Pricing and Matching with Frictions*

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April 24, 2000

Abstract

We analyze markets where n buyers each want to buy 1 unit and m sellers each want to sell 1 or more units of a good. Sellers set prices, then buyers choose which seller to visit. In symmetric equilibrium, all sellers with the same quality and capacity post the same price, and buyers randomize. Thus, more or fewer buyers may show up than a seller can accommodate – this is what we call *frictions*. We solve for prices as functions of n and m. As n and m grow, prices converge to the prediction of a popular model, but for finite n and m they differ. We also solve for the endogenous matching function. It displays decreasing returns for finite n and m, but converges to constant returns as n and m grow. We argue that the standard matching function in the literature is misspecified and discuss implications for the Beveridge curve.

*Earlier versions of this paper were presented at the meetings of the Society for Economic Dynamics and Control in Los Angeles and at the Conference on Dynamic Economics at the University of Essex. We have benefitted from the input of Daron Acemoglu, Melvyn Coles, Jan Eeckhout, Jeremy Greenwood, Michael Kremer, George Mailath, Alan Manning, Mike Peters, Andy Postlewaite and Robert Shimer. The editor and an anonymous referee also gave us some very helpful comments. The National Science Foundation and the Social Sciences and Humanities Research Council of Canada provided financial support.

1. Introduction

We analyze markets where n buyers each want to buy 1 unit and m sellers each want to sell 1 or more units of an indivisible good. First sellers set prices, then each buyer chooses which seller to visit. There is no search problem in the usual sense, because buyers know the price and capacity of each seller with certainty. Still, in equilibrium, there is a chance that more buyers will show up at a given location than the seller can accommodate, in which case some customers get rationed, and simultaneously fewer buyers show up at another location than the seller there can accommodate, in which case he gets rationed. This is what we mean by *frictions*. We are interested in the relationship between these frictions and pricing decisions, and in the number of successful matches between buyers and sellers.

We derive prices and the equilibrium matching function with the number of buyers and sellers as arguments, and compare the results to predictions of other models. In the case of homogeneous sellers, e.g., in a symmetric equilibrium they all charge the same price p, and buyers simply choose a seller at random – in some sense endogenizing the specification that is assumed in the undirected search literature. More generally, we derive the closed form for prices as a function of n and m. In the limit as n and m get big, p converges to the price generated by a simplified version of the model that is standard in the literature. However, for finite n and m the standard version does not give the correct answer. In terms of our endogenous matching function, for finite n and m it exhibits decreasing returns to scale – i.e., frictions get worse as the market gets thicker – but as the market grows we converge to constant returns.

Sellers here can be thought of as offering combinations of a price and a probability of service. Examples of related work include Butters (1977), Peters (1991), Montgomery (1991), McAfee (1993), Burdett and Mortensen (1998), and Lagos (2000). Indeed, our model with 2 sellers and 2 buyers is isomorphic to Montgomery's model, where he has 2 workers and 2 firms. However, the analyses differ in our $n \times m$ model and in an $n \times m$ version of his model. Intuitively, we take into account explicitly the strategic interaction between sellers, while previous analyses following Montgomery assume that each firm sets a price or wage taking as given some measure of aggregate market conditions (see also Lang [1991], Moen [1994], Shimer [1996], Acemoglu and Shimer [1996,1999] and Mortensen and Wright [1998]). Again, the two methods give different answers for finite n and m, but converge to the same limit as the market grows.

We also allow sellers to differ in capacity, exogenously in one version of the model and endogenously in another version. This leads to several new insights. For example, the effect on prices of an increase in supply can be very different depending on whether this increase occurs along the intensive or the extensive margin (that is, a change in the number of goods per seller or in the number of sellers). Moreover, we show that the matching function depends not only on the number of buyers and the number of goods for sale, but also on how those goods are distributed across sellers. For example, it makes a difference if there is one large seller or many small sellers. As with the effect on prices, we show that the effect of an increase in supply on the number of successful matches in equilibrium can be quite different depending on whether it occurs along the intensive or extensive margin.

This suggests that the standard matching function used in the literature is misspecified. In the typical labor market application, as in Pissarides (1990) or Mortensen and Pissarides (1995), e.g., job creation depends on the number of unemployed workers and the number of vacancies. Our results imply that it should also depend on whether there are many firms each with a few vacancies or a few firms with many vacancies. This allows us to propose a new explanation of the shifts in the Beveridge curve (the locus of observed points in vacancy–unemployment space) documented by Blanchard and Diamond (1989) and Jackman, Layard and Pissarides (1989). Our explanation is that these shifts may be due to changes in the firm-size distribution. In fact, the relative number of small firms has increased over time (Stanworth and Gray [1991]), which in our model implies the observed shifts in the Beveridge curve.

The rest of the paper is organized as follows. Section 2 examines the case of two homogeneous buyers and sellers. Section 3 presents results for the general case with n buyers and m sellers.

Section 4 describes the alternative and more standard method for analyzing these kinds of models, and shows that it gives the wrong answer for finite n and m but the right answer in the limit. Section 5 extends the model to allow sellers to differ and discusses the difference between changes in supply along the intensive and extensive margins in terms of implications for prices and the matching function. Section 6 presents a brief summary, and some ideas for future research.

2. The 2×2 Case

In general there are n buyers and m sellers, but we begin with $n = m = 2$. Label the buyers 1 and 2 and the sellers A and B. Each buyer wants to buy 1 unit of an indivisible good and is willing to pay up to his reservation price, which is normalized to 1. If he buys at price p he obtains utility $u = 1 - p$; if he does not buy he obtains 0. For now sellers are homogeneous, and each wants to sell 1 unit at a price above his reservation price of 0. If he sells at price p he obtains payoff $\pi = p$; if he does not sell he obtains 0.

The process of exchange proceeds in two stages. First, each seller j posts a price p_j taking as given the price of his competitor (more generally, his $m - 1$ competitors). At the second stage each buyer chooses a probability of visiting each seller, taking as given prices and the strategies of other buyers. If two or more buyers show up at the same location the good is allocated randomly at the posted price.[1] Let θ_i be the probability buyer i visits seller A and $1 - \theta_i$ the probability he visits B. Let U_{ij} be his expected payoff if he visits seller j and $U_i = \max\{U_{iA}, U_{iB}\}$. To compute U_i, observe that if buyer 1 visits seller A his expected payoff is $1 - p_A$ times the probability he gets served. If buyer 2 also visits A, which occurs with probability θ_2, buyer 1 gets served with probability $1/2$; if buyer 2 does not visit A, buyer 1 gets served for sure. So $U_{1A} = [\theta_2/2 + 1 - \theta_2](1 - p_A) = \frac{1}{2}(2 - \theta_2)(1 - p_A)$. Similarly, $U_{1B} = \frac{1}{2}(1 + \theta_2)(1 - p_B)$. Buyer 2's payoffs are symmetric.

[1] By assumption a buyer who is rationed cannot sample a second location within the period, but the basic message depends only on there being some cost to doing so. Also, we assume that sellers cannot make price contingent on how many buyers show up. Coles and Eeckhout (2000) generalize our model so that sellers are allowed to condition prices on the number of buyers, and show is that there is always an equilibrium where prices do *not* depend on the number of buyers, and therefore the restriction is not binding.

Consider the second stage game. By comparing U_{1A} and U_{1B} taking θ_2 and (p_A, p_B) as given, we see that the best response of buyer 1 is to go to seller A with probability

$$\theta_1 = \begin{cases} 0 & \text{if } \theta_2 > \theta(p_A, p_B) \\ 1 & \text{if } \theta_2 < \theta(p_A, p_B) \\ [0,1] & \text{if } \theta_2 = \theta(p_A, p_B) \end{cases} \tag{2.1}$$

where $\theta(p_A, p_B) = \frac{1+p_B-2p_A}{2-p_A-p_B}$. Buyers 2's best response is symmetric. Notice a buyer does not necessarily go the seller with a lower price, since the probability of rationing needs to be taken into account. In any case, equilibrium in the second stage game is as follows. First, if $p_A \geq \frac{1+p_B}{2}$ the unique equilibrium is $(\theta_1, \theta_2) = (0, 0)$ (both buyers go to B). Second, if $p_A \leq 2p_B - 1$ the unique equilibrium is $(\theta_1, \theta_2) = (1, 1)$ (both go to A). Finally, if $2p_B - 1 < p_A < \frac{1}{2}(1 + p_B)$ there are exactly 3 equilibria: two pure strategy equilibria, $(\theta_1, \theta_2) = (0, 1)$ and $(\theta_1, \theta_2) = (1, 0)$, plus a mixed-strategy equilibrium $\theta_1 = \theta_2 = \theta(p_A, p_B)$. See Figure 2.1.

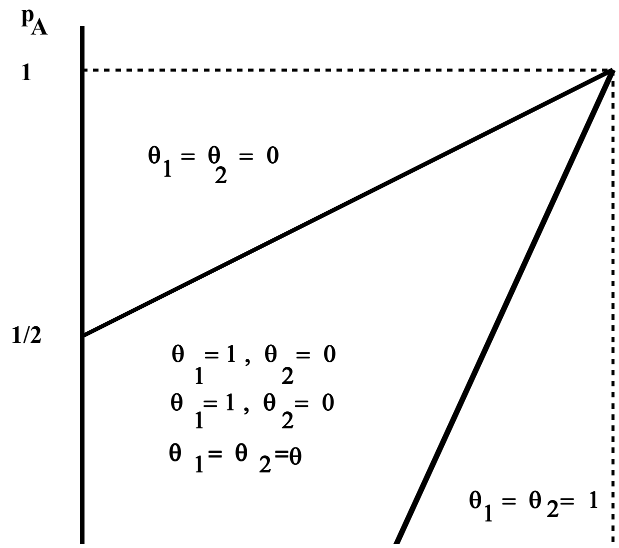

Figure 2.1: Equilibria in Second Stage Game

Consider now the first-stage game. Expected profit of seller A is as follows. If $p_A \leq 2p_B - 1$ then $\pi_A = p_A$ (since he gets both customers for sure); if $p_A \geq \frac{1}{2}(1 + p_B)$ then $\pi_A = 0$ (since he gets no customers); and if $2p_B - 1 < p_A < \frac{1}{2}(1 + p_B)$ then, since there are multiple equilibria at the second stage, there are two possibilities. If buyers play either of the two pure strategy equilibria then $\pi_A = p_A$, and if buyers play the mixed-strategy equilibrium then

$$\pi_A = p_A \frac{3(1-p_B)(1+p_B-2p_A)}{(2-p_A-p_B)^2}. \tag{2.2}$$

Given buyers play the mixed-strategy equilibrium at the second stage, the conditional best response of seller A (i.e., conditional on mixed-strategy equilibrium at the second stage) and his profit function are:

$$p_A^*(p_B) = \frac{(2-p_B)(1+p_B)}{7-5p_B} \tag{2.3}$$

$$\pi_A^*(p_B) = \frac{(1+p_B)^2}{4(2-p_B)}. \tag{2.4}$$

The conditional best response and profit functions of B are symmetric.

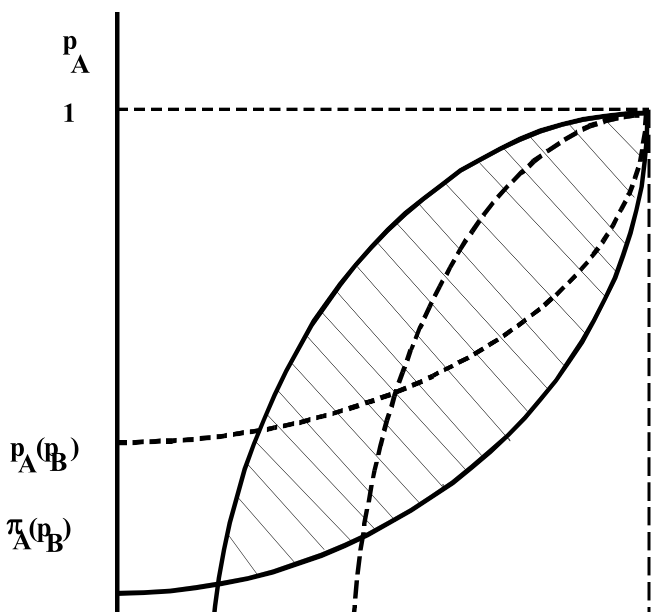

Figure 2.2: Equilibria in First Stage Game

Figure 2.2 shows the conditional best response and profit functions. It is easily verified that p_A^* and π_A^* are upward sloping and convex, and p_A^* lies above π_A^*, as shown. Clearly, p_B^* and π_B^* are mirror images of p_A^* and π_A^*. Also, p_A^* and p_B^* intersect at $(\frac{1}{2}, \frac{1}{2})$ while π_A^* and π_B^* intersect at

$(\frac{1}{5}, \frac{1}{5})$. Finally, given p_B, note that if seller A sets $p_A = p_A^*(p_B)$ then his expected profit is no less than $\pi_A^*(p_B)$, no matter what happens at the second stage: he earns $\pi_A = \pi_A^*(p_B)$ if buyers play the mixed-strategy equilibrium, and $\pi_A = p_A^*$ if buyers play a pure-strategy equilibrium. These results are useful for establishing the following.

Proposition 2.1. *A pair (p_A, p_B) is an equilibrium in the pricing game iff it is in the shaded region in Figure 2.2 between $p_A = \pi_A^*(p_B)$ and $p_B = \pi_B^*(p_A)$.*

Proof: First suppose (p_A, p_B) is such that $p_A < \pi_A^*$. Since expected profit for seller A is no more than p_A in any equilibrium and no less than π_A^* in any equilibrium where he sets p_A^*, he has a profitable deviation (no matter what equilibrium is played at the second stage). Hence, (p_A, p_B) cannot be an equilibrium if it is below the π_A^* curve. By symmetry, there is no equilibrium to the left of the π_B^* curve. Therefore, any equilibria must be in the shaded region. We now show that any (p_A, p_B) in this region is an equilibrium, with the following additional features. At the second stage the buyers play pure strategies along the equilibrium path – i.e., upon observing (p_A, p_B) one buyer goes to seller A and the other goes to B with probability 1 – and if there is any deviation by a seller, at the second stage they play: $\theta_1 = \theta_2 = 1$ if $p_A \leq 2p_B - 1$; $\theta_1 = \theta_2 = 0$ if $p_A \geq \frac{1}{2}(1 + p_B)$; and $\theta_1 = \theta_2 = \theta(p_A, p_B)$ if $2p_B - 1 > p_A > \frac{1}{2}(1 + p_B)$, where $\theta(p_A, p_B)$ was defined immediately after (2.1).

Clearly buyers' strategies constitute equilibrium at the second stage after a deviation (recall Figure 2.1). If there is no deviation buyers play pure strategies, which implies $\pi_A = p_A$ and $\pi_B = p_B$. Consider a deviation by seller A, to p_A^d, say. There are three cases to consider. (1) If $p_A^d \leq 2p_B - 1$, then at the second stage we must have $\theta_1 = \theta_2 = 1$ and therefore $\pi_A \leq 2p_B - 1 < p_A$. This is not a profitable deviation. (2) If $p_A^d \geq \frac{1}{2}(1 + p_B)$ then $\theta_1 = \theta_2 = 0$ and therefore $\pi_A = 0 < p_A$. This is not a profitable deviation. (3) If $2p_B - 1 < p_A^d < \frac{1}{2}(1 + p_B)$ then $\theta_1 = \theta_2 = \theta(p_A^d, p_B)$. The best deviation of this sort is $p_A^d = \frac{(2-p_B)(1+p_B)}{7-5p_B}$, which generates profits $\pi_A < p_A$. Hence, there is no profitable deviation for seller A. By symmetry there is no profitable deviation for B. This completes the proof. ■

The set of equilibria can be partitioned into the case where $(p_A, p_B) = (\frac{1}{2}, \frac{1}{2})$ and buyers play mixed strategies at the second stage, to which we return below, and a large set of prices (p_A, p_B) where buyers play pure strategies and go to different sellers with probability 1.[2] All of these except $p_A = p_B = 1$ are sustained by an implicit *threat* from the market: starting at any (p_A, p_B) in the relevant region, if a seller changes his price, buyers trigger to the mixed-strategy equilibrium. Since the mixed-strategy equilibrium is bad for sellers, they will not adjust their price. The case $p_A = p_B = 1$ is different. In this case, if a seller deviates by lowering his price by any $\varepsilon > 0$ both customers come to him with probability 1; there is no incentive for him to do so, however, since he is already getting a buyer with probability 1 and this exhausts his capacity.

All of these pure-strategy equilibria require a lot of coordination, in the sense that every buyer has to somehow know where everyone else is going. This may not be so unreasonable when $n = m = 2$, but it seems hard to imagine for general n and m, which is what we want to consider below. Moreover, in the equilibria supported by threats, buyers have to also coordinate on where to trigger after a deviation, which even harder to imagine in a large market. The case $p_A = p_B = 1$ does not have this latter difficulty, since it is not supported by triggers; however, this case is not particularly robust. Suppose, as a simple example, we introduce just a little noise: when seller j tries to set p_j the price that actually gets posted is uniformly distributed on $(p - \varepsilon_j, p + \varepsilon_j)$. For arbitrarily small $\varepsilon_B > 0$, it is *not* an equilibrium for seller A to set $p_A = 1$, because with probability $\frac{1}{2}$ we have $p_B < 1$ and this implies $\pi_A = 0$.

The only other equilibrium is the one where at the second stage each buyer picks a location at random, and $p_A = p_B = \frac{1}{2}$, as shown by the intersection of the conditional best response functions in Figure 2.2. This equilibrium requires no coordination whatsoever – indeed, since buyers pick sellers at random, in some sense this endogenizes what is assumed in the typical undirected search model. Additionally, it is the unique symmetric equilibrium, in the sense that $\theta_1 = \theta_2$ and $p_A = p_B$. Also, this equilibrium *is* robust to perturbations such as introducing noise. All of these

[2] Note that the set of equilibrium prices is larger than the area between the conditional best response functions in Figure 2.2; in fact, it is the area between the profit functions.

considerations, as well as the fact that this equilibrium generates interesting implications, suggest that in the general model discussed below it is worth concentrating on symmetric equilibria where buyers randomize. We say these equilibria are characterized by *frictions*, in the sense that with positive probability one seller gets more customers while another gets fewer than he can service (see also Lagos [2000]). The main point of what follows is to study pricing and matching with frictions.[3]

3. The $n \times m$ Case

Suppose now there are n buyers and m sellers. As discussed above, we focus on symmetric equilibria where all sellers charge the same price and all buyers use the same mixed strategy. Given all sellers post the same p, the mixed strategy each buyer uses must be to visit all sellers with equal probability, $\theta = 1/m$, since otherwise a given buyer would deviate by going to a seller that has the lowest expected number of customers. We have the following results.

Proposition 3.1. *The unique symmetric equilibrium has every buyer visit each seller with probability $\theta^* = 1/m$, and all sellers set*

$$p^* = p(m,n) = \frac{m - m(1 + \frac{n}{m-1})\left(1 - \frac{1}{m}\right)^n}{m - (m + \frac{n}{m-1})\left(1 - \frac{1}{m}\right)^n}. \tag{3.1}$$

Proof: To begin, let Φ be the probability that at least one buyer visits a particular seller when all buyers visit him with the same (but arbitrary) probability θ. Since $(1-\theta)^n$ is the probability all n buyers go elsewhere, $\Phi = 1 - (1-\theta)^n$. Next, let Ω be the probability a given buyer gets served when he visits this seller. Since the probability of getting served conditional on visiting this seller times the probability that this buyer visits him equals the probability of this seller serving the particular buyer, we have $\Omega\theta = \Phi/n$. Hence

$$\Omega = \frac{\Phi}{n\theta} = \frac{1-(1-\theta)^n}{n\theta}. \tag{3.2}$$

[3] Of course, an equilibrium with frictions fails to maximize the total available surplus: in the example, the number of possible successful matches is 2 but the expected number in the mixed-strategy equilibrium is 1.5. Notice, however, that the equilibrium actually does remarkably well in the following sense. Given that both buyers go to seller A with some arbitrary probability θ, the probability of two showing up at the same location is $\theta^2 + (1-\theta)^2 \geq 1/2$. In equilibrium, $\theta^* = 1/2$, which minimizes this quantity.

Now suppose every seller posts p and one contemplates deviating to p^d. Let the probability that any given buyer visits the deviant be θ^d. Then the probability he visits each of the non-deviants is $\frac{1-\theta^d}{m-1}$. By (3.2), a buyer who visits the deviant gets served with probability

$$\Omega^d = \frac{1-\left(1-\theta^d\right)^n}{n\theta^d}, \tag{3.3}$$

and a buyer who visits a non-deviant gets served with probability

$$\Omega = \frac{1-\left(1-\frac{1-\theta^d}{m-1}\right)^n}{n\frac{1-\theta^d}{m-1}}. \tag{3.4}$$

In a symmetric equilibrium in the second stage game, buyers are indifferent between visiting the deviant and any other seller: $\Omega(1-p) = \Omega^d(1-p^d)$. Inserting Ω and Ω^d and rearranging, this condition becomes

$$\frac{1-p}{1-p^d} = \frac{\left(1-\theta^d\right)\left[1-\left(1-\theta^d\right)^n\right]}{(m-1)\theta^d\left[1-\left(1-\frac{1-\theta^d}{m-1}\right)^n\right]} \equiv \Psi(\theta^d). \tag{3.5}$$

Notice that $\Psi(\theta^d)$ is a strictly decreasing function with

$$\lim_{\theta^d\to 0}\Psi(\theta^d) = \frac{n/(m-1)}{1-\left(1-\frac{1}{m-1}\right)^n} \geq 1 \text{ and } \lim_{\theta^d\to 1}\Psi(\theta^d) = \frac{1}{n}. \tag{3.6}$$

Hence, whenever $\Psi(1) < \frac{1-p}{1-p^d} < \Psi(0)$, there is a unique $\theta^d = \theta^d\left(p^d, p\right) \in (0,1)$ that makes buyers indifferent between sellers posting p and p^d. If $\frac{1-p}{1-p^d}$ is too small or too big, no $\theta^d \in (0,1)$ makes buyers indifferent, and either all or no buyers visit the deviant. Hence,

$$\theta^d = \begin{cases} 0 & \text{if } \frac{1-p}{1-p^d} > \Psi(0) \\ 1 & \text{if } \frac{1-p}{1-p^d} < \Psi(1) \\ \theta^d(p^d,p) & \text{otherwise.} \end{cases} \tag{3.7}$$

Expected profit of the deviant is $\pi(p^d,p) = p^d\left[1-\left(1-\theta^d\right)\right]^n$ where θ^d is given by (3.7). Clearly, the best deviation satisfies $p^d > 0$, and hence satisfies the first order condition

$$\frac{\partial\pi}{\partial p^d} = 1-(1-\theta^d)^n + p^d n\left(1-\theta^d\right)^{n-1}\frac{\partial\theta^d}{\partial p^d} = 0. \tag{3.8}$$

Assuming $\theta^d \in (0,1)$, we can differentiate (3.7) and then insert the symmetric equilibrium conditions $p^d = p$ and $\theta^d = 1/m$ to derive

$$\frac{\partial\theta^d}{\partial p^d} = \frac{-(m-1)^2\left[1-\left(\frac{m-1}{m}\right)^n\right]}{m^2\left[(m+n-1)\left(\frac{m-1}{m}\right)^n - m+1\right](1-p)}. \tag{3.9}$$

Inserting this into (3.8) and solving, we arrive at (3.1). As remarked above, given all sellers post p buyers must visit each with the same probability $\theta^* = \frac{1}{m}$. This completes the proof. ■

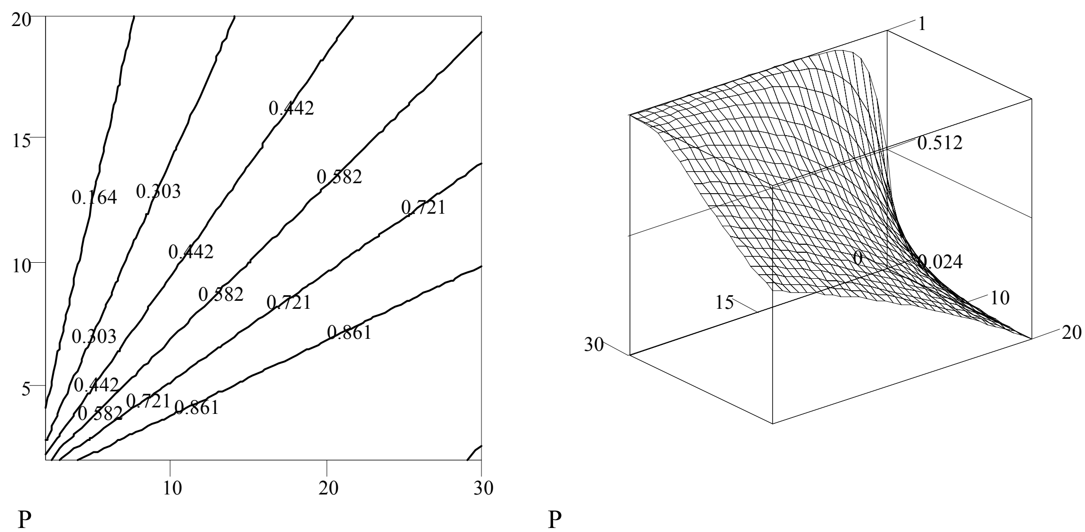

Figure 3.1: Contour and 3D Plots of p versus n and m

Figure 3.1 shows a contour plot (the level sets) and 3D plot of $p(m, n)$. Naturally, price is increasing in n and decreasing in m, and $p^* \to 1$ as $n \to \infty$ while $p^* \to 0$ as $m \to \infty$. Notice that p^* does not jump discontinuously from the monopsony price to the monopoly price as the buyer-seller ratio $b = n/m$ crosses 1, however, as would be predicted by a simple frictionless model – so, there is a sense here in which *frictions smooth things out.*[4] Figure 3.2 shows how the price varies with m for three buyer-seller ratios, which are from top to bottom $b = 2$, 1, and 0.5. Observe that p^* falls with m in a sellers' market (where b is relatively large) and rises with m in a buyers' market (where b is small).[5] Figure 3.2 shows p converging. Perhaps surprisingly, the

[4]By a frictionless model we mean the simplest Walrasian model in which p is 0 or 1 as n is above or below m. Of course, some Walrasian models with indivisible goods but no other frictions would predict that prices vary smoothly with n and m once randomization is introduced via lotteries or sunspots (see Rogerson [1988] or Shell and Wright [1993], for example). Models with indivisibilities and lotteries or sunspots have some similarities with the model presented here, in the sense that our frictions also introduce randomization, but the predictions of the present model also differ in many respects.

[5]See Cao and Shi (2000) for additional discussion, and explicit conditions describing when prices rise or fall with market size.

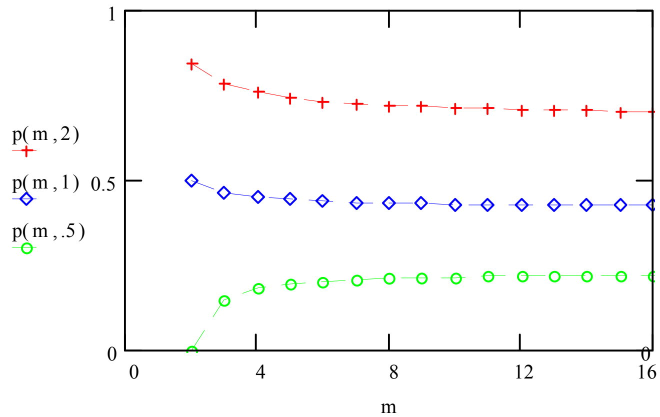

Figure 3.2: Price versus Market Size

limit is a very simple function of b.

Proposition 3.2. *Let $b = n/m$ be fixed. Then the limit of as $m, n \to \infty$ is*

$$\bar{p} = 1 - \frac{b}{e^b - 1}. \tag{3.10}$$

Proof: Eliminating $n = bm$ from (3.1), we have

$$p^* = \frac{m - 1 - [m(b+1) - 1]\left(\frac{m-1}{m}\right)^{mb}}{m - 1 - (m + b - 1)\left(\frac{m-1}{m}\right)^{mb}}. \tag{3.11}$$

Taking the limit yields the result. ■

Given $p(m, n)$, equilibrium values of other variables can easily be computed.[6] One thing in which we are especially interested is the expected number of sales, or successful buyer-seller matches,

$$M^* = M(m, n) = m\left[1 - \left(1 - \frac{1}{m}\right)^n\right]. \tag{3.12}$$

[6] For example, expected profit and utility are

$$\begin{aligned} \pi^* &= \pi(m, n) = \left[1 - \left(1 - \frac{1}{m}\right)^n\right] p(m, n) \\ U^* &= U(m, n) = \frac{m}{n}\left[1 - \left(1 - \frac{1}{m}\right)^n\right][1 - p(m, n)]. \end{aligned}$$

As m gets large holding b fixed these converge to $\bar{\pi} = 1 - (1 + b)e^{-b}$ and $\bar{U} = e^{-b}$.

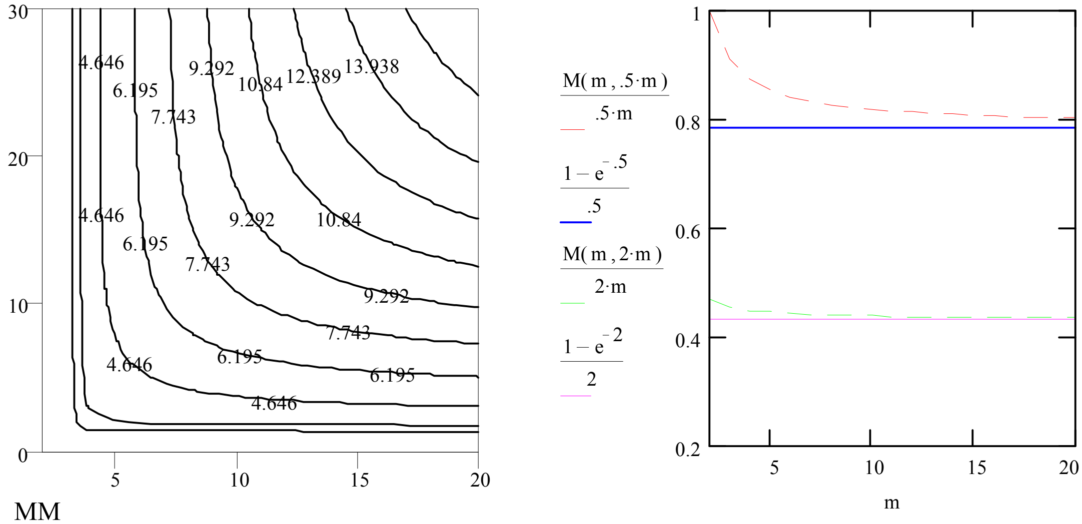

Figure 3.3: Iso-Matching Curves and Buyer Arrival Rates

This is the *equilibrium matching function*, analogous to the specification in the labor literature where hires are a function of the number of vacancies and unemployed workers. The left panel of Figure 3.3 shows the level sets; e.g., with $m = n = 20$ we can expect about 14 sales. The probability of a successful match, or the *arrival rate*, for a buyer is $A_b = M(m,n)/n$. The right panel shows A_b versus market size holding b fixed, and indicates that doubling n and m increases $M(m,n)$ by less than two-fold, or *decreasing returns to scale*. Intuitively, there are more frictions in bigger markets. However, A_b converges to $\frac{1-e^{-b}}{b}$, so for large markets matching exhibits approximately constant returns. We will have more to say about matching functions below, in the generalized version of the model with heterogeneous sellers. First, we present a different method of solving the model needed to analyze the heterogeneous-seller case.

4. An Alternative Solution Method

Here we present the analogue of the approach in Montgomery (1991) and some of the other papers mentioned in the introduction. The key to this method is to assume sellers take as given that they must offer buyers a certain level of expected utility U, which later we determine endogenously. Thus, suppose that an individual seller chooses p and buyers respond by coming to him with probability θ; then p and θ must generate an expected utility of at least U if he is to get any customers. Recalling from Section 3 that the probability a seller gets at least one buyer is $1-(1-\theta)^n$ and the probability a buyer who visits him gets served is $\Omega = \left[1-(1-\theta)^n\right]/n\theta$, the seller's problem is

$$\begin{aligned} \max \quad & \pi = p[1-(1-\theta)^n] \\ \text{st} \quad & (1-p)\Omega \geq U. \end{aligned} \tag{4.1}$$

Solving the constraint at equality for p, substituting into the objective function, and maximizing with respect to θ, we arrive at

$$(1-\theta)^{n-1} = U. \tag{4.2}$$

Since sellers all take U as given, they all choose the same θ and p, and therefore any possible equilibrium is symmetric and entails $\theta = \frac{1}{m}$. Inserting this into (4.2) determines U, and then the constraint in (4.1) can be solved for

$$p^a = 1 - \frac{n\left(1-\frac{1}{m}\right)^{n-1}}{m\left[1-\left(1-\frac{1}{m}\right)^n\right]}. \tag{4.3}$$

This is not the same as (3.11). If $n = m = 2$, for example, (4.3) yields $p^a = \frac{1}{3}$, while we know from (3.1), and also from Section 2, that the correct answer is $p^* = \frac{1}{2}$.

The problem with the method leading to (4.3) is that it ignores elements of strategic interaction between sellers. At least for small values of n and m, it does not really make sense for sellers to take parametrically market conditions as summarized by U, because any change by a deviant seller in his price – and thus in the probability that buyers visit him – implies a change in the probability that buyers visit other sellers and thus a change in market utility. Our method takes this into account: in (3.5), the left-hand side is the expected utility a buyer gets from visiting a deviant seller, the right-hand side is the expected utility a buyer gets from visiting a non-deviant seller, and both sides depend on θ^d. This is not the case with the constraint in (4.1), where the right-hand side is a fixed number U independent of θ^d.

Intuitively, our model captures competition between sellers for the probability of getting customers, while the model leading to (4.3) captures only competition between an individual seller and the market. One might imagine that this distinction vanishes as the market gets big. This is correct. To see it, set $n = mb$ in (4.3), let $m \to \infty$, and observe that $p^a \to 1 - b/(e^b - 1)$, which is the same as the result in (3.10). In addition to putting some previous literature into perspective, this result will be useful for the analysis in the next section.

5. Heterogeneous Sellers

We are interested in what happens where sellers differ. Heterogeneity in terms of different quality of goods is easy to handle, and potentially interesting given that consumers would then have to trade off quality as well as price and the probability of service.[7] However, we are more interested here in sellers that differ in capacity. We will first illustrate what happens with an example, and then proceed to the general case. So, to begin, consider the 2×2 model where seller A now has the option of producing a second unit at cost c_2, while B still has 1 unit. Assume A sets the same price for both units. Intuitively, in this case we expect $p_A > p_B$, since seller A never rations. However, we still expect buyers will go to B with positive probability, since he is cheaper, even though he may ration.

Generalizing Section 2, it is easy to show the symmetric equilibrium in the second stage is for both buyers to go to seller A with probability $\theta = \frac{1-2p_A+p_B}{1-p_B}$. Note that $p_A = p_B$ implies $\theta = 1$; B will obviously have to cut his price to compete. The conditional reaction functions are

$$p_A = \frac{p_B + 1}{4} \text{ and } p_B = \frac{p_A}{2 - p_B}. \tag{5.1}$$

The solution is $(p_A, p_B) = (.293, .172)$, which implies $\theta = 0.707$. Expected profit for A is $\pi_A =$

[7]Consider the 2×2 case where the utility from the good of seller A is $u_A = \beta$ and the utility from B is $u_B = 1$. Generalizing the analysis in Section 2, the symmetric equilibrium in the second stage is for both buyers to go to A with probability $\theta = \frac{2\beta-1+p_B-p_A}{1+\beta-p_A-p_B}$, while the conditional reaction functions are

$$p_A = \frac{(2\beta - 1 + p_B)(1 + \beta - p_B)}{5 + 2\beta - 5p_B} \text{ and } p_B = \frac{(2 - \beta + p_A)(1 + \beta - p_A)}{2 + 5\beta - 5p_A}.$$

For instance, when u_A falls from 1 to 0.67, p_A falls to 0.29, p_B increases to 0.57, and customers respond by visiting the low quality seller A with probability 0.42.

$0.414 - c_2$, and since $\pi_A = 0.375$ when A had only 1 unit of capacity, he is willing to produce the second unit iff $c_2 \leq 0.039$. This may seem low, given that A can in principle corner the market with a second unit and given that the good is in principle worth $u = 1$ to a buyer; notice, however, the response of B to capacity expansion by A is a rather drastic price cut.

To pursue this further, suppose *both* sellers choose capacity $k \in \{0, 1, 2\}$, where the costs of the first and second unit are c_1 and c_2. Given capacity, they post prices and buyers decide who to visit, as above. Assuming a symmetric equilibrium once (k_A, k_B) is determined, payoffs in the capacity game are described in Table 4.1. Figure 5.1 shows the equilibria in (c_1, c_2) space.[8] In terms of economic results, we have the following: If c_2 is much bigger than c_1 the unique outcome is $(k_A, k_B) = (1, 1)$, and if c_2 is much smaller than c_1 then we have $(k_A, k_B) = (2, 0)$ or $(0, 2)$. In the intermediate region these equilibrium coexist. We have $(k_A, k_B) = (1, 0)$ or $(0, 1)$ iff $c_2 \geq 1$ and c_1 is big. It is never an equilibrium to have $(k_A, k_B) = (2, 2)$, since this implies $\pi_A = \pi_B = 0$, but we can get $(k_A, k_B) = (2, 1)$ or $(1, 2)$ when c_1 and c_2 are small.

Table 4.1: Payoffs in the Capacity Game

		A: 0	A: 1	A: 2
B	0	$0, 0$	$1 - c_1, 0$	$2 - c_1 - c_2, 0$
B	1	$0, 1 - c_1$	$.375 - c_1, .375 - c_1$	$.414 - c_1 - c_2, .086 - c_1$
B	2	$0, 2 - c_1 - c_2$	$.086 - c_1, .414 - c_1 - c_2$	$-c_1 - c_2, -c_1 - c_2$

It would seem interesting to endogenize capacity along these lines in the general case of n buyers and m sellers. However, to focus more clearly on some implications for matching, here we assume exogenously that $m_L \in (0, m)$ sellers each have 1 unit for sale and $m_H = m - m_L$ each have 2 units. We focus on symmetric equilibria, where all high capacity sellers charge p_H and all low capacity sellers charge p_L, and buyers use mixed strategies. We will solve the model using the method in Section 4, where sellers take as given the utility U that they must provide buyers. For finite market size we know that this method does not give the right answer, but we will see

[8] To save space, when there are two equilibria that are merely re-labellings, such as $(k_A, k_B) = (2, 0)$ and $(k_A, k_B) = (0, 2)$, only one is shown in the figure.

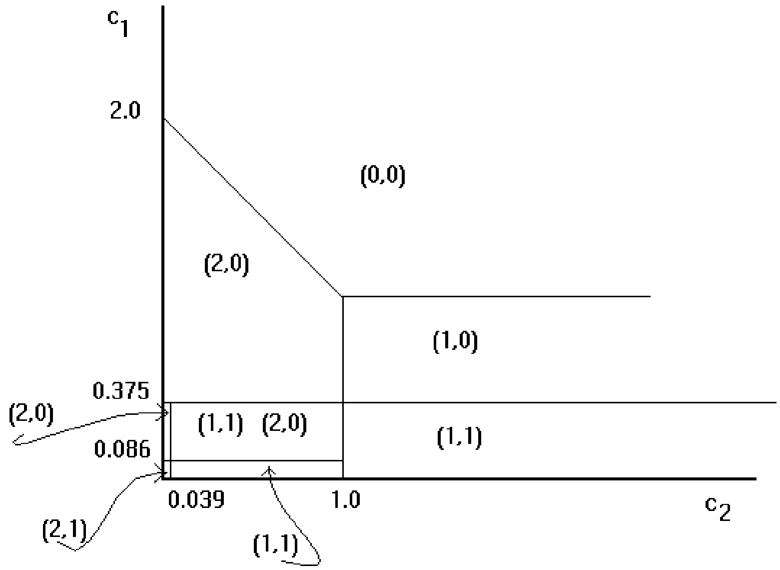

Figure 5.1: Equilibrium in the Capacity Game

that it converges to the right answer as the market gets large, just as in the case of homogeneous sellers.

To begin the analysis, note that a low capacity seller solves

$$\begin{aligned} \max \quad & \pi_L = p_L[1-(1-\theta_L)^n] \\ \text{st} \quad & (1-p_L)\Omega_L \geq U, \end{aligned} \tag{5.2}$$

where $\Omega_L = \frac{1-(1-\theta_L)^n}{n\theta_L}$. As this is the same as (4.1) in the previous section, the solution satisfies:

$$(1-\theta_L)^{n-1} = U. \tag{5.3}$$

For a high capacity seller, one can show the relevant problem is

$$\begin{aligned} \max \quad & \pi_H = p_H \left\{2[1-(1-\theta_H)^n] - n\theta_H(1-\theta_H)^{n-1}\right\} \\ \text{st} \quad & (1-p_L)\Omega_H \geq U, \end{aligned} \tag{5.4}$$

where $\Omega_H = \frac{2}{n\theta_H}[1-(1-\theta_H)^n] - (1-\theta_H)^{n-1}$, after a little analysis.[9] Substituting from the

[9]A buyer who visits a high capacity seller gets the good for sure if the seller is visited by either no other buyers or exactly one other buyer, the probability of which is $(1-\theta_H)^{n-1} + (n-1)\theta_H(1-\theta_H)^{n-2}$. If the seller is visited by $k \geq 2$ other buyers, he randomly chooses 2 to serve, so each buyer gets served with probability

constraint and maximizing with respect to θ_H, we have (compare with 5.3):

$$(1-\theta_H)^{n-1} + \theta_H(n-1)(1-\theta_H)^{n-2} = U. \tag{5.5}$$

When all sellers were homogeneous, we could insert the result $\theta = \frac{1}{m}$ into (4.2) and solve for U. Here we have to solve (5.3), (5.5), and the adding up condition $m_L\theta_L + m_H\theta_H = 1$ simultaneously for U, θ_H and θ_L. The result is the following.

Proposition 5.1. *Consider the model with n buyers and m sellers where $m_L \in (0, m)$ sellers have 1 unit for sale and $m_H = m - m_L$ have 2 units, and sellers take U parametrically. Let $b = n/m$ and $h = m_H/m_L$ be fixed. Then as n and m grow, we have in the limit*

$$p_L = 1 - \frac{b(1-x)}{(1-h)\left[\exp\left(\frac{b-bx}{1-h}\right) - 1\right]} \tag{5.6}$$

$$p_H = 1 - \frac{(1+\frac{bx}{h})\frac{bx}{h}}{2\exp\left(\frac{bx}{h}\right) - 2 - \frac{bx}{h}} \tag{5.7}$$

where $x = m_H\theta_H$. This is the same as the answer one gets by first solving the model for finite n and m taking strategic interaction into account and then taking the limit.

Proof: Combining (5.3), (5.5) and the adding up condition yields

$$(1-\theta_H)^{n-1} + \theta_H(n-1)(1-\theta_H)^{n-2} = \left(1 - \frac{1-m_H\theta_H}{m-m_H}\right)^{n-1}. \tag{5.8}$$

Rewriting this in terms of x, we have

$$(1-\frac{x}{hm})^{bm-1} + \frac{x}{hm}(bm-1)(1-\frac{x}{hm})^{bm-2} = \left[1 - \frac{1-x}{m(1-h)}\right]^{bm-1}. \tag{5.9}$$

$1 - C_k^2/C_{k+1}^2 = 2/(k+1)$. Thus, a buyer who visits a high capacity seller gets the good with probability

$$\begin{aligned}\Omega_H &= (1-\theta_H)^{n-1} + (n-1)\theta_H(1-\theta_H)^{n-2} + \sum_{k=2}^{n-1}\frac{2}{k+1}C_{n-1}^k\theta_H^k(1-\theta_H)^{n-1-k} \\ &= \frac{2}{n\theta_H}[1-(1-\theta_H)^n] - (1-\theta_H)^{n-1}.\end{aligned}$$

Hence, profit for a high capacity seller is

$$\pi_H = p_H\left[n\theta_H(1-\theta_H)^{n-1} + 2\sum_{k=2}^{n} C_n^k{\theta_H}^k(1-\theta_H)^{n-k}\right],$$

where the first term in brackets is the probability that the seller is visited by only one buyer, and the second is the probability that he is visited by at least two buyers. This simplifies to the expression in the text.

Letting $m \to \infty$ and rearranging, we get

$$1 + \frac{bx}{h} = \exp\left[\frac{b(x-h)}{h(1-h)}\right]. \tag{5.10}$$

There is a unique $x \in (0,1)$ satisfying (5.10). Given x, we then solve for U, p_H and p_L. From (5.3) we have

$$U = \left(1 - \frac{1-x}{m(1-h)}\right)^{bm-1}. \tag{5.11}$$

Notice that $U \to \exp\left(\frac{bx-b}{1-h}\right)$ as $m \to \infty$. From the constraints in (5.2) and (5.4) we have

$$1 - p_L = \frac{\frac{b-bx}{1-h}\left(1 - \frac{1-x}{m(1-h)}\right)^{bm-1}}{1 - \left(1 - \frac{1-x}{m(1-h)}\right)^{bm}} \tag{5.12}$$

$$1 - p_H = \frac{\frac{bx}{h}\left(1 - \frac{1-x}{m(1-h)}\right)^{bm-1}}{2\left[1 - \left(1 - \frac{x}{mh}\right)^{bm}\right] - \frac{bx}{h}\left(1 - \frac{x}{mh}\right)^{bm-1}}. \tag{5.13}$$

Taking the limits yields (5.6) and (5.7). As indicated above, this method neglects the strategic effect that a seller can have on U, but in the Appendix we consider the model taking into account all relevant strategic considerations, and show that the solutions generated by the two methods do indeed converge to the same limit as the m expands holding fixed $b = \frac{n}{m}$ and $h = \frac{m_H}{m}$. This is accomplished, even though we cannot actually solve for p when n and m are finite, by showing the equilibrium conditions for the finite model converge as $m \to \infty$ to the conditions in the model where U is taken parametrically. ■

The responses of prices to a change h are depicted as the downward sloping curves in Figure 5.2, for $b = 0.5$ in the left and $b = 1.5$ in the right panel. Observe that p_H is above p_L. Also, prices fall as we increase h and increase as we raise b (to see the later effect, compare the two panels). All of this is quite intuitive. Also shown is probability of visiting a high capacity seller, $x = m_H\theta_H$, which is the curve that goes from the origin to $(1,1)$. Notice, for example, that if $\frac{1}{2}$ the sellers are high capacity, a buyer visits a high capacity location with probability $x > \frac{1}{2}$, as we saw in the 2×2 example. The other upward sloping curve is the arrival rate for a buyer, A_b, which is naturally increasing in h.[10]

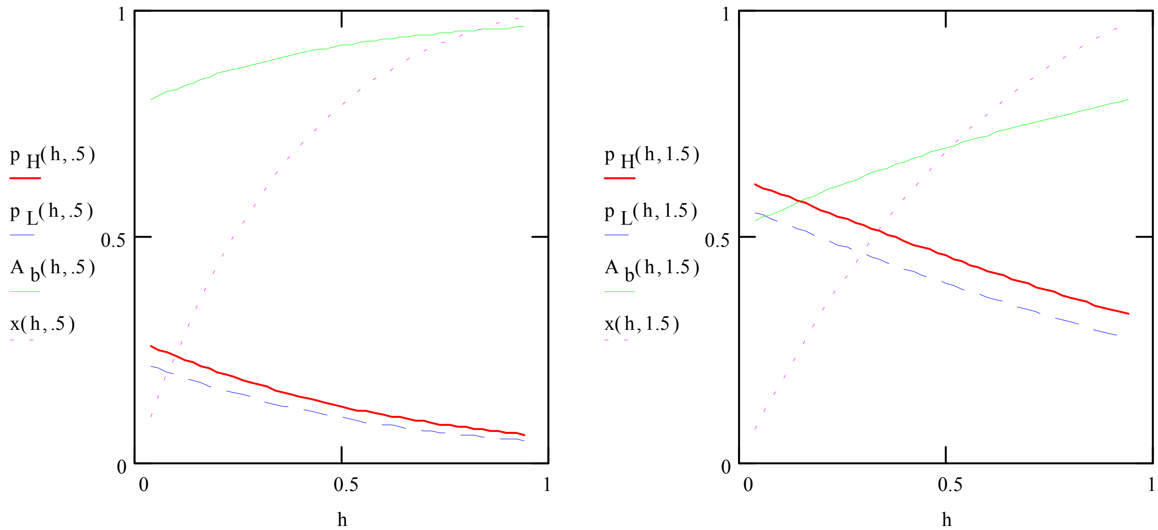

Figure 5.2: Prices and Probabilities

An increase in h constitutes an increase in available supply along the intensive margin, as it implies that there are the same number of sellers but more of them have high capacity. An decrease in b constitutes an increase in supply along the extensive margin, as there are more sellers per buyer, holding capacity per seller fixed. Figure 5.3 shows the difference in the impact of changes in supply along the two margins on p_H (p_L looks similar). The solid curves indicate the impact of increasing supply along the extensive margin, and the dotted curves the intensive margin, with the increase in the total available supply the same. Also shown is the arrival rate A_b. A key observation is that both variables are more responsive to supply changes along the intensive margin; that is, the price decreases more and the arrival rate for buyers increases more when we increase the number of goods per seller than when we increase the number of sellers.

[10] To derive the arrival rate in this version of the model, note that the probability a buyer gets served is given by $A_b = m_H\theta_H\Omega_H + m_L\theta_L\Omega_L$. Inserting Ω_H and Ω_L and rearranging yields:

$$\begin{aligned} A_b &= \frac{2h}{b}\left[1-\left(1-\frac{x}{mh}\right)^{bm}\right] - x\left(1-\frac{x}{mh}\right)^{bm-1} + \frac{1-h}{b}\left[1-\left(1-\frac{1-x}{m(1-h)}\right)^{bm}\right] \\ &\to \frac{2h}{b}\left[1-\exp\left(-\frac{bx}{h}\right)\right] - x\exp\left(\frac{x}{mh}\right) + \frac{1-h}{b}\left[1-\exp\left(\frac{bx-b}{1-h}\right)\right]. \end{aligned}$$

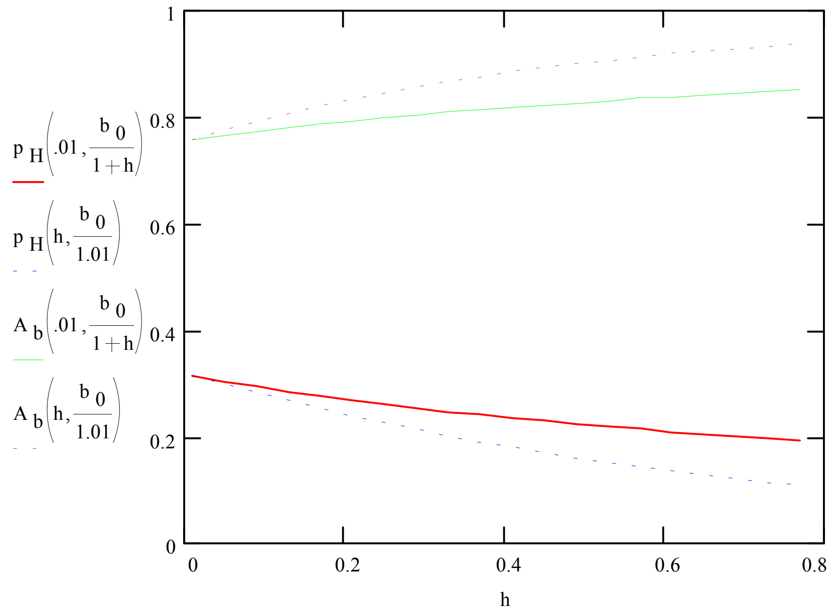

Figure 5.3: Changes in Supply Along Intensive and Extensive Margins

Consider the implications for the typical application in search theory to the labor market, as in Pissarides (1990) or Mortensen and Pissarides (1995). Those models assume that the number of successful meetings between employers and workers is a function of the number of unemployed workers and the number of employers with vacancies. Thinking of employers as the analogue of sellers in our model (they post wages in exchange for jobs), what we have found is that it makes a difference whether there are many employers with 1 vacancy each or few employers with more than 1 vacancy. In other words, the standard specification of the matching function is incomplete if firms may post more than 1 vacancy. In particular, the iso-matching curves (recall Figure 3.3) shift out as we increase the number of sellers with low capacity, holding the total number of units for sale fixed. Intuitively, frictions are more problematic when there are more locations with limited capacity.

These results imply that the number of matches will fall, given the vacancy and unemployment rates, if the firm-size distribution shifts toward more small firms. That is, the Beveridge curve (the locus of observed vacancy - unemployment pairs) will shift out as the relative number of small firms increases. It is a matter of fact that the Beveridge curve has shifted out in the post war period, as documented by Blanchard and Diamond (1989) for the US and by Jackman, Layard and Pissarides (1989) for the UK. Blanchard and Diamond say that about "half [of the shift] is due to an unexplained deterministic trend" (pp. 4-5), and that the evidence suggests "trend changes in matching, which we find in our estimation of the matching function for the period 1968 to 1981, account for a good part of this deterministic trend" (pp. 47,50). Jackman et al. say that "shifts in the [Beveridge] curve reflect the efficiency with which the labor market matches unemployed workers to job vacancies, and the outward shift in the UK seems mainly attributable to the fall in the effectiveness of the unemployed as job seekers" (p. 377).

Our model helps to make sense of these trend shifts to the extent that the size-distribution has shifted toward smaller firms during the period. There is evidence to this effect at least for the 1980s; see Stanworth and Gray (1991). According to our theory, the relative increase in the number of small firms means that the efficiency of the matching technology, as a function of the vacancy and unemployment rates, is diminished. Hence, our model predicts the observed outward shift in the Beveridge curve is the result of the change in the firm-size distribution. Of course, one would like to have more evidence for the shift in the firm-size distribution, and it would be interesting to measure quantitatively just how much this can account for the shift in the Beveridge curve, but this goes beyond the scope of the current project.[11]

6. Conclusion

We have studied pricing and allocations in markets with frictions. The frictions here are not due to a lack of information as to which seller is offering which good, how many goods, or at what price – all of this is known with certainty. The frictions arise when all sellers set the same price (conditional on quality and capacity), and buyers randomize over who they visit, since this means that sometimes more and sometimes fewer buyers show up than a seller can accommodate.

[11]We do not provide an explanation here for why the size distribution has changed, or why it does not become degenerate over time, as this would requires a dynamic story beyond the scope of this paper. See Shi (2000) for a version of one such model specifically applied to the labor market.

The results can be summarized as follows. First, we gave a complete characterization of the set of equilibria for the 2×2 case, including those supported by trigger strategies. Second, we argued that the equilibria where buyers randomize are interesting outcomes on which to focus for several reasons: they are symmetric; they do not require unreasonable coordination; they are robust to minor perturbations such as introducing noise; and they generate lots of economic implications. They also endogenize what is usually assumed in the undirected search literature. We derived the closed form for $p(m, n)$ for the general case, and showed that it equals the price generated by a simple alternative model in the limit, but not for finite m and n, because the simple model neglects certain strategic elements of price posting. We also derived the closed form for the endogenous matching function $M(m, n)$ and showed it exhibits decreasing returns for finite m and n, although it converges to a function with constant returns in the limit.

We endogenized capacity in the 2×2 case and showed how the outcome depends on production costs. In the general case, when m_H and m_L sellers start with high and low capacity, we found equilibrium prices and arrival rates for large markets, and showed how they depend on the ratio of high to low capacity sellers as well as the ratio of sellers to buyers. Interestingly, we found prices and arrival rates are both more sensitive to changes in the former than the latter ratio. In particular, our endogenous matching function is less efficient when there are more low capacity sellers, holding total available supply constant, simply because the frictions are more severe. Given a shift towards relatively more small employers, this would explain the observed shifts in the Beveridge curve over time. In future research it may be interesting to pursue this idea quantitatively. It would also be good to pursue dynamic versions of the model, allowing buyers and sellers to stay in the market for more than one period, and perhaps also allowing entry by new buyers and sellers.

7. Appendix

Suppose there are finite numbers of agents: n buyers, m_H sellers with 2 units, and $m_L = m - m_H$ sellers with 1 unit. However, here each seller takes as given the reaction functions of other agents, rather than market utility U, as in (5.2) and (5.4). The goal is to show that the two models give the same answer when m and n are large.

We look for an equilibrium where all high capacity sellers charge p_H, all low capacity sellers charge p_L, and all buyers go to each high capacity seller with probability $\theta_H > 0$ and each low capacity seller with probability $\theta_L > 0$, where $m_H\theta_H + m_L\theta_L = 1$. The probabilities of a buyer getting served, conditional on arriving at a low and at a high capacity seller, are given by

$$\Omega_L = \frac{1-(1-\theta_L)^n}{n\theta_L} \text{ and } \Omega_H = \frac{2}{n\theta_H}[1-(1-\theta_H)^n] - (1-\theta_H)^{n-1}. \tag{7.1}$$

Since buyers visit all sellers with positive probability, we require $(1-p_H)\Omega_H = (1-p_L)\Omega_L$, or

$$\frac{1-p_H}{1-p_L} = \frac{\frac{1}{n\theta_L}[1-(1-\theta_L)^n]}{\frac{2}{n\theta_H}[1-(1-\theta_H)^n] - (1-\theta_H)^{n-1}}. \tag{7.2}$$

Now suppose we are in such an equilibrium, and consider a deviation by one seller. First, consider a high capacity seller who deviates to p_H^d. Let θ_H^d be the probability that a buyer visits the deviant, where

$$\theta_H^d + (m_H - 1)\theta_H + m_L\theta_L = 1. \tag{7.3}$$

If he visits the deviant, a buyer gets served with probability

$$\Omega_H^d = \frac{2}{n\theta_H^d}[1-(1-\theta_H^d)^n] - (1-\theta_H^d)^{n-1}. \tag{7.4}$$

Given that we settle on an equilibrium where buyers go to all sellers with positive probability, we require $(1-p_H^d)\Omega_H^d = (1-p_H)\Omega_H$, or

$$\frac{1-p_H}{1-p_H^d} = \frac{\frac{2}{n\theta_H^d}[1-(1-\theta_H^d)^n] - (1-\theta_H^d)^{n-1}}{\frac{2}{n\theta_H}[1-(1-\theta_H)^n] - (1-\theta_H)^{n-1}}, \tag{7.5}$$

in addition to (7.2).

Conditions (7.3), (7.5) and (7.2) implicitly define a function $\theta_H^d = \theta_H^d(p_H^d; p_H, p_L)$. Taking this function and (p_H, p_L) as given, the deviant high capacity seller seeks to maximize

$$\pi_H^d = p_H^d \left\{ 2[1-(1-\theta_H^d)^n] - n\theta_H^d(1-\theta_H^d)^{n-1} \right\}. \tag{7.6}$$

Note that this is the same as the objective function in (5.4), but here the seller takes into account the reaction function $\theta_H^d = \theta_H^d(p_H^d; p_H, p_L)$, rather than the constraint that buyers have to receive expected utility of at least U. An interior solution satisfies

$$\frac{\partial \pi_H^d}{\partial p_H^d} + \frac{\partial \pi_H^d}{\partial \theta_H^d} \cdot \frac{\partial \theta_H^d}{\partial p_H^d} = 0 \tag{7.7}$$

Similarly, consider a low capacity seller who deviates to p_L^d. Let θ_L^d be the probability that a buyer visits the deviant, where

$$\theta_L^d + m_H \theta_H + (m_L - 1)\theta_L = 1. \tag{7.8}$$

Given that we settle on an equilibrium where buyers go to all sellers with positive probability, we require $(1 - p_L^d)\Omega_L^d = (1 - p_L)\Omega_L$, or

$$\frac{1 - p_L}{1 - p_L^d} = \frac{\frac{1}{n\theta_L^d}[1 - (1 - \theta_L^d)^n]}{\frac{1}{n\theta_L}[1 - (1 - \theta_L)^n]}, \tag{7.9}$$

in addition to (7.2). Conditions (7.8), (7.9) and (7.2) implicitly define $\theta_L^d = \theta_L^d(p_L^d; p_H, p_L)$. Taking this and (p_H, p_L) as given, the deviant low capacity seller seeks to maximize

$$\pi_L^d = p_L^d[1 - (1 - \theta_L^d)^n]. \tag{7.10}$$

An interior solution satisfies

$$\frac{\partial \pi_L^d}{\partial p_L^d} + \frac{\partial \pi_L^d}{\partial \theta_L^d} \cdot \frac{\partial \theta_L^d}{\partial p_L^d} = 0. \tag{7.11}$$

A symmetric mixed-strategy equilibrium is a list $(p_H, p_L, \theta_H, \theta_L)$ satisfying the first order conditions for high and low capacity sellers (7.7) and (7.11), the condition (7.2) that makes buyers indifferent between visiting the two types of sellers, and the identity $m_H \theta_H + m_L \theta_L = 1$, all evaluated at $(p_H^d, p_L^d, \theta_H^d, \theta_L^d) = (p_H, p_L, \theta_H, \theta_L)$. This system is complicated, in general, but simplifies considerably when $m \to \infty$ holding $h = m_H/m$ and $b = n/m$ constant. Taking limits and inserting $(p_H^d, p_L^d, \theta_H^d, \theta_L^d) = (p_H, p_L, \theta_H, \theta_L)$, after some algebraic manipulations, we can reduce (7.7) and (7.11) to

$$\frac{p_H}{1 - p_H} = \frac{-bz + 2\left[\frac{\exp(bz) - 1}{bz} - 1\right]}{1 + bz}, \tag{7.12}$$

$$\frac{p_L}{1 - p_L} = \frac{1 - h}{b(1 - hz)}\left[\exp\left(\frac{b(1 - hz)}{1 - h}\right) - 1\right]. \tag{7.13}$$

where $z = \lim_{m\to\infty} m\theta_H$. Also, we can simplify (7.2) to

$$\frac{1-p_H}{1-p_L} = \frac{(1-h)z}{1-hz} \frac{2\left[1-\exp\left(-bz\right)\right] - bz\exp\left(-bz\right)}{1-\exp\left(-\frac{b(1-hz)}{1-h}\right)}. \tag{7.14}$$

These can be combined to yield one equation in z. Then, upon solving for z, one can solve for p_H and p_L. At this stage, it is a matter of routine algebra to verify that the answer is the same as the solution for the model in the text when $m \to \infty$ holding h and b constant. ■

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