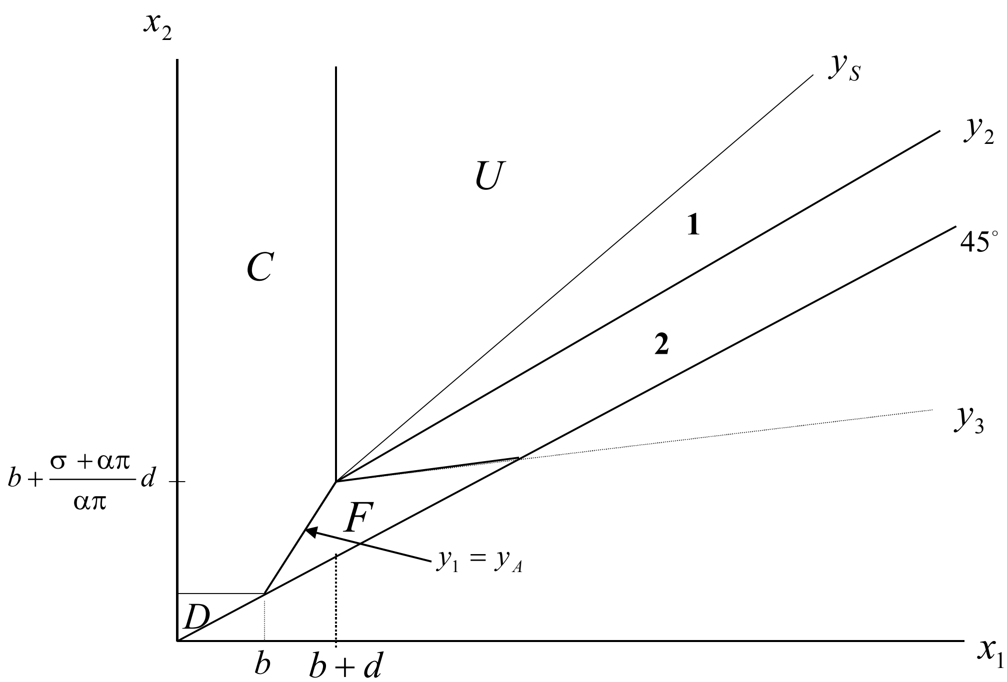


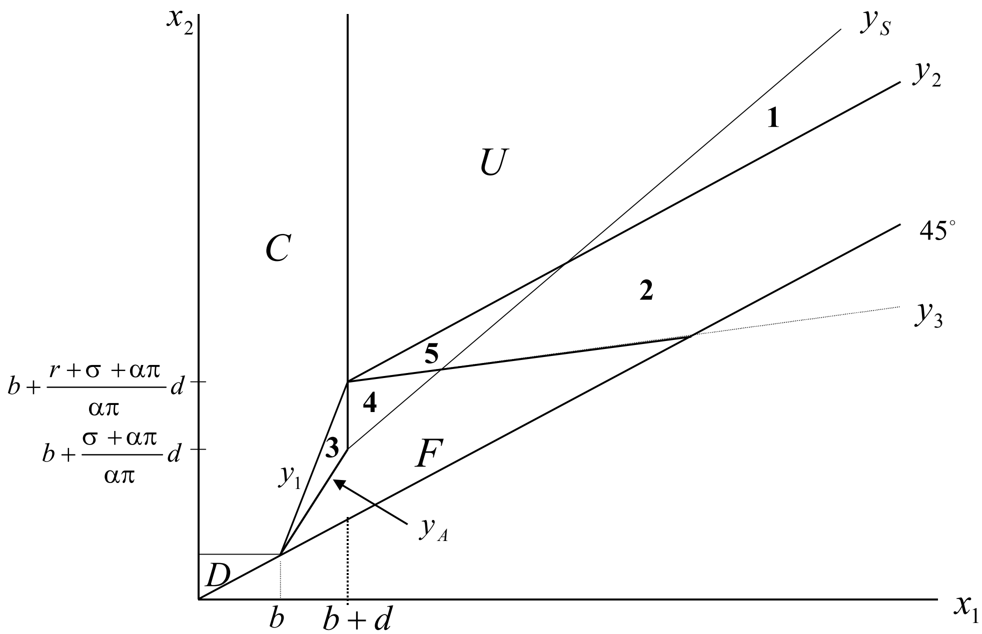
x_2
y_4
U,P
y_2
U
C
F,U,P
45°
F,U
y_3
$b+\frac{r+\sigma+\alpha\pi}{\alpha\pi}d$
F
y_1
D
b
$b+d$
x_1

Figure 1:



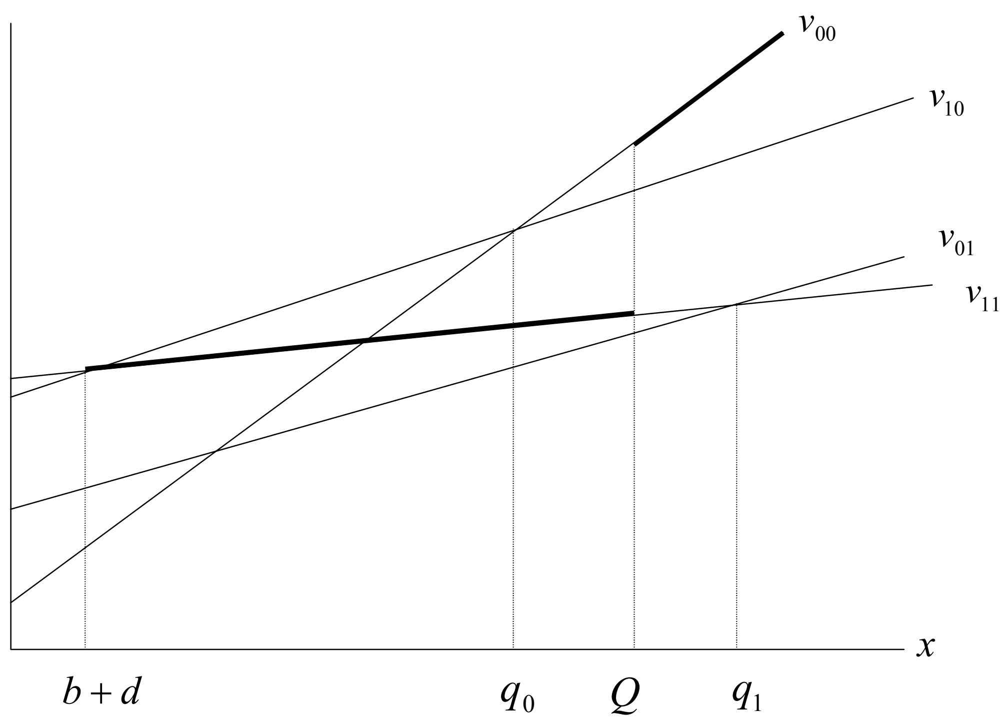
x_2
y_S
y_2
U
1
C
45°
2
y_3
$b+\frac{\sigma+\alpha\pi}{\alpha\pi}d$
F
$y_1 = y_A$
D
x_1
b
$b+d$

Figure 2:



x_2
y_S
y_2
1
U
C
45°
2
y_3
5
$b+\frac{r+\sigma+\alpha\pi}{\alpha\pi}d$
4
$b+\frac{\sigma+\alpha\pi}{\alpha\pi}d$
3
F
y_1
y_A
D
b
$b+d$
x_1

Figure 3:



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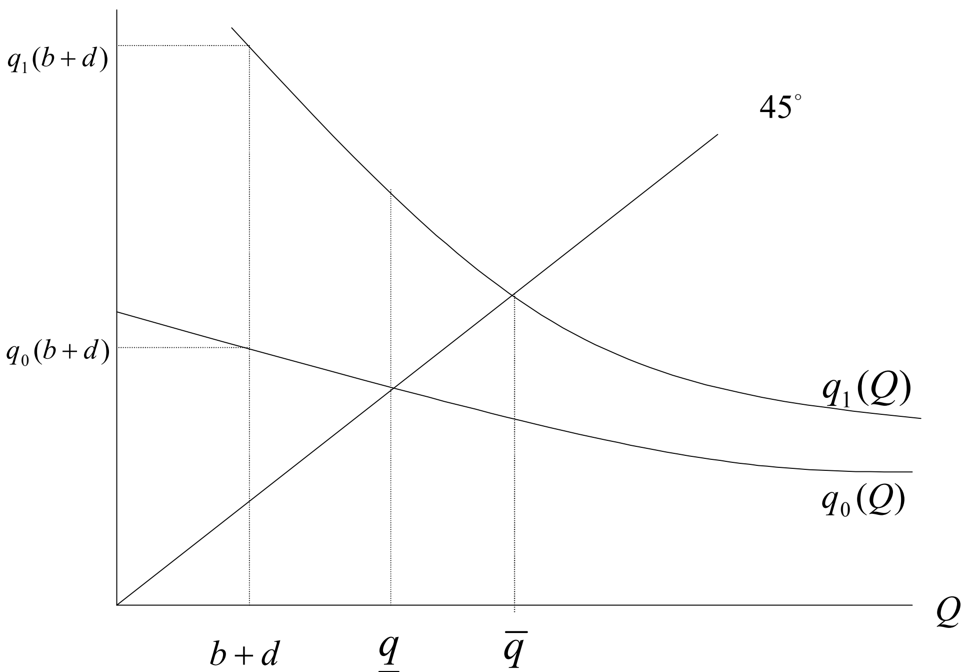

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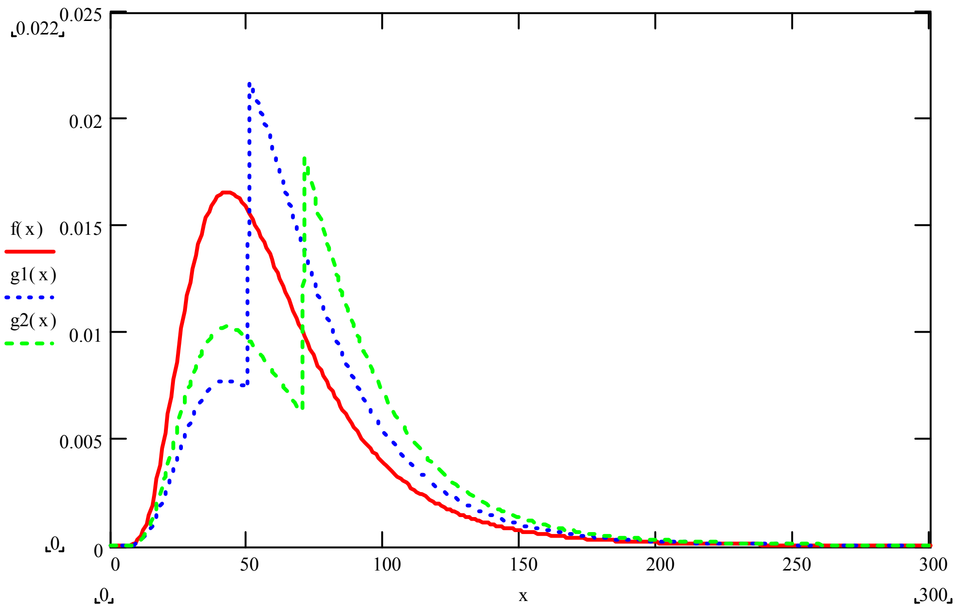

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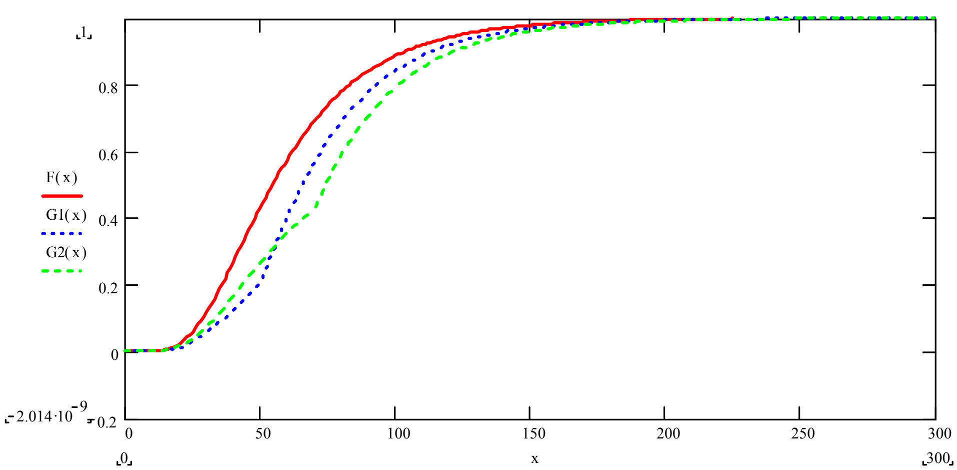

Figure 7: