Cognition and Wealth: The Importance of Probabilistic Thinking

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1. Introduction

This paper is motivated by proposed reforms of Social Security which expand the domain of household choice and by corresponding trends in the private sector away from defined benefit pension plans toward defined contribution plans, including growth in tax-sheltered IRA's and 401k plans which allow individual choice in contribution rates and in the timing and magnitude of payouts. Whether such reforms will be beneficial or harmful depend on how well individuals and households plan their finances. Subjective probabilities are a key ingredient in any economic model of optimal financial planning, yet little is known about the capacity of individuals to utilize probabilistic thinking in ways that would enable them to exploit the advantages of expanded choice. In this paper, we utilize a large battery of subjective probability questions that have been administered in the Health and Retirement Study (HRS) to investigate how probabilistic thinking affects portfolio choices and net worth.

Proponents of expanded choice argue that household welfare will be improved because plan characteristics can be better matched to individual preferences and circumstances. In the case of pension plans, it is argued that higher returns available on investment portfolios will enable households to have better and more secure retirements than can be afforded by pay-as-you–go tax and transfer programs such as Social Security, defined benefit plans or fixed income securities. For example, Poterba and Wise (1998) note that estimates of the the long term average annual rate of return on a diversified portfolio of stocks range from 8.1 to 9.6 percent compared with returns of about 4.6 percent for a portfolio of long term bonds or 3.8 percent for a portfolio of short term Treasury bills. Compounded over a lifetime of savings, differences in expected returns of this magnitude create very large variations in household resources available for retirement or bequests. While the higher expected returns from stocks may be offset by higher risk of losses, simulated portfolios suggest that the value at retirement of a portfolio of stocks accumulated over a lifetime may stochastically dominate a bond portfolio (Wise cite??). Moreover, flexibility about whether and when to annuitize their portfolio after retirement, together with an expanded range of choice of the features of health insurance, allow households to reduce their exposure to poor health. Poor health affects household income and wealth primarily through medical expenditures and (mostly uninsurable) losses in labor income (Smith 1999b). Even insurance against inflation risk, a traditional advantage of Social Security benefits, is now attainable for households who choose to hold indexed Treasury bonds.

Although expanded choice offers many important potential benefits to households, critics argue that large segments of the population will fail to make choices that exploit these potential benefits and, consequently, the expansion of choice will expose these segments of the population to greater risks of poverty and exacerbate the already very large inequalities in wealth among older households (cites??). Some of the reasons given for these worries fit within the conventional life cycle model of expected utility maximization. For example, in the presence of incomplete insurance and annuity markets, risk aversion might lead low income households to choose less risky portfolios with lower expected returns than higher income households. There may also be variation in taste parameters, such as time preference, such that persons with high rates of time preference save at low rates (and also choose lower investments in human capital and less healthy lifestyles) which leave them with poor health and few resources in old age.

Those who emphasize "equality of opportunity" may view such outcomes as an acceptable consequence of the exercise of free choice. Those who value "equality of outcomes" stress the value of placing constraints on choice *ex ante*, both to protect others from the bad consequence of their actions and to protect themselves from higher taxes to fund redistributional programs needed to offset these consequences.

This paper utilizes a large battery of subjective probability questions that have been administered to a sample of over 20,000 individuals in the 1998 wave and earlier waves of the Health and Retirement Study (HRS) and its companion study, Asset and Health Dynamics of the Oldest Old (AHEAD). Our goal is to develop a measure of competence in probabilistic thinking which is based on the degree to which an individual's probabilistic beliefs are precise or imprecise and examines the empirical relationship between this measure and measures of asset accumulation and portfolio composition. We provide a theoretical justification for our approach using a simple model in which people with imprecise probability beliefs behave in a more risk averse manner than those with more precise beliefs. Our model represents a possible resolution of the "Ellsberg Paradox" (Ellsberg 1961), which alleges that individuals display "uncertainty aversion." Unlike most models of uncertainty aversion, our model is compatible with rationality as defined by the axioms underlying the Ramsey-Savage theory of personal probabilities (Ramsey 1926, Savage 1954), Baysesian statistical theory and the Von Neuman-Morgenstern subjective expected utility (SEU) model .[1] In addition to providing a link between probabilistic thinking and uncertainty aversion, the model also provides a framework which could be used to study how individuals may acquire more precise information about probabilities, how they may increase the precision of their beliefs through experimentation, and how information acquisition is related to preference parameters such as risk aversion and time preference. These further implications are not pursued in this paper.

The plan of the paper is as follows. Section 2 provides a discussion of the subjective probability questions in the HRS and some background about the elicitation of data on expectations in a survey context. The distribution of responses to these probability questions suggests that there may be considerable heterogeneity among respondents in the precision their probability beliefs and/or their competence in probabilistic thinking. In Section 3, we discuss the idea of uncertainty aversion in the context of the Ellsberg paradox. In Section 4 we develop a rational model of uncertainty aversion. In Section 5, we consider how survey responses to probability questions are related to the precision of probability beliefs. Section 6 presents an econometric analysis relating measures of imprecise probability beliefs to the share of risky assets in household portfolios and to the growth rate of household assets. A summary and conclusions are presented in Section 7.

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[1] After formulating our model, we discovered that Scheeweiss (1999) independently proposed essentially the same resolution of the Ellsberg Paradox. He does not, however, discuss the implications of the model for learning or apply it empirically.

2. Subjective Probability Questions in the HRS

This paper makes use of a unique body of data on large number of subjective probability questions asked to respondents in several longitudinal waves of the Health and Retirement Study.[2] In this paper, we analyze questions asked in the 1998 wave which surveyed a national probability sample of over 22,000 respondents representing the U.S. population over age 50, as well as questions asked to subsets of these persons in other waves.[3] These probability questions cover a wide range of topics ranging from personal life expectancy and date of retirement to beliefs about the rate of inflation, future Social Security policy and other macro-level variables.

The format of the probability questions is presented in Table 1. There is an introduction which states that the respondent will be asked some questions about how likely they think various events might be. For each question, the respondent is told to give a number between 0 and 100 where "0" means "no chance at all" and "100" means the event is absolutely sure to happen. In several waves of the survey, a "warm up" question is asked about the chance that tommorow will be sunny,[4] followed by a list of questions on a variety of topics. Several examples of probability questions are given in Table 1, classified by whether they are about general events (e.g., social security

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[2] See Dominitz and Manski (1999) for a discussion of the history of methods of eliciting expectations data in surveys.

[3] The initial HRS sample, consisting of 12,654 persons born in 1931-41, was first surveyed in 1992 when respondents were 51-61 years of age and has been resurveyed in 1994, 1996 and 1998. The AHEAD survey, consisting of 8,221 persons born before 1924, were first surveyed in 1993 when they were 70 years of age and up and were followed up in 1995. Beginning with 1998, the AHEAD survey of persons born before 1924 has been integrated into the HRS which also incorporated a new cohort born in 1924-30 who were entering their 70's and another new cohort born in 1942-47 who were entering their 50's. Thus, the 1998 wave of the HRS represents the entire U.S. population over age 50. See Juster and Suzman (1995), Soldo, et. al. (1997) and Willis (1999) for more detailed descriptions of the HRS and AHEAD studies.

[4] Basset and Lumsdaine (1999a) have used this question to investigate whether some individuals are persistently "optimistic" or "pessimistic". They find that persons who give high probabilities that the weather will be sunny tomorrow tend to give higher probabilities of "good" outcomes on other topics. This question was omitted from HRS-1998, but has been put back into HRS-2000 which is currently in the field.

generosity or rate of inflation), events with personal knowledge (e.g., survival to age 75, income will keep up with inflation) or events subject to personal control (e.g., leaving an inheritance or working at age 62). In HRS-1998, there were seventeen probability questions. Some questions were asked to a subset of respondents (e.g., the probability of working at age 62 was only asked of those less than age 62). Others contain additional sub-questions, depending on the answer (e.g., if the probability of leaving an inheritance is larger than zero, the respondent is asked about the probability of leaving inheritances larger than $10,000 and, subject to that probability being positive, the probability of leaving more than $100,000).

Analysis of the subjective probability questions has demonstrated that they contain considerable useful information. For example, on average, subjective probabilities of life expectancy match life tables surprisingly well and co-vary with variables such as smoking, drinking, health conditions or education in ways that would be expected from studies of actual mortality (Hurd and McGarry, 1995). As another example, Smith (1999a) finds that average values of expected bequest probabilities behave in sensible ways and appear to provide genuine information about behavior.

Although the subjective probability responses in HRS seem to "work well" when averaged across respondents, individual responses appear to contain considerable noise and are often heaped on "focal values" of "0", "50" and "100".[5] The degree of heaping is apparent in Figure 2 which presents histograms of the answers to the six probability questions listed in Table 1. A comprehensive tabulation of focal answers to all

[5] Several researchers who have analyzed the HRS data emphasize the large number of focal answers and show that the likelihood of such answers is correlated with education and cognitive measures (see, especially, Hurd, McFadden and Gan (1998)). In addition, Bassett and Lumsdaine (1999b) find other evidence of noise and unobserved hetereogeneity in answers to these questions.

probability questions in HRS-1998 is presented in Table 2. On average, only 5% of respondents refused to answer the probability questions. However, 52% of questions were heaped on either "0" or "100" and an additional 15% were heaped on "50".

Probabilities and probabilistic thinking are crucial ingredients of economic models of decisionmaking about asset accumulation and portfolio choice. In this paper, we hypothesize that a persistent tendency to give focal answers to probability questions indicates that a respondent has imprecise beliefs about the true value of a probability. Further, we hypothesize that imprecise beliefs about probabilities will lead an individual to make conservative financial choices which yield lower wealth and lower rates of wealth accumulation than otherwise similar individuals who have more precise probability beliefs. In the next two sections, we explore a more formal theoretical framework to understand why imprecise probabilistic thinking might lead to such a relationship.

3. Probabilistic Thinking, Imprecise Beliefs and Uncertainty Aversion

Despite much criticism, subjective expected utility (SEU) theory remains the dominant framework for economic models of decisionmaking under uncertainty. (See Starmer, 2000, for a survey of alternatives to SEU theory). SEU theory assumes that individuals decide on a given course of action by choosing that action which yields the highest expected utility. For example, consider a decision between actions *A* and *B* by individual *i*. The expected utilities associated with these actions are

(1) $$EU_{ia} = \sum_{j=1}^{\Omega_a} p_{ij}^a U_{ij}^a \quad \text{and} \quad EU_{ib} = \sum_{j=1}^{\Omega_b} p_{ij}^b U_{ij}^b,$$

where there are Ω_a discrete states of the world that might occur if action *A* is chosen and Ω_b states if *B* is chosen. p_{ij}^a represents *i*'s beliefs about the probability that the *j*th state of the world will occur and U_{ij}^a is the utility *i* believes that he will receive in that state. Similarly, p_{ij}^b and U_{ij}^b represent *i*'s beliefs about probabilities and utilities when *B* is chosen. The individual's choice of action is given by the decision rule

(2) $$Choose\ A\ if\ \ EU_i^a > EU_i^b;\ Otherwise\ Choose\ B.$$

Taken literally, the SEU model presumes that individuals competently perform some extremely demanding tasks before making any given decision. The individual must be capable of imagining a large number of states of the world. Within each state, the individual solves an optimization problem in which he maximizes his (possibly state-dependent) utility subject to a set of constraints embodying his income and wealth endowments, prices, state of health and all other data relevant to the optimization problem. The outcome of this optimization yields the level of utility associated with that state. Finally, the SEU model assumes that the individual has a coherent and well defined set of beliefs about the probabilities that each state will occur.

The latter assumption has been called into question at least since Frank Knight (1921) introduced the distinction between "risk" and "uncertainty" where risk refers to a situation in which a specific probability can be attached to a given outcome while uncertainty refers to a situation in which a probability cannot be specified. For example, examination of a symmetric coin may lead an individual to conclude that heads and tails have equal probability, that there are no other possible outcomes and, consequently, that the probability of heads is one half. Or, a person may have examined actuarial tables to determine his probability of death this year. In contrast to these risks, uncertainty may attach to questions in which there is neither a quantitative model of the random process, such as the simple physical model of a coin described above, nor is there adequate data to form a statistical estimate of the event from past frequencies of occurrence. Examples range from mundane questions about the probability that a new start-up firm will be in business in ten years or that a given marriage match will succeed to large questions such as the probability of nuclear war or the probability that there is intelligent life elsewhere in the universe.

The Ramsey-Savage theory of personal probability appeared to resolve this problem by eliminating the distinction between risk and uncertainty within a Baysesian framework in which individual preferences are assumed to fulfill certain axioms which lead to rational, maximizing behavior (Savage, 1954). According to this theory, the probabilities that enter into decision problems are subjective. Individuals are assumed to have a set of prior beliefs about the probability of any given event given by the distribution function, $f(p)$. The probability of the event is simply given by the expected value of this distribution, $\overline{p} = \int_0^1 pf(p)dp$. The probabilities entering the von Neumann-Morgenstern expected utility function in (1) have this interpretation.

An influential challenge to this resolution was presented by Ellsberg (1961) in an example that has come to be known as the "Ellsberg Paradox". In one version of the example, subjects are confronted with choices about betting on the outcome of draws from two urns, Urn I and Urn II, each containing red and/or black balls. The subject chooses which urn to play and the color of the ball to bet on. If a ball of the chosen color is drawn, the subject wins $1; otherwise, he wins nothing. Urn I contains 50 red balls and 50 black balls. Urn II contains 100 red and black balls in unknown proportion.

When asked about which color they preferred to bet on, most subjects indicated indifference regardless of which urn was under consideration. This suggests that subjects believe that red and black are equiprobable in either urn. When asked which urn they preferred to draw from, some subjects indicated indifference but a majority indicated a preference for Urn I, explaining that they preferred the greater certainty about the probability of drawing a ball of given color. The latter preference clearly violates the Savage axioms.

The paradox that Ellsberg identified, and that subsequent experimental research has verified, is that most people prefer a known risk to an uncertain one of equal expected value. Ellsberg often found uncertainty aversion even in a decidedly non-random sample of the founders of SEU theory. He reports that G. Debreu, R. Schlaiffer and P. Samuelson do not violate the Savage axioms, while J. Marshak and N. Dalkey violate them "cheerfully and even with gusto" and "others sadly but persistently, having looked into their hearts, found conflicts with the axioms and decided, in Samuelson's phrase, to satisfy their preferences and let the axioms satisfy themselves." (Ellsberg, pp. 655-56). Interestingly, Ellsberg places Savage himself in the latter group. Among the violators, he writes, "What is at issue might be called the *ambiguity* [italics in original] of this information, a quality depending on the amount, type, reliability and `unanimity' of information, and giving rise to one's degree of 'confidence' in an estimate of relative likelihoods." (Ellsberg, 1961, p. 657.)

Ellsberg and most of the subsequent literature react to this paradox by partially giving up on the Savage axioms, instead suggesting non-rational preferences that allow for "uncertainty aversion" in ambiguous situations. An example, is the maxmin expected utility (MMEU) function originally proposed by Gilboa and Schmeidler (1989). Intuitively, these preferences suggest that a decision maker who is uncertain about the true probabilities governing the outcomes of his decision may focus special attention to thinking about the consequences of the "worst case scenario" (among all "reasonable" scenarios) and will make conservative choices according to a maxmin criterion.

4. A Rational Model of Uncertainty Aversion and Learning about Probabilities

In this section, we propose an alternative model of uncertainty aversion. Given repeated sampling, individuals are fully rational in the sense that they behave in accord with the SEU model, but they may have more or less precise beliefs about the "true values" of the probabilities upon which their decisions are based.[6] Ellsberg's examples all concern "one shot" bets on a single roll of a die or a single ball drawn from an urn. Our model departs from the Ellsberg framework by assuming that most individuals implicitly form their preferences about choices among uncertain prospects in a context of real world choices whose consequences determine a sequence of random outcomes. Obvious examples include the returns from investments in human or physical capital, the choice of a marriage partner, or the returns from a stock held more than one period. In this model, it is easy to show that uncertainty aversion is simply a consequence of risk

[6] An advantage of retaining the SEU model is that we can freely draw upon the entire body of conventional economic theory in our application whereas theories such as the MMEU theory mentioned above are not (yet) fully embedded in a broader economic model. We exploit this advantage later in this section when we show how utility aversion and learning may be related.

aversion.[7] In addition, we show how this model may also shed light on decisions to engage in experimentation and other forms of learning to reduce uncertainty.

The basic idea of our model may be illustrated by considering a variant of the Ellsberg example in which a subject chooses between betting on one of two dice. The subject is allowed to inspect the first die and sees that it is a symmetric cube with each of its six sides uniquely numbered one through six. He is also allowed to roll the die and observe the outcome as much as he wishes. On the basis of physical examination and observation, assume that the subject concludes that the probability of any outcome from one through six on any given roll is equal to one sixth. Accordingly, we label the first die "fair".

In contrast, the subject is only allowed to see that the second die is a symmetric cube but is not allowed to see any of the numbers painted on its side nor is he allowed to experiment with it before placing a bet. He is only told that the die must come up with some number between one and six when rolled and that he is free to bet on any of these numbers. In this situation, the individual may have imprecise beliefs about the probability of any given outcome because the properties of the random device are not known, the principles governing its operation are not understood and statistical evidence about its behavior is not available. The subject is justified in worrying that the die is "loaded", with some outcomes having probabilities that exceed one sixth while other outcomes have lower probabilities. Nonetheless, given the symmetry of the die and his freedom to bet on any number of his choice, it is reasonable to assume that the subject believes that the expected probability of any given outcome is one sixth. That is, let the

[7] As noted earlier, Scheeweiss (1999) independently proposed essentially the same resolution of the Ellsberg Paradox.

density function $f(p_n)$ denote the subject's prior beliefs about the true value of the probability that the number n turns up on a single roll of the "loaded die". Then,

(3) $$\bar{p}_n = \Pr(outcome = n) = \int_0^1 p_n \, f(p_n) dp_n = 1/6; \quad n = 1,2,\ldots,6.$$

According to SEU theory, it is clear from (3) that a rational bettor should be indifferent between betting on a single roll of the fair or the loaded die. Assuming that a winning bet pays \$6, the expected payoff is \$1 in either case.

If the payoff is based on multiple throws of the die, however, it is easy to show that a risk averse individual would prefer to bet on the fair die. The reason is that the distribution of outcomes for the loaded die is riskier, according to the definition of Rothschild and Stiglitz (1970), because it is a mean preserving spread of the distribution of outcomes for the fair die. To see this, suppose that once a die is chosen it must be rolled twice and assume that the subject always bets on an ace (i.e., $n = 1$). The payoffs are \$12 for two aces, \$6 for one ace and nothing for no aces. In the case of a fair die, the probabilities of these outcomes are *1/36*, *10/36*, and *25/36*, respectively, and the expected payoff is \$2. The expected payoff from two rolls of the loaded die is also \$2, but the probabilities of the two extreme outcomes of two aces or no aces are increased relative to the fair die. That is, let p_1 denote the true probability of an ace $q_1 = 1 - p_1$ be the complementary probability. Given that x^2 is a convex function of x, it follows from Jensen's inequality that $E(p_1^2) \geq (1/6)^2 = 1/36$ and $E(q_1^2) \geq (5/6)^2 = 25/36$ with the strict inequality holding if there is prior uncertainty about the probabilities. Hence, the loaded die is riskier and a risk averse person will prefer the fair die when payoffs depend on multiple realizations of the die. Alternatively, he would demand a risk premium to bet on the loaded die.

While our model suggests that imprecise probability beliefs lead risk averse individuals to make conservative choices, it can also be used to show that risk averse individuals may prefer the more uncertain alternative if they are in a position to learn from observation. To illustrate this most simply, assume that the subject is told that the loaded die has the same number painted on each face but is not told the number. Given symmetry across possible outcomes, the expected probability of an ace on a single roll of the loaded die is *1/6*. If the subject must choose a die and roll it twice, the probability of two aces is *1/6*, the probability of no aces is *5/6*, the probability of one ace is zero and the expected payoff is $2. As before, the loaded die is riskier and a risk averse subject will prefer to bet on the fair die.

Now suppose that we change the terms of the game by allowing the subject to choose one of the die, roll it once, and then either continue with that die or switch to the other die for the second throw. Assume the subject chooses the fair die on the first roll. Then, regardless of the outcome, he will be indifferent between the two die on the second toss and the distribution of payoffs will be identical to a sequence of two rolls of the fair die. Alternatively, suppose that he chooses to roll the loaded die on the first toss. If he rolls an ace, he knows that the probability of rolling an ace on the second toss is one and, hence, he will choose to roll the loaded die again. If he does not obtain an ace, there is a zero probability that the loaded die will be an ace on the second roll so it is optimal to switch to the fair die. Thus, for two tosses, beginning with the loaded die, the distribution of returns is $12 with probability 1/6, $6 with probability 5/36 and zero with probability 25/36. The expected return is $2.83 if the individual begins with the loaded die compared to an expectation of only $2 when he begins with the fair die. The difference in expected return of 83 cents represents the monetary value of the learning opportunity presented by the loaded die. Moreover, the distribution of returns associated with choosing the loaded die on the first roll dominates the distribution from choosing the fair die first, so that a subject would prefer the loaded die no matter how risk averse he is.

This model may be used as a metaphor for a life cycle model of decisionmaking under uncertainty. Assume that an individual lives for two periods, with the roll of a die determining his income in each period. Some kinds of decisions faced by the individual require a lifetime bet. Examples might include the decision to go to college or have a child. We might think of such decisions as analogous to a game in which a person must choose one die at the beginning of a game and continue to roll it until the game ends. Holding expected payoffs constant, our analysis suggests that risk averse individuals will choose those options about which their probability beliefs are most precise. To the extent there is heterogeneity across individuals in the degree of risk aversion or in precision of beliefs, self-selection will tend to induce correlations between observed choices, on the one hand, and information and risk aversion parameters, on the other hand. For example, other things equal, high school seniors with more precise beliefs about the payoffs to college may tend to go on to college while their less well informed classmates do not.

Other kinds of decisions involve more flexible choices analogous to our second game in which it is possible to switch from one die to another after observing the first outcome. For example, Johnson (1978) presents a model of job shopping by young workers who are confident about how well they will do in some jobs but uncertain about their suitability for other jobs. The model predicts that workers will choose the more uncertain jobs, continuing with the job if it is a good match and leaving for the more certain job if it is a bad match. In labor market equilibrium, the option to obtain valuable information will lead to a lower starting wage for uncertain jobs. Since the benefits of information occur in the second period, workers with high rates of time preference will tend to choose "safer" jobs and, other things equal, will have learned less about their productivity by the beginning of the second period.

In this section, we established a theoretical connection between the precision of probabilistic beliefs and decisions about risky alternatives. In order to move toward an empirical specification of this relationship, in the next section we attempt to relate the degree of precision about probabilistic beliefs to survey responses to probability questions.

5. Precision of Probability Beliefs and Survey Responses

As we discussed in Section 2, survey responses to subjective probability questions in the HRS tend to behave quite reasonably when averaged across individuals, but are quite noisy with considerable heaping on "focal" answers of "0", "50" and "100". We have hypothesized that heaping is associated with respondents' ambiguity or uncertainty about true probabilities and a correspondingly diffuse distribution of their prior beliefs. In this section, we first develop a simple formal model of the relationship between the information that a respondent has about the probability of a given outcome and the shape of the density function of his prior beliefs. We then propose a specific testable hypothesis about how answers to survey questions about subjective probabilities are related to the prior density and, finally, present empirical evidence that responses in the HRS are consistent with this model.

Assume that the information that an individual has about the likelihood that a given discrete outcome will occur is given by the probit function,

(4) $$p = \Pr(I > 0 \mid x, d) = \Pr(xb + d > u) = F(xb + d)$$

where I is an index function,

(5) $$I = xb + d - u \, .$$

In this function, x is a vector of variables that determine the likelihood of an outcome, d is a normally distributed variable with mean zero and variance s_d^2 which reflects the individual's uncertainty about the true value of the index, and u is a standard normal random variable. For example, if p is the individual's belief about the probability of leaving an inheritance, x would include variables such as age, sex, income, wealth, health history, marital status, number of children and other variables that are predictive of the value of the estate at death, the date of death and the existence of heirs. The effects of these variables, given by the probit coefficients b , might be those that would be estimated by in a scientific study of bequest behavior or they might simply reflect personal beliefs that are not supported by scientific analysis.[8] In addition, of course, the individual may possess personal information about the likelihood of leaving a bequest that would not be known to an outside observer.

The random variable d indicates the range of the individual's uncertainty about the true probability. If $s_d^2 = 0$, the individual has sharp priors which are identical to the predicted survival probabilities that would be produced by an actuarial analysis given by

$p^* = F(xb)$ where $F()$ is the normal cdf. We shall call p^* the "true personal probability."[9] As s_d^2 increases, the individual's beliefs about personal probabilities become more and more diffuse. The precision of a given person's beliefs may depend on his or her education, cognitive ability and experience in observing and making decisions in various domains. Thus, let $s_d = s_d(z)$ where z is vector of variables determining the precision of beliefs. Note that some elements of z may overlap with elements of x. For example, increases in education tend to increase the true personal probability of survival and also decrease uncertainty about the "true" probability.

Letting $s_e = \sqrt{1+s_d^2}$, $F()$ denote the standard normal c.d.f., and following Lillard and Willis (1978), the c.d.f. of the prior distribution of subjective probabilities associated with (4) is

(6) $$G(p) = F\left(\frac{s_e}{s_d} F^{-1}(p) - \frac{s_e}{s_d} xb\right),$$

and the density function is

(7) $$g(p) = \frac{s_e f\left(\frac{s_e}{s_d} F^{-1}(p) - \frac{s_e}{s_d} xb\right)}{s_d f(F^{-1}(p))}.$$

A matrix of density functions corresponding to different values of true personal probabilities, measured by p^* on the horizontal axis, and different degrees of uncertainty, measured by s_d on the vertical axis, is illustrated in Figure 2. The graph contained in

[8] See Camerer (1995) for survey of "calibration studies" which attempt to determine how well or poorly subjective probabilities elicited in surveys correspond to "objective" probabilities based on evidence.

[9] The term "true personal probability" simply refers to the subjective probability belief that an individual would have if he had no uncertainty. A separate question that we do not address in this paper is the degree to which such personal probabilities coincide with "objective probabilities" based on expert opinion, scientific research, or cognitive processing of personal experience according to a Bayesian model.

each cell in the matrix depicts the density function in (7) corresponding to a given pair of values of $p^* = F(xb)$ and s_d. The bottom row of density functions illustrates (almost completely) precise probability beliefs with $s_d = .01$ for nine values of xb ranging between –2 and 2 and corresponding values of p^* ranging between .022 and .978. For each value of xb, there is a column of nine graphs associated with increasing values of s_d up to a maximum of $s_d = 100$, reflecting ever increasing uncertainty about the true value of the subjective probability.

The effect of increasing uncertainty on the shape of the density function depends on the value of xb. Consider first the case of $p^* = F(0) = .5$. As s_d increases over the range $0 < s_d < 1$, the density function has a symmetric, unimodal shape whose variance grows as s_d increases. When $s_d = 1$, the density function becomes uniform. For values of $s_d > 1$, the density function becomes U-shaped with the density increasingly concentrated near the extremes of $p = 0$ and $p = 1$. Now consider the case of $p^* = F(-.0.25) = 0.401$. For values of $0 < s_d \le 0.75$, the density function is a right-skewed unimodal function with a mode near *0.4* for small values of s_d. As s_d increases to *0.75*, the mode decreases but remains well above zero. At $s_d = 1$, the density function becomes J-shaped with its single mode near zero. As s_d increases above one, the density function takes on an asymmetric U-shape with the larger mode near zero and the smaller mode near one. As the degree of uncertainty continues to increase, the U-shaped density functions become more and more symmetric near zero and one so that, for large values of s_d, the modes near zero and one are approximately equal. This pattern is repeated for smaller values of p^*, but the range of s_d over which the function is unimodal shrinks and the range over which it is J-shaped or U-shaped increases. These patterns are repeated in mirror image for values of $p^* = F(xb) > 0.5$.

We now wish to address the following question. How do responses to survey questions in HRS about subjective probabilities differ across individuals who have varying degrees of uncertainty about true probabilities? That is, assuming that density functions such as those depicted in Figure 3 are in the minds of respondents, how do they respond to survey questions about subjective probabilities like those discussed in Section 2? One possible hypothesis is that all individuals give a fully Bayesian response and report the expected value of their prior density. In this case, no matter how diffuse their priors, they would return an exact answer, $\overline{p} = E(p) = \int_0^1 g(p)dp$.[10] This would be inconsistent with evidence of heaping on focal values presented in Section 2.

[10] In the probit model presented in this section, $\overline{p}$ is a decreasing function of s_d holding xb constant, with $\overline{p}$ approaching *0.5* as s_d approaches infinity. That is, as uncertainty grows, the Baysesian prior probability approaches 50-50. This is not a general implication of Bayesian models with uncertainty. For example, assume the prior density *p* is given by a beta distribution, $g(p) = \frac{1}{B(a,b)} p^{a-1}(1-p)^{b-1}$ where $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}dt = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ with expected value $E(p) = \frac{a}{a+b}$ and variance $s_d^2 = \frac{ab}{(a+b)^2(a+b+1)}$ which is a decreasing function of *a* and *b*. As Heckman and Willis (1977) show, when covariates are introduced into this model with the parameterization $a = \exp(xb_a)$ and $b = \exp(xb_b)$, the expected value of the probability is given by the logistic function $\overline{p} = 1/(1+e^{-xb})$ where $b = b_a - b_b$. In this model, it is easy to hold $\overline{p}$ constant while increasing the amount of uncertainty. However, the pattern of shapes displayed by a matrix of beta density functions associated with differing levels of $\overline{p}$ and s_d is very similar to that displayed in the heterogeneous probit model in Figure 2.

An alternative hypothesis, which we shall call the "modal choice hypothesis," is consistent with heaping. According to this hypothesis, respondents respond by reporting that probability which is most likely among all possible values of their true subjective probability. Specifically, a respondent would choose an exact (i.e., non-focal) value given by the mode of *g(p)* when his prior distribution has a unimodal "triangular" shape, a modal value of "0" or "100" when the distribution is J-shaped or U-shaped with one mode much larger than the other, and a value of "50" if the individual is not sure which mode is larger.

According to the modal choice hypothesis, subjective probability responses tend to have a systematic pattern as p^* varies. This pattern, which can be discerned in the matrix of density functions shown in Figure 2, is drawn explicitly in Figure 3 where the (p^*, s_d) plane is divided into four areas corresponding to those combinations of p^* and s_d in which the respondent gives (a) an exact answer, (b) a focal answer of "0", (c) a focal answer of "100", or (d) a focal answer of "50". The area in which an exact answer is given has an upper boundary given by an inverted U-shaped curve which attains a maximum at $p^* = 0.5$ and $s_d = 1$ where it is tangent to the U-shaped lower boundary of the area in which a 50-50 answer is given. At the point of tangency, the prior distribution is uniform; for $s_d < 1$, the prior densities are unimodal, and for $s_d > 1$ they are bimodal with modes of equal size near zero and one. As p^* deviates in either direction from *0.5*, the values of s_d below which the prior distribution is triangular with a single mode decrease so that as p^* nears zero or one a respondent will not give an exact answer unless s_d is very small. Similarly, the lower boundary of the region in which 50-50 answers are given has an inverted U-shape. The reason is that as p^* deviates in either direction from *0.5*, the critical value of s_d above which the prior distribution is U-shaped (and has approximately equal size modes near zero and one) increases. The regions between these two areas generate focal answers equal to zero if $p^* < 0.5$ or one if $p^* > 0.5$ because, as can be seen in Figure 2, the prior distributions in these two regions are either J-shaped with a single mode near zero or one or they are U-shaped with the larger mode near zero or one.

The modal choice hypothesis is consistent with the "reasonable" behavior of subjective probabilities that we discussed in Section 2 when responses are averaged across samples of respondents who give a mixture of exact and focal answers. That is, as xb increases in a population of individuals with varying degrees of uncertainty, the average value of exact answers increases as does the fraction of heaped answers that reflect higher focal values.

In Figure 4, we present evidence that focal values of the subjective probability questions do contain such information. Specifically, we have estimated a set of regressions with subjective probability responses from the 1992 and 1998 waves of the HRS on the left hand side and a set of demographic characteristics (age, sex, race and education) on the right hand side. The regressions are estimated on two different samples: (a) the full sample of respondents who responded to a given question with either an exact or focal answer and (b) the subsample of respondents who gave an "exact" (i.e., non-focal) answer. Predicted values for each regression are then sorted into centile groups. Within each group, the value of the responses of those who gave focal answers (0.0, 0.5 and 1.0) is regressed on the predicted probability. If the average value of the focal answers behaves in the same way as the exact answers, a plot of the predicted focal answer vs. the average predicted probability within the centile group should fall along a 45 degree line.

These plots are shown relative to a 45 degree line in the matrix of graphs in Figure 4 for nine questions from 1992 in Panels A and B and sixteen questions from 1998 in Panels C and D. The plotted lines are very close to the 45 degree line in Panels A and C, where the prediction equations are based on the full sample. While deviations from the 45 degree line are more prominent when the prediction equations are based only on the subsample of respondents who gave exact answers, all are positively sloped and there is no regular pattern to the deviations. We conclude that focal answers to subjective probability questions contain similar information to that contained in exact answers when averaged across respondents.

A more direct test of the modal choice hypothesis is to see whether the propensity to give exact answers follows the inverted U-shaped pattern depicted in Figure 3. In order to perform this test, we wish to estimate the relationship

(7) $$\Pr(ExactAnswer) = f(p^*(x)) + u,$$

where x is a set of covariates that influence the level of p^* and u is an error term which is independent of x. We estimate a parametric model in which $f()$ is quadratic and also a non-parametric lowess (robust locally weighted regression) function to test for the inverted U-shape.

Before proceeding, however, it is important to note that our theory suggests that we may have a serious identification problem in estimating the model. Specifically, in order to obtain unbiased estimates of (7) we want to vary p^* independently of s_d.

However, as we discussed earlier, for most of the probability questions in the HRS it is quite likely that variables such as age, race, sex, health and education which affect the level of a probability are also correlated with the precision of an individual's probability beliefs. Fortunately, one of the questions – the warm-up question asking respondents to give the probability that tomorrow will be sunny[11] – allows us to vary p^* by using information on the month of interview together with a primary sampling unit (psu) indicator to construct a measure of the average value of the probability of a sunny day within each month-psu cell. We use all cells with at least three responses. This measure produces a very wide range of average probabilities which presumably vary independently of average values of s_d.

The "sunny" question was asked in HRS 1994, AHEAD 1993 and AHEAD 1995. The results of our estimates of (7) are shown graphically in Figure 5 for questions from these three waves of data, with the quadratic estimate on the left and the corresponding lowess estimate on the right. The dots in the graphs on the right show the values of the psu-month cells that were used to estimate both the quadratic and lowess models. The results for all three waves and for both the parametric and non-parametric estimates all possess the inverted U-shape with a maximum near $p^* = 0.5$ that is predicted by the modal choice hypothesis.

6. Uncertainty and Wealth

[11] See Table 1 for the text of this question.

In Section 5, we developed theory and evidence which link variations in the degree of uncertainty about personal probabilities across individuals to the propensity of individuals to give exact or focal answers to survey questions about their probability beliefs in the HRS. In this section, we address the empirical link between uncertainty and financial choices that we discussed theoretically in Section 4. Specifically, we utilize responses to a large number of probability questions to construct indices of the propensity of individuals to give focal answers and then use these measures in regression equations explaining the share of household assets held in risky assets in 1998 and the growth of assets between 1992 and 1998.[12] According to the theory presented in Section 4, increased uncertainty leads individuals to behave more risk aversely. Thus, we predict that individuals who have a higher propensity to give focal answers will hold a smaller share of risky assets. We then examine the effect of focal answers on the growth of assets to determine whether increased uncertainty has a negative impact on economic status, either by leading individuals to choose portfolios with relatively low rates of return or because they have a lower propensity to save. (Unfortunately, we cannot distinguish between the effect on the rate of return, which is clearly related to the theory presented in this paper via risk aversion, and the savings propensity, whose relation to the theory is less clear.)

The first set of regressions, presented in Table 4, examines the determinants of the share of risky assets in the portfolios of 12,339 households in the 1998 wave of the HRS. Descriptive statistics for this sample are presented in Table 3. These households include 6,954 couple households, 5,318 single female households and 1,927 single male

[12] See Table 3 for the descriptive statistics of these samples.

households.[13] The dependent variable is defined as a ratio of the value of risky assets to total household gross worth, excluding the value of housing, where risky assets are defined as the sum of the values of (non-housing) real estate, business assets, stocks plus the full value of IRA accounts if the respondent reported that these accounts contain mostly stocks, one half the value of the IRA if it was reported that the IRA contained about equal amounts of stocks and interest-earning assets, and zero if the entire account was in interest-earning assets.

The independent variables of main interest for this paper are measures of the propensity of respondents to give focal answers to subjective probability questions in the HRS. We specify three variables, all of which are constructed in a similar way. The first is the fraction of answers to subjective probability questions in HRS-1998 that are focal (i.e., "0", "50" or "100") for up to 17 questions that were answered by each respondent.[14] In preliminary analysis, we found little difference in the effects of focal answers for husbands and wives in couple households so, for simplicity, we use the average fraction of focal answers for couple households in the regressions reported in this paper. This variable is used in the regressions reported in columns (1a) and (1b) of Table 4. In columns (2a) and (2b), focal answers are disaggregated into a propensity to give focal answers at the extremes of "0" and "100" and a propensity to give an intermediate answer of "50". Finally, in columns (3a) and (3b), the propensities are completely disaggregated with variables measuring the propensity to give "0", "50" or "100". On average, 68.7

[13] There are 14,209 household level observations in HRS-1998. Of these, our estimation sample of 12,339 was obtained by excluding 1231 cases in which the share of risky assets could not be defined because the household had either zero or missing gross worth and 630 had missing values on focal answers.

[14] Skip patterns caused variations in the number of probability questions that were asked of respondents and there were refusals to some questions.

percent of answers are focal, 54.0 percent are either “0” or “100”, 40.5 percent are “0”, 13.5 percent are “100”, and 14.8 percent are “50” (see Table 3).

The other independent variables are intended as controls. Since the composition of a portfolio is likely to vary with its magnitude, we control for the log of net worth and also enter a dummy variable for negative net worth. We also control for basic demographics including age of household head, marital status (single male or single female relative to married couple), and race. We estimate the equations with and without controls for education and immediate and delayed word recall to help control for cognitive capacity. We do so, in part, to determine how sensitive the estimated effect of the propensity to give focal answers is to cognitive capacity. The first three colums (marked “a”) do not control for education and word recall and the second three columns (marked “b”) include these variables.

The results reported in Table 4 show that households with a higher propensity to give focal answers have a significantly smaller fraction of risky assets in their portfolio, as predicted by the theory. The magnitude of this effect is moderate, with an elasticity of -0.138. The mean fraction of risky assets is 26.7 percent. Using the estimates reported in column (1a), an increase in the propensity to give focal answers from one standard deviation below the mean to one standard deviation above the mean would reduce the fraction of risky assets from 28.5 percent to 24.8 percent. These results are only slightly affected by controls for education and word recall and there is not much difference between the effects of the propensity to give focal answers at the extremes of “0” or “100” compared to the effects of a propensity to give a “50-50” answer. When the types of focal answers are completely disaggregated, the variables measuring the propensities to answer “0” or “50” remain negative and highly significant, but the propensity to give “100” becomes positive and insignificant.[15] Both log net worth and the existence of zero or negative net worth have extremely large and significant coefficients. None of the other demographic or cognitive control variables are significant.

The second set of regression of results uses a sample of 4174 households who were respondents in the original HRS cohort in 1992 who also responded to the fourth wave of HRS in 1998. These regressions estimate the effects of the propensity to give focal answers in the 1992 baseline interview on the annual rate of growth of household net worth between 1992 and 1998 (calculated by dividing the difference in log net worth in 1998 and 1992 by six). Note that these households who contained at least one individual aged 51-61 in 1992 are much younger (56.7, on average) than the HRS-1998 sample (68.1, on average) used in the regressions reported in Table 4. Descriptive statistics are given in Table 3. The design of these regressions is identical to those reported in Table 4.

On average, HRS households experienced 4.88 percent annual growth in net worth between 1992 and 1998. During this time the CPI rose at an annual rate of 3.78 percent, implying that the real annual rate of increase was 1.10 percent. The regressions in Table 5 show that the propensity to give focal answers has a very large and highly significant effect on the rate of growth of assets. Using the coefficient of –0.1984 on fraction of answers that are focal reported in Column (1a), the estimated elasticity of nominal growth with respect to focal propensity is –2.4 and the elasticity with respect to real growth is –10.8. An increase in the fraction of focal answers from one standard

[15] Since this sign reversal also occurs more strongly in the regressions reported in Table 5, we shall delay discussion of its interpretation until we report those results.

deviation below to one standard deviation above the mean implies a decrease in the nominal annual growth rate of net worth from 9.6 percent to 0 percent and from highly positive to negative real growth rates. In contrast to the results for risky assets discussed earlier, the addition of a control for education has a large impact on the estimated effect of focal propensity, cutting the coefficient size in half in column (1b) compared to column (1a). (The word recall variables remain insignificant.) The effect of controlling for education probably occurs because we have not controlled for household permanent income in these regressions. In a future version of this paper, we plan to do this. Assuming that the column (1b) estimates are more realistic, the implied elasticities are -1.2 for nominal growth and -5.2 for real growth.

The results reported in Table 5 are also much more sensitive to the way in which we parameterize the propensity to give focal answers than was true for the regressions in Table 4. In columns (2a) the propensity to give "0 or 100" and the propensity to give "50" are both significantly negative; in column (2b) controlling for education reduces the the coeffienct of "0 or 100" by one half and reduces the coefficient of "50" to insignificance. Finally, in columns (3a) and (3b) we see that negative effect of "0" answers becomes extremely large in absolute value while the effect of a "100" answer become significantly positive.

These results raise the possibility that we need to consider more than just the degree of respondent uncertainty in interpreting the relationship between focal answers to probability questions and financial choices. For example, it is possible that a propensity to give an answer of "100" may be related to "optimism" and that more optimistic people have higher rates of asset accumulation because the put more weight on the future.

Bassett and Lumsdaine (1999a) show that HRS respondents who think its more likely to be sunny tommorow, controlling for place and time of year, also tend to give more optimistic answers to other probability questions. We plan to explore what role optimism plays in a future version of this paper.

7. Summary and Conclusions

This paper was motivated by the question of how well older Americans will be able to take advantage of trends in both the private and public sectors which expand the scope of individual choice in financial decisionmaking. We have not attempted to address this question in general. Rather, we have focused attention on one aspect of decisionmaking – probabilistic thinking – which plays a crucial role in economic models of saving decisions and portfolio choice. To our knowledge, this is the first attempt to provide empirical evidence on the relationship between financial behavior and probabilistic thinking for a nationally representative sample of households. We are able to do so because the Health and Retirement Study asks a large number of subjective probability questions to its respondents which, in effect, creates a psychometric test of probabilistic thinking for the more than 20,000 HRS respondents.

One of the most prominent features of responses to these probabilistic questions is the large proportion – about 60 percent – of answers for which the probability that the event in question will occur is reported to be zero, fifty-fifty or one hundred percent rather than a more exact answer such as nine percent or seventy percent. While there are other possible interpretations of such heaping on focal answers, we emphasize the possibility that they reflect the respondent's uncertainty about the true value of the probability. We then refer to a large literature originating with the Ellsberg paradox

(Ellsberg 1961) in which it is hypothesized that individuals display "uncertainty aversion" that is inconsistent with conventional subjective expected utility theory. We suggest a new theoretical approach to this issue in which we are able to deal with uncertainty within the conventional SEU framework and show, within this framework, that uncertainty aversion is a consequence of risk aversion. This, in turn, implies that individuals who are more uncertain would tend to choose less risky portfolios and perhaps receive lower returns. We also note that our theory has clear implications for endogenous learning which may reduce uncertainty, but we do not pursue this point in this paper.

We then turn to an attempt to provide an explicit link between the theoretical concept of uncertainty and the characteristics of survey response to subjective probability questions. The first step in building this link is to propose a simple formal model of the determinants of an individual's subjective prior distribution of probabilities. This model is formulated as a heterogeneous probit model in which the hetereogeneity term reflects an individual's uncertainty about the true probability. A graphical representation of these prior density functions for varying levels of the true probability and degrees of uncertainty is presented. After examining these prior density functions, we propose an empirical hypothesis to descibe the determinants of whether a respondent gives a focal or exact answer to a probability question. We call this the "modal choice" hypothesis. According to this hypothesis, when asked to state a probability the respondent chooses the most likely probability, given his prior density, which is the mode of the distribution. This hypothesis has two testable empirical implications. First, it implies that when averaged across individuals the mean value of focal responses should vary in the same way as the mean value of exact responses. We find this to be broadly true in the HRS data. Second, the hypothesis implies that the probability of giving an exact (i.e., non-focal) answer to a probability question is a nonlinear, inverted U-shaped function of the level of the true probability. Using data from questions in three different waves of HRS about the likelihood that tommorow will be sunny, the empirical relationship conforms to this prediction.

In the final section of the paper, we construct a measure of the propensity of an individual to give focal answers by calculating the fraction of all probability questions that he or she was asked that received a focal response. We then estimate the effect of this propensity to the fraction of risky assets in a household's portfolio in 1998 and to the rate of growth of assets from 1992 to 1998. The propensity to give focal answers is found to have a highly significant negative effect on both the fraction of risky assets and the rate of growth of net worth. The magnitude of the effect is modest on risky asset holding but very large on the rate of growth of assets.

We believe that this paper provides clear evidence that there is considerable heterogeneity in the precision of probabilistic thinking in the population and that more precise probabilistic thinking leads individuals to be willing to take more risks and to enjoy higher growth in wealth. These results provide some justification for fears that have been expressed about expanding the scope for choice through individual accounts because significant portions of the population will be unable to exploit the benefits of choice. However, it would be a mistake in our view to jump to policy conclusions too quickly. In particular, we believe that it is important to explore the degree to which individuals reduce their uncertainty through experience with financial management and, as a consequence, become better able to manage their own affairs for their own benefit. We hope to pursue this notion in future research by exploiting the longitudinal information in the HRS.

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Table 1. Illustration of Probability Questions in HRS

1. Basic Introduction
Next I have some questions about how likely you think various events might be. When I ask a question I'd like for you to give me a number from 0 to 100, where "0" means that you think there is absolutely no chance , and "100" means that you think the event is absolutely sure to happen.

2. Sunny Day Warm Up Question
Let's try an example and start with the weather. What do you think are the chances that it will be sunny tomorrow?("0" means 'a 0 percent chance of sunny weather.' "100" means 'a 100 percent chance of sunny weather.' And you can say any number from 0 to 100.)

3. Types of Probability Questions

A. General Events

Social Security to Be Less Generous
How about the chances that Congress will change Social Security so that it becomes less generous than now?

Double Digit Inflation
And how about the chances that the U.S. economy will experience double-digit inflation sometime during the next 10 years or so?

B. Events with Personal Information

Survival Probability
(What is the percent chance) that you will live to be 75 or more?

Income Will Keep Up with Inflation
What do you think are the chances that your income will keep up with inflation for the next five years?

C. Events with Personal Control

Will leave inheritance
And what are the chances that you (and your (husband/wife/partner)) will leave an inheritance totaling $10,000 or more?

Will Work at Age 62
IF R IS WORKING FOR SOMEONE ELSE (NOT SELF-EMPLOYED): Thinking about work in general and not just your present job, what do you think the chances are that you will be working OTHERWISE: What do you think the chances are that you will be working full-time after you reach age 62?

Table 2. Fraction of Probability Questions Heaped and Refused
HRS 1998, Full Sample

Variable	Fraction of Heaped Answers to 0	to 100	0 or 100	50-50	Refusal Rate	Number of Cases
General Events						
Soc.Security To Be Less Genrous	0.08	0.17	0.25	0.22	0.05	4,608
Major Economic Depression	0.14	0.05	0.19	0.26	0.09	4,605
2-Digit Inflation in the U.S.	0.10	0.07	0.17	0.32	0.14	18,884
Subtotal	**0.10**	**0.08**	**0.19**	**0.29**	**0.12**	**28,097**
Events With Personal Information						
Live To Be 75	0.05	0.21	0.26	0.25	0.05	9,905
Live To Be 85	0.10	0.10	0.19	0.22	0.07	9,441
Receive Financial Help	0.74	0.02	0.75	0.04	0.02	18,884
Receive Inheritance	0.75	0.06	0.81	0.05	0.01	18,884
Income To Keep Up with Inflation	0.17	0.11	0.28	0.24	0.06	18,884
Lose Job	0.57	0.02	0.60	0.12	0.02	6,014
Find Job (In Case Losing One)	0.20	0.19	0.40	0.15	0.02	6,014
Find Job (If Looking For One)	0.11	0.14	0.25	0.20	0.00	844
Health to Limit Work	0.16	0.04	0.21	0.34	0.05	7,595
Subtotal	**0.40**	**0.09**	**0.49**	**0.16**	**0.04**	**96,465**

Table 2 (continued)

Variable	Fraction of Heaped Answers to 0	to 100	0 or 100	50-50	Refusal Rate	Number of Cases
Events With Personal Control						
Go To Nursing Home	0.56	0.01	0.57	0.15	0.07	11,744
Move in 2 Years	0.66	0.04	0.70	0.11	0.02	11,951
Give Financial Help	0.43	0.14	0.57	0.13	0.02	18,884
Leave Any Inheritance	0.65	0.10	0.76	0.08	0.11	3,907
Leave Inheritance >$10,000	0.18	0.47	0.65	0.09	0.03	18,884
Leave Inheritance >$100,000	0.31	0.29	0.60	0.10	0.05	15,541
Work Sometimes in the Future	0.77	0.02	0.80	0.05	0.01	11,237
Work at Age 62	0.27	0.20	0.46	0.14	0.01	5,433
Work at Age 65	0.25	0.11	0.35	0.16	0.02	4,042
Subtotal	**0.44**	**0.18**	**0.62**	**0.11**	**0.03**	**101,623**
Total	**0.39**	**0.13**	**0.52**	**0.15**	**0.05**	**226,185**

Table 3. Descriptive Statististics for the Samples used in Tables 4 and 5

	Table 4. Determinants of Fraction of Total Assets which are Risky		Table 5. Determinants of Average Yearly Percentage Change in Net Worth	
Number of Observations	12339		4174	
	Mean	Standard Deviation	Mean	Standard Deviation
Dependent Variable	0.2674	0.3371	0.0488	0.2373
Fraction of Answers which are Focal	0.6873	0.2183	0.5830	0.1564
Fraction of Answers at 0 or 1	0.5397	0.2427	0.3956	0.1582
Fraction of Answers at 0	0.4045	0.2341	0.2535	0.1327
Fraction of Answers at 1	0.1352	0.1312	0.1421	0.0928
Fraction of Answers at .5	0.1475	0.1218	0.1874	0.0835
Log of Net Worth	10.2178	3.2532	10.8156	1.8332
1998 Zero or Negative Net Worth	0.0541	0.2263		
Single Female	0.3279	0.4695	0.1770	0.3818
Single Male	0.1275	0.3335	0.0745	0.2626
Household Age	68.0715	10.4612	56.7053	4.7905
Hispanic	0.0512	0.2205	0.0630	0.2430
Black	0.1282	0.3343	0.1222	0.3275
Education	12.1850	2.9737	12.6174	2.6673
Immediate Word Recall	5.3768	1.8125	7.7522	2.2281
Delayed Word Recall	4.3212	2.0670	5.7183	2.4321

Note: Variables used in Table 4 use 1998 data, those used in Table 5 use 1992 data.

Table 4. Determinants of Share of Risky Assets in Portfolio in 1998*

	Share of Total Assets (Less Housing) which are Risky					
	(1a)	(2a)	(3a)	(1b)	(2b)	(3b)
Fraction of Answers which are Focal	**-0.0535**			**-0.0470**		
	(-4.215)			(-3.622)		
Fraction of Answers at 0 or 1		**-0.0512**			**-0.0441**	
		(-3.951)			(-3.334)	
Fraction of Answers at 0			**-0.0825**			**-0.0746**
			(-5.473)			(-4.747)
Fraction of Answers at 1			**0.0160**			**0.1467**
			(0.762)			(0.697)
Fraction of Answers at .5		**-0.0728**	**-0.0739**		**-0.0685**	**-0.0709**
		(-3.156)	(-3.208)		(-2.965)	(-3.066)
Log of 1998 Net Worth	**0.0817**	**0.0818**	**0.0792**	**0.0804**	**0.0805**	**0.0786**
	(63.614)	(63.472)	(55.277)	(58.117)	(58.078)	(52.919)
Zero or Negative Net Worth	**0.6330**	**0.6336**	**0.6128**	**0.6227**	**0.6232**	**0.6076**
	(36.508)	(36.521)	(33.896)	(34.871)	(34.888)	(33.073)
Single Female	**0.0126**	**0.0126**	**0.0135**	**0.0092**	**0.0090**	**0.0107**
	(2.131)	(2.117)	(2.281)	(1.512)	(1.486)	(1.759)
Single Male	**0.0072**	**0.0069**	**0.0057**	**0.0064**	**0.0061**	**0.0052**
	(0.908)	(0.875)	(0.715)	(0.808)	(0.769)	(0.655)
Household Age, 1998	**-0.0034**	**-0.0035**	**-0.0031**	**-0.0032**	**-0.0033**	**-0.0029**
	(-12.826)	(-12.622)	(-10.388)	(-10.999)	(-10.955)	(-9.419)
Hispanic	**0.0043**	**0.0038**	**0.0059**	**0.0097**	**0.0092**	**0.0094**
	(0.367)	(0.319)	(0.501)	(0.806)	(0.763)	(0.776)
Black	**0.0027**	**0.0023**	**0.0029**	**0.0045**	**0.0041**	**0.0042**
	(0.346)	(0.290)	(0.371)	(0.570)	(0.512)	(0.532)
Education				**0.0016**	**0.0017**	**0.0010**
				(1.586)	(1.623)	(0.972)
Immediate Word Recall				**0.0026**	**0.0026**	**0.0022**
				(1.056)	(1.060)	(0.914)
Delayed Word Recall				**0.0003**	**0.0003**	**0.0002**
				(0.135)	(0.153)	(0.103)
Constant	**-0.3360**	**-0.3304**	**-0.3287**	**-0.3786**	**-0.3731**	**-0.3605**
	(-14.737)	(-14.071)	(-14.006)	(-13.623)	(-13.225)	(-12.687)
Number of Observations	**12339**	**12339**	**12339**	**12339**	**12339**	**12339**
Adjusted R-Squared	**0.3119**	**0.3119**	**0.3127**	**0.3121**	**0.3121**	**0.3128**

*t-values in parenthesis

Table 5. Determinants of Growth of Assets from 1992 to 1998*

	Yearly Percent Change in Net Worth					
	(1a)	(2a)	(3a)	(1b)	(2b)	(3b)
Fraction of Answers which are Focal	**-0.1984**			**-0.0975**		
	(-8.727)			(-4.132)		
Fraction of Answers at 0 or 1		**-0.2077**			**-0.1093**	
		(-8.786)			(-4.474)	
Fraction of Answers at 0			**-0.5202**			**-0.3993**
			(-18.007)			(-12.577)
Fraction of Answers at 1			**0.3159**			**0.2914**
			(8.434)			(7.772)
Fraction of Answers at .5		**-0.1466**	**-0.0962**		**-0.0310**	**-0.0236**
		(-3.429)	(-2.327)		(-0.725)	(-0.563)
Log of 1992 Net Worth	**-0.0478**	**-0.0479**	**-0.0616**	**-0.0584**	**-0.0585**	**-0.0666**
	(-23.669)	(-23.703)	(-29.343)	(-27.450)	(-27.504)	(-30.838)
Single Female	**-0.0751**	**-0.0749**	**-0.0743**	**-0.0961**	**-0.0960**	**-0.0892**
	(-7.943)	(7.923)	(-8.145)	(-10.110)	(-10.099)	(-9.589)
Single Male	**-0.0343**	**-0.0332**	**-0.0357**	**-0.0534**	**-0.0520**	**-0.0489**
	(-2.601)	(-2.513)	(-2.797)	(-4.037)	(-3.923)	(-3.773)
Household Age, 1992	**-0.0012**	**-0.0011**	**0.0023**	**-0.0003**	**-0.0001**	**0.0023**
	(-1.684)	(-1.542)	(3.154)	(-0.370)	(-0.197)	(3.200)
Hispanic	**-0.1274**	**-0.1262**	**-0.1033**	**-0.0695**	**-0.0678**	**-0.0674**
	(-8.801)	(-8.709)	(-7.359)	(-4.692)	(-4.577)	(-4.652)
Black	**-0.0856**	**-0.0840**	**-0.0795**	**-0.0774**	**-0.0755**	**-0.0751**
	(-7.864)	(-7.681)	(-7.535)	(-7.176)	(-6.961)	(-7.083)
Education				**0.0208**	**0.0209**	**0.0147**
				(13.340)	(13.367)	(9.253)
Immediate Word Recall				**0.0005**	**0.0006**	**-0.0009**
				(0.199)	(0.245)	(-0.371)
Delayed Word Recall				**0.0016**	**0.0016**	**0.0014**
				(0.783)	(0.740)	(0.693)
Constant	**0.7863**	**0.7752**	**0.7204**	**0.5109**	**0.4961**	**0.5457**
	(17.002)	(16.529)	(15.887)	(10.070)	(9.662)	(10.846)
Number of Observations	**4174**	**4174**	**4174**	**4131**	**4131**	**4131**
Adjusted R-Squared	**0.1371**	**0.1373**	**0.197**	**0.1750**	**0.1755**	**0.2121**

*t-values in parenthesis

Figure 1. Histograms of Probability Responses in HRS

A. General Events

Social Security less generous

Double digit inflation

B. Events with Personal Information

Survival to 75

Income increase faster than inflation

C. Events with Personal Control

Leave inheritance

Work at age 62

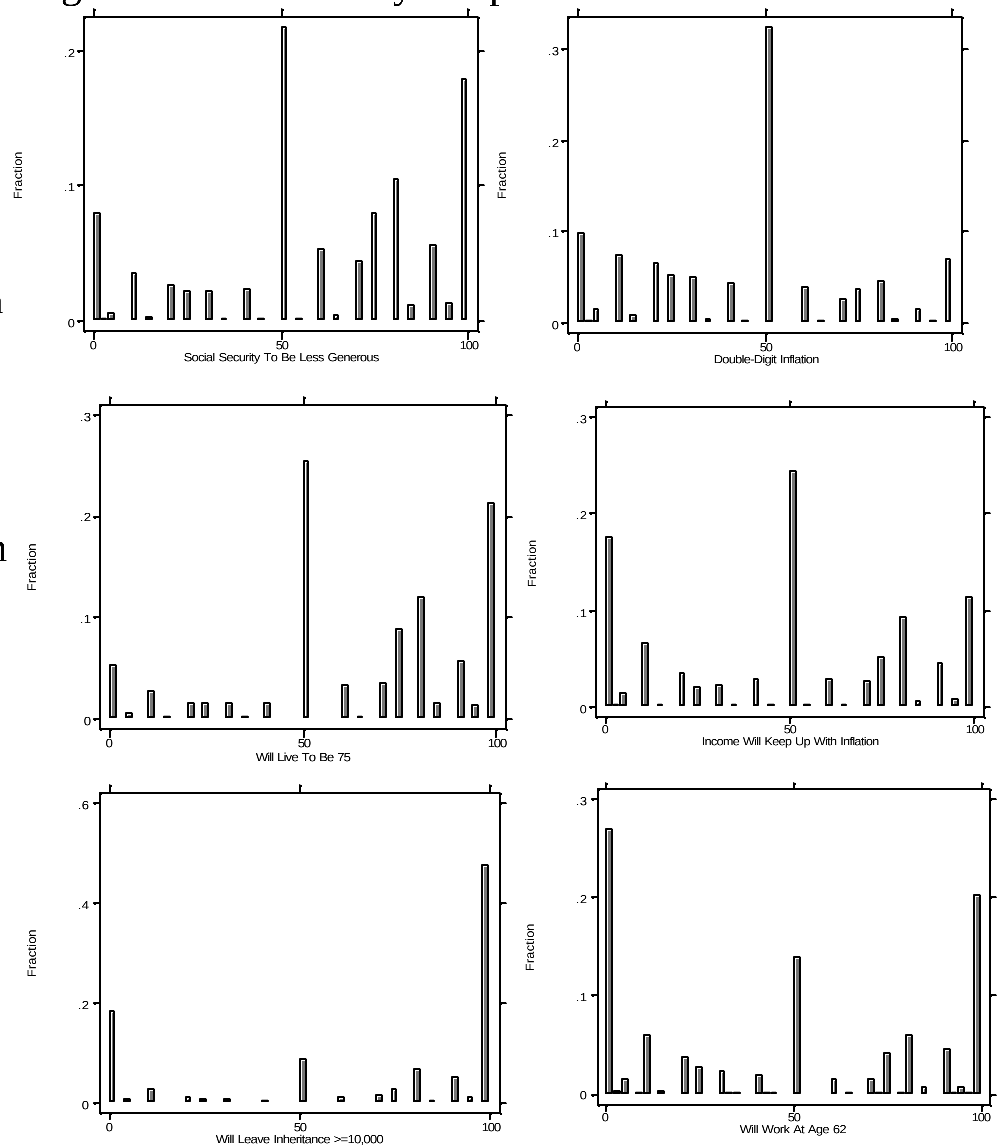


Figure 2. Distribution of Prior Probabilities with Varying Expected Values and Precision.

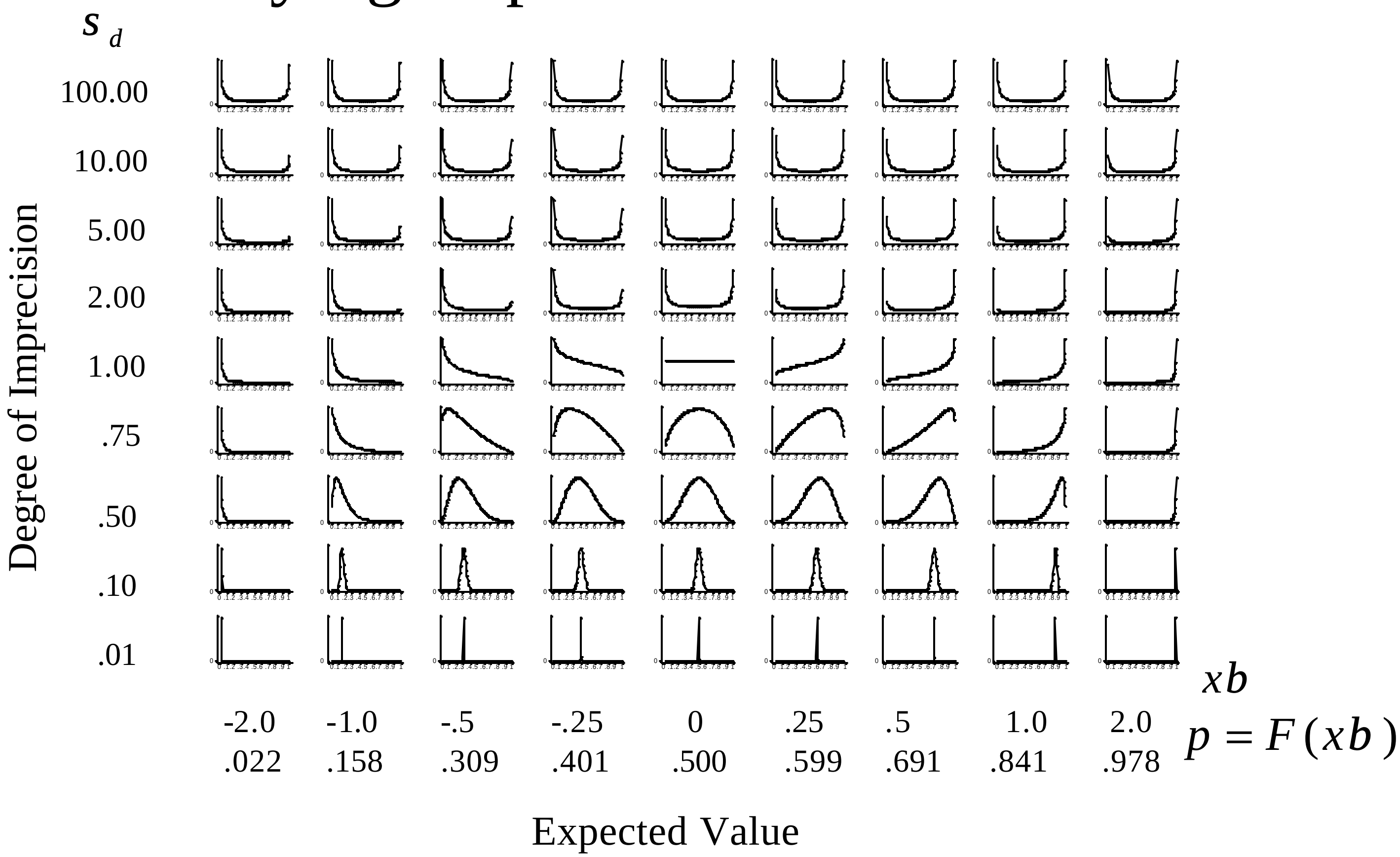


Figure 3. Relation Between Prior Distribution and Heaping under Modal Choice Hypothesis.

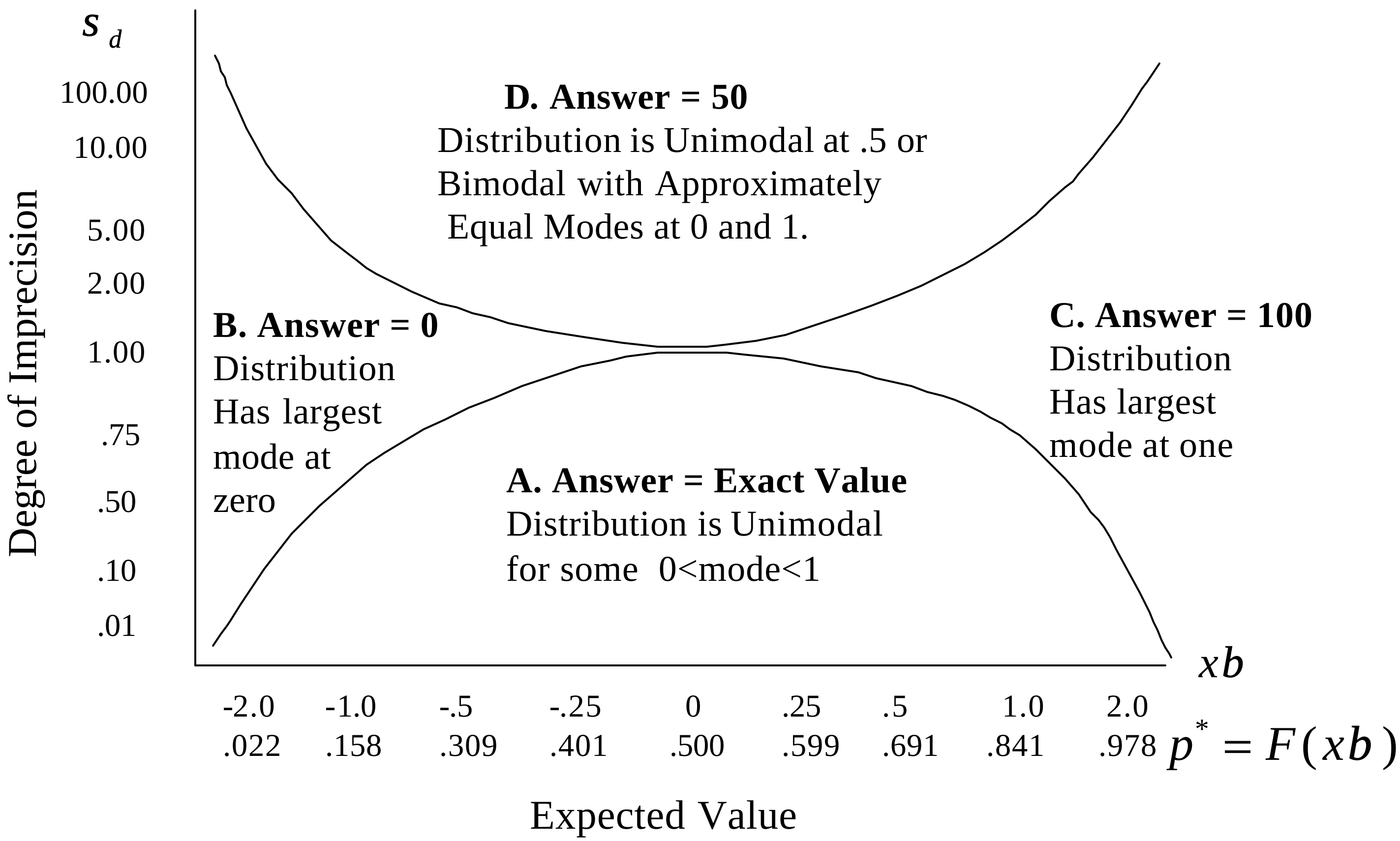
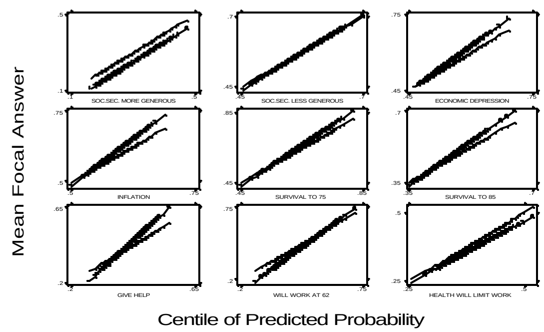
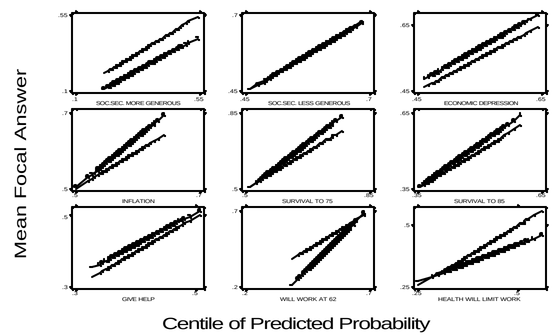


Figure 4. Mean Focal Answers vs. Predicted Probabilities

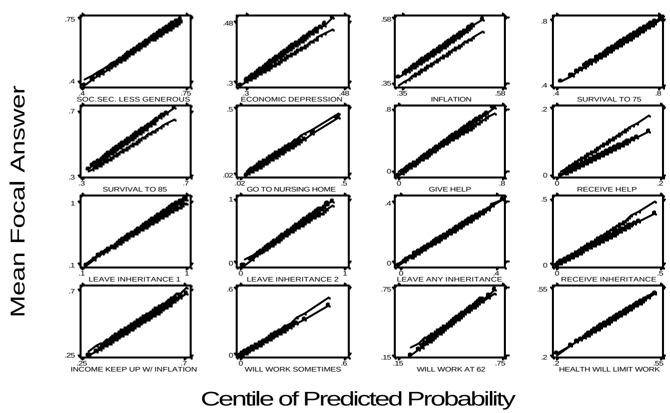
A. Prediction Based on Full Sample, 1992



B. Prediction Based on Sample with Exact Answers, 1992



C. Prediction Based on Full Sample, 1998



D. Prediction Based on Sample with Exact Answers, 1998

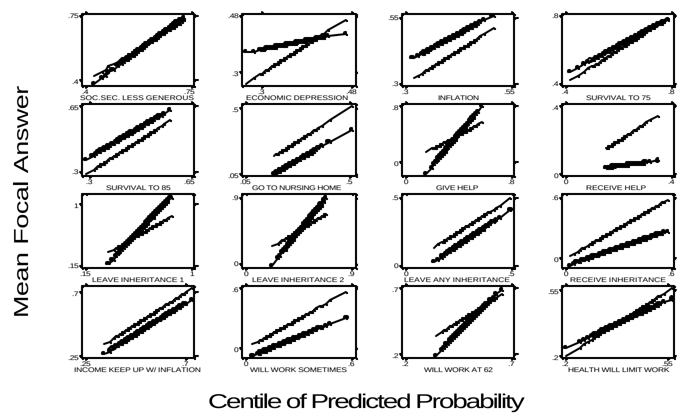
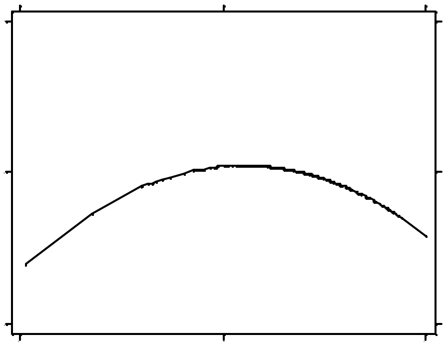
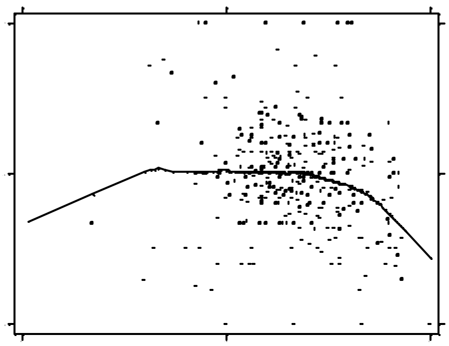


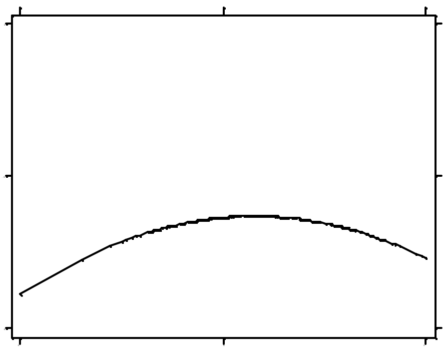
Figure 5. Sunny Question.
Mean probability by sample stratum and month of the interview (horizontal axis), and fraction of exact answers (vertical axis). Restricted to cells with at least 3 observations



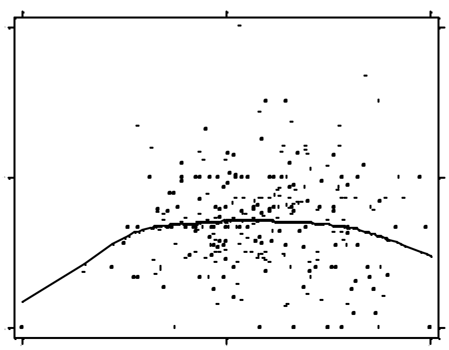
Quadratic Model
HRS 1994



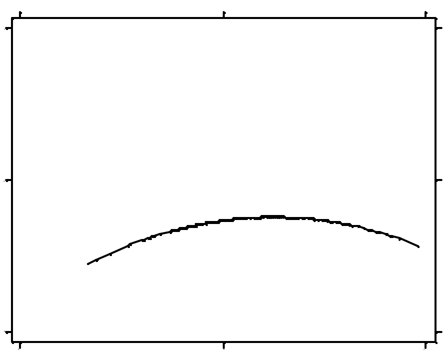
Nonparametric Model & Scatterplot
HRS 1994



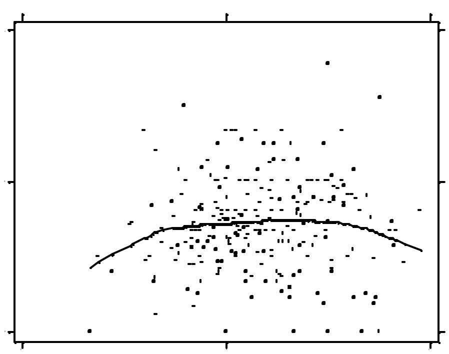
Quadratic Model
AHEAD 1993



Nonparametric Model & Scatterplot
AHEAD 1993



Quadratic Model
AHEAD 1995



Nonparametric Model & Scatterplot
AHEAD 1995