Multi-issue Bargaining and Linked Games:

Ricardo Revisited or No Pain No Gain

Ignatius J. Horstmann
University of Western Ontario

James R. Markusen
University of Colorado, NBER and CEPR

Jack Robles
University of Colorado

There has been much discussion about what issues should be included and negotiated together in international "trade" negotiations. Different countries, firms, and activist groups have quite different views as to which items should be treated simultaneously, including trade, investment, environment, labor, and even human rights issues. Some arguments for or against linking have been made on moral grounds, and economic analysis seems lacking. This paper analyzes two countries bargaining over two issues, and contrasts outcomes when the issues are negotiated separately to when they are negotiated simultaneously. A key concept is referred to as "comparative interest", analogous to Ricardian comparative advantage. We provide general results and note in particular situations where a county can benefit by agreeing to include an agenda item for which, viewed by itself, the country cannot possibly receive a positive payoff.

First Draft, May 2000

1. Introduction

Considerable controversy exists over what issues should and should not be included in multilateral trade negotiations. Some US and European groups, for example, want environmental and labor standards included with trade negotiations. Other groups from these same countries want to link trade with investment liberalization and intellectual-property protection. Some developing countries want competition policy included with any negotiations on investment liberalization. But often a linked negotiation desired by one group is opposed by some other group, and some writers oppose any linkage (especially trade with environment/labor standards) "on principle". The latter see any European or US attempt to link trade and environment in negotiations with developing countries as morally wrong, and simply assume that the developing countries must be worse off with such a linkage (an assumption generally shared by those developing countries).

This paper attempts to provide a bargaining-theoretic framework for understanding who gains and loses by linking games. We begin by considering linking two issues, one of which is more important to one country than the other and vice versa for the other issue, generating a pattern of "comparative interest". This is analogous to Ricardian comparative advantage but somewhat more complicated in that negative payoffs are an important part of the problem. We solve for the bargaining outcome in the linked game versus when the two are negotiated and implemented by entirely disjoint sets of bureaucrats.

General results establish that linking is Pareto improving in a wide range of circumstances, including situations in which one (or both players) cannot receive a positive payoff in one game (e.g., a country should not necessarily refuse to add an item to the agenda just because it cannot receive any positive payoff from that game viewed in isolation). The conditions for Pareto improvement in the linked game have an analogy to the existence of comparative advantage in a Ricardian trade model.

In general, sufficient conditions for linking to be Pareto improving are (A) that a pattern of "comparative interest" exists, (B) that any negative payoff for a country occurs in the game in which the country has a comparative disinterest and (C) that the maximum payoff in the game of comparative disinterest is non-negative. When two games are of different "sizes", a concept we make rigorous, the player with the comparative interest in the larger game often takes all the surplus from linking.

These results carry a clear policy message. Countries should not refuse to include an issue in negotiations simply because the country cannot receive a positive payoff from that issue viewed in isolation. The key question is whether or not the issue yielding negative payoffs is an issue of comparative interest or disinterest. If the latter is the case, the country can generally gain, or at least be no worse off, by agreeing to include that issue.

2. Model and Notation

There are two players and two issues. Players are denoted 1 and 2 and issues are also denoted 1 and 2. The payoff frontier in utility space on each issue (also referred to as a game) is linear. Let U_i denote the utility of player i. Utility payoffs in game i are given by:

$$(1) \qquad U_i = a_i - b_i U_j \qquad i,j = 1,2 \qquad a_i, b_i > 0$$

We assume a pattern of "comparative interest" analogous to Ricardian comparative advantage. We label the two games such that player 1 has a comparative interest in game 1 and player 2 has a comparative interest in game 2. Formally, this is given by:

$$(2) \qquad -\left[\frac{dU_1}{dU_2}\right]_{game\,1} > -\left[\frac{dU_1}{dU_2}\right]_{game\,2} \qquad or \qquad b_1 > \frac{1}{b_2}$$

We allow for a possible cost to an agent in the agent's game of comparative disinterest, allowing for a maximum payoff that is non-positive in that game. So the general form of (1) is given by (3).

$$(3) \qquad U_i = a_i - b_i(U_j + c_i) \qquad i,j = 1,2 \qquad b_i, c_i > 0 \qquad U_j \geq -c_i, \quad U_i \geq 0$$

Under the restriction on U_j that it not be less than $-c_i$, and U_i is non-negative, we have

Player i's maximum payoff from game i:	a_i
Player i's minimum payoff from game i:	0
Player j's maximum payoff from game i:	$a_i/b_i - c_i$
Player j's minimum payoff from game i:	$-c_i$

This basic model is shown in Figures 1A and 2A. Figures 1 shows the case where the cost parameters c_i are zero. The analogy to Ricardian comparative advantage here is apparent. Figure 2 shows the case where the cost parameters are large, such that the maximum payoff to an agent in his/her game of comparative disinterest is strictly negative.

We will assume a Nash bargaining solution to the model, in which each agent's threat point in unlinked or linked games is zero; that is, an agent can walk away from the table. Thus the solution to a game maximizes the product of the agents' utilities: Max U_1U_2 . We like to think of this as the limit of the Rubenstein alternating-offers model as discounting goes to zero (the discount factor goes to 1). This may aid in interpreting certain findings concerning the distribution of gains from linking presented later in the paper.

Figure 2 gives a simple case of two symmetric games in which the cost parameters are zero. The solution to each unlinked game is the mid-point of the utility frontier, giving a total payoff from the two unlinked games by vector addition as shown. The payoff frontier for the linked games is found by a simple graphically technique familiar to all trade economists. Equilibrium in the linked game is at the vertex, giving clear welfare gains to both players over the sum of payoffs in the two unlinked games. The source of the gains is familiar from Ricardian comparative advantage, and akin to gains from trade through specialization according to comparative advantage. By linking the games together, each agent can trade off a share of the comparative disinterest game for an increased share of the comparative interest game.

Figure 3 extends to argument by adding costs c_1 and c_2 which are positive but relatively small. Thus an agent can now receive a negative payment in the agent's game of comparative disinterest. The important point for theory and for policy is that the addition of these small, positive c_i's has no effect on the total payoffs to the two unlinked games, since agent's threat points are zero. But payoffs *increase* the gains in the linked game.[1] The ability to gain by trading off shares of your game of comparative disinterest against gains in your game of comparative interest extends to taking losses in the former. The point is not whether or not you take losses in a game, but whether $1 of loss gets you more than $1 of gain in the other game.

Figures 4 makes the point that linking can be mutually beneficial even if each player can only achieve a strictly negative payoff in his/her game of comparative disinterest. In this case, the total payoffs from the two unlinked games are zero to each agent: in each game, one agent refuses to play.

Figure 5 alters the thrust of the argument to consider what sorts of circumstances preclude Pareto improving gains from linking. In the top panel, we reverse our earlier assumptions, and here assume that each player's game of comparative interest is the game in which the player receives a non-positive payoff. We could call this "negative comparative interest". There is no gain from linking, and the payoffs in both the linked and unlinked games are zero.

The bottom panel of Figure 5 shows a case where the c_i's are so large as to preclude gains from linking. Here it is so costly to add an agent's game of comparative disinterest that this cost outweighs potential gains in the game of comparative interest. The case of negative comparative interest was implicitly ruled out by our earlier assumptions, but the lower panel of Figure 5 has not been rule out to this point.

---

[1] Technically, the intersection of the linked-game frontier with the positive orthant is at a greater value of U_1 than the sum of the intersections of the two unlinked games with the positive orthant.

1. Conditions for Pareto Improvement from Linking

We can now turn to a more formal analysis and present *sufficient* conditions for linking the two games to be Pareto improving.

Proposition 1

If: (1) there exists a pattern of comparative interest, $b_1 > 1/b_2$

(2) the minimum payoff to an agent in his/her game of comparative interest is non-negative

(3) the maximum payoff to an agent in his/her game of comparative disinterest is non-negative

Then: the solution to the linked game is Pareto superior to the combined payoffs to the two games negotiated separately.

Assumption (2) requres that a_1 and a_2 are strictly greater than zero.. Assumption 3 implies the restritions that:

(4) $a_2/b_2 \ \& \ c_2 \ \$ \ 0, \qquad a_1/b_1 \ \& \ c_1 \ \$ \ 0$

The top panel of Figure 5 violates assumption 2. The bottom panel violates assumption 3.

Under the assumptions noted, agent 1's payoffs from the two games negotiated separately is given as the maximum of zero and the sum of agent1's maximum payoffs in the positive orthant of payoff space for each game, divided by two. In order to easy the nightmare of notation, refer to Figure 6 where we have shown a situation where assumptions (2) and (3) hold as strict inequalities (i.e., in each case replace “non-negative” with “positive”). Here we present a proof of the proposition, referring to an appendix where a lemma is proved.

The sum of payoffs to player 1 in the two unlinked games is given by:

$$(5) \qquad U_1^{/} = \left[a_1 + a_2/b_2 - c_2 - c_1 b_1\right]/2 \geq 0$$

Assumptions (2) and (3) ensure that this value is non-negative, hence it is greater than or equal to the default value of walking away from one or both games.

In the linked game, the intersection of the payoff frontier with the U_1 axis is given by the formula shown in Figure 6. Since this frontier is concave under our assumptions, then the *minimum* payoff that agent 1 can obtain in the linked game is given by half that value.[2] Thus the minimum payoff for agent 1 in the linked game is:

$$(6) \qquad U_1^{*} = \left[a_1 + a_2/b_2 - c_2 - c_1/b_2\right]/2 \geq 0$$

which is non-negative under assumptions (1) and (3). Subtracting (5) from (6), we have the games from linking.

$$(7) \qquad U_1^{*} - U_1^{/} = c_1(b_1 - 1/b_2)/2 \geq 0 \qquad > 0 \quad if \;\; c > 0$$

since $b_1 > 1/b_2$ by assumption (1). Thus if player 1 receives the *minimum* payoff noted in the linked game (5), player 1 is strictly better off or at least no worse off ($c_1 = 0$) by linking.

Player 1 receives the minimum payoff noted above when the bargaining solution lies on the *X2* locus shown in Figure 6. But if the solution lies on this locus, it must be on the mid-point of the locus between the U_1 and U_2 axes. This in turn allows us to solve for U_2, when U_1 takes on its *minimum* value: $U_2 = U_1 b_2$.

---

[2]This is proved in Lemma 1 in Appendix 1. The minimum payoff noted in (6) occurs when the U_1 axis-value of point *XX* in Figure 6 is less than half the vlaue of U_1 where the linked-game frontier crosses the U_1 axis. Equivalently, it occurs when the Nash "utility function" $U_1 U_2$ is tangent on the *X2* locus. Later, we will refer to this as a situation where game 1 is "small" relative to game 2.

$$(8) \qquad U_2^{*} = b_2 U_1^{*} = \left[a_1 b_2 + a_2 - c_2 b_2 - c_1\right]/2 \geq 0$$

The sum of payoffs to player 2 in the two unlinked games is given by the corresponding equation to that for player 1 in (5).

$$(9) \qquad U_2' = \left[a_2 + a_1/b_1 - c_1 - c_2 b_2\right]/2 \geq 0$$

Subtracting (9) from (8), we get:

$$(10) \qquad U_2^{*} - U_2' = (a_1 b_2 - a_1/b_1)/2 > 0 \qquad \textit{since} \;\; b_2 > 1/b_1 \;\; \textit{by} \;\; (1)$$

Thus if player 1 gets the minimum gain, player 1 is no worse off from linking and player 2 is strictly better off. This occurs when the Nash utility function U_1U_2 is tangent on the *X2* locus in Figure 6 as noted in a footnote. An equivalent result obviously holds when the equilibrium lies on the *X1* locus in Figure 6. Finally, when the equilibrium lies at point *XX* in Figure 6 and the slope of U_1U_2 lies strictly between the slopes of *X2* and *X1*, then both players are strictly better off.[3] This completes the proof of Proposition 1.

[3] This occurs when point *XX* in Figure 6 has a U_1 value greater than half the value of U_1 where the frontier crosses the U_1 axis (equation (6) above), and similarly for the U_2 value of *XX*. Note that this must be the case if the games are symmetric as in Figures 2 and 3.

4. Distribution of Gains from Linking

Having established that assumptions (1), (2), and (3) of Proposition 1 are sufficient to ensure that linking is Pareto improving, it is of interest to establish some conditions under which both players receive positive gains and conditions under which all of the surplus created by linking is captured by one player.

The key concept is the "size" of a game. This is defined and shown in Figure 7, where we show the frontier of the linked game. Game 1 is defined as "small" if the game 1 triangle as defined in Figure 7 fits inside the game 2 triangle similarly defined. In such a situation, player 1 does not view an increased share of game 1 (his/her game of comparative interest) as preferable to an increased share of game 2.

Proposition 2 Given assumptions (1), (2), and (3) of Proposition 1

A: If: c_1 and $c_2 > 0$

Then: linking improves the welfare of both agents

B: If: $c_1 = c_2 = 0$ and game 1 is small ($a_2/b_2 > a_1$, $a_2 > a_1/b_1$)

Then: player 2 captures all the surplus from linking

C: If: $c_1 = 0$ and $c_2 > 0$ and game 1 is small ($a_2/b_2 > a_1 - c_2$, $a_2 - c_1 > a_1/b_1$)

Then: player 2 captures all the surplus from linking

As in the case of Proposition 1, we present here a relatively informal proof of Proposition 2 and leave the more formal proof to an appendix.

Part (A) of Proposition 2 has essentially already been proved. Equation (7) gives the *minimum* gain to player 1 from linking, and this is strictly positive if $c_1 > 0$. An equivalent result holds for player 2. This establishes part (A).

Parts (B) and (C) also follows directly from what we have already shown. If $c_1 = 0$, then the minimum gain to player 1 from linking is zero (equation (7)). This minimum gain occurs when the equilibrium is on the *X2* locus in Figure 6 as we noted earlier. This in turn requires than the point *XX* is at a value of U_1 equal to or less than half the value where *X2* crosses the U_1 axis. But that happens when game 1 is small in the sense defined in Figure 7. That establishes parts (B) and (C).

Figure 8A illustrates a "knife edged" case where game 1 just satisfies the definition of small as an equality, $c_1 = c_2 = 0$, and player 2 captures all the gains. In the linked game, player 1 gets all of game 1, but that is exactly equal to what player would have received in total from the two unlinked games. Player 2 receives all of game 2 in the linked game which, however, is larger than what player 2 would have received in total from the unlinked games.

We are unsure of the correct intuition behind the result of Proposition 2B and Figure 8A. Think of the Nash solution as the Rubenstein problem of alternating offers as discounting goes to zero (the discount factor goes to 1). Shares in games 1 and 2 are equivalent from player 1's point of view. but not to player 2. Player 1 has no interest in holding up an agreement on game 2 to extract a larger share in game 1. Player 2 on the other hand has a definite interest in holding up an agreement on game 1 to extract a larger share in game 2. In the (limit of the) Ruberstein solution, player 2 is able to attract all of the surplus from linking.

Figure 8B presents an (interior) case in which game 1 is strictly small and again $c_1 = c_2 = 0$. Player 2 gets all the surplus.

Figure 8C is another attempt at intuition, looking explicitly at the Rubenstein solution when game 1 is small. Let the maximum value of U_1 on the Pareto frontier be anchored at $U_1 =$

1. The maximum value of U_2 if the upper (flatter) segment of the frontier were extended to the U_2 axis is given by U_{2A}.

Let δ denote the discount factor (one over one plus the discount rate). Assume that the game ends at time T and that player 1 makes the last offer, demanding and receiving x, which is a point between zero and 1 on the U_1 axis. At time T-1 player 2 makes an offer, denoted y, also defined as a point on the U_1 axis. Player 2 will want to extract as much surplus as possible subject to player 1 not wanting to delay resolution. Thus player 2's offer should be $y = \delta x$. At time T-2 player 1 makes an offer and similarly tries to extract as much surplus as possible without player 2 rejecting and delaying the game another period. If player 2 delays until T-1, the present value at T-2 of what he/she can expect to get at T-1 is given by $\delta U_{2A}(1 - y)$. Thus player 1's demand x should satisfy $U_{2A}((1 - x) = \delta U_{2A}(1 - y)$. This is summarized as follows:

T:	Player 1 demands and receives x	
T - 1:	Player 2 offers y	
T - 1:	$y = \delta x$	(player 2's best offer at T-1)
T - 2:	$U_{2A}((1 - x) = \delta U_{2A}(1 - y)$	(player 1's best offer at T-2)

The last two equations have two unknowns, and the solution is given by:

$$(11) \qquad x = \frac{1}{1 + \delta} \quad \rightarrow \quad x = \frac{1}{2} \quad as \quad \delta \rightarrow 1$$

The important point for our purposes is that the solution in (11) does not depend on U_{2A}. We could slide U_{2A} out to the right holding the maximum value of U_1 anchored at $U_1 = 1$, and the solution value for x would not change. At time T-2, what player 1 has to offer player 2 to prevent delay is independent of U_{2A}. Thus as U_{2A} gets bigger, all the extra gains are captured by 2.

It should be emphasized again that Propositions 1 and 2 present sufficient conditions. Figures 9A, 9B, 10A and 10B present some other outcomes that are not addressed by these Propositions. Figures 9A and 9B shows a case where linking is Pareto improving but player 2 captures all the gains. The reason is the large value of c_2, and is not related to “smallness”. Figure 9B serves to emphasize that strictly negative payoffs for player 1 in game 2 is not sufficient for linking to lower player 1's welfare.

Figure 10A makes the point that player 1 may have strictly positive gains from linking even though game 2 has no positive payoff for player 1. Figure 10B shows a case where linking worsens player 1's welfare due to a large negative value of c_2. This last case violates assumption (3) of Proposition 1 since player 1's maximum payoff in game 2 is strictly negative in Figure 10B.

5. Summary and Conclusions

This paper is motivated by the fact that several prominent economists, as well as many public figures, have made questionable comments about the vice of linking together different issues in international "trade" negotiations. In particular, there seems to be a notion that countries (meaning poor countries) should never be asked to include agenda items for which the country cannot receive a positive payoff for that item viewed in isolation.

Our analysis suggests that linking is likely to be a virtue rather than a vice. An agent or country should not refuse to include an agenda item simply because it cannot, by itself, yield a positive payoff to that agent. The important question is whether or not the item yielding the negative payoffs is an item of comparative interest or disinterest. If the negative-payoff agenda item is an issue of comparative disinterest, then the agent can typically gain by linking it to an item of comparative interest which yields positive payoffs.

We also presented an analysis of the distribution of the surplus created by linking. When one item/game is "small" relative to the other in a well-defined sense, the agent with the comparative interest in the large item may capture all the gains.

The US- Canada free trade negotiations and later the NAFTA negotiations may provide an example. The US wanted to include tough provisions on services and investment, while Canada and Mexico preferred to stick with goods only. If Canada and Mexico had not agreed to include services and investment, our guess is that the negotiations would have failed, since there was little support (rightly or wrongly) in the US for free trade in goods, especially with Mexico. By agreeing to include issues in which Mexico and Canada perceived (rightly or wrongly) that they had nothing to gain, those two countries improved their welfare through trade concessions that were worth more than what they gave up on services and investment. We close by conjecturing that further progress on multilateral negotiations and the collapse of the Seattle talks in particular may be due to a misplaced view by some countries concerning accepting issues on which they cannot gain viewing those issues in isolation. Unfortunately, this attitude is encouraged by some NGOs and other activists, and even by a few economists.

REFERENCES

Busch, Lutz-Alexander and Ignatius J. Horstmann (1997), "Bargaining Frictions, Bargaining Procedures and Implied Costs in Multiple-Issue Bargaining", Economica 64, 669-80.

Fershtman, Chaim (1990), "The Importance of the Agenda in Bargaining", Games and Economic Behavior 2, 224-238.

Nash, John (1950), "The Bargaining Problem", Econometrica 18, 155-162.

Rubenstein, Ariel (1982), " Perfect Equilibrium in a Bargaining Model", Econometrica 50, 97-110.

APPENDIX 1

Lemma 1

If $(U_1, U_2) = (X, 0)$ is part of a convex bargaining set, with zero for defection values, then the lowest payoff the Nash Bargaining Solution (NBS) can assign to player one is $X/2$.

Proof

Let $(f(U_2), U_2)$ be the Pareto frontier of the bargaining set.

Let $X = f(0)$, and let Y solve $X/2 = f(Y)$.

The NBS (U_1^*, U_2^*) is the solution to $max\ U_1 U_2\ \ subject\ to\ \ U_1 = f(U_2)$

Let $g(U_2) = -(X/(2Y))U_2 + X$. Then $U_1 = g(U_2)$ is the line through $(X, 0)$ and $(X/2, Y)$.

The NBS of $[(U_1, U_2) | U_1 \leq g(U_2)]\ \ is\ \ (X/2, Y)$ since

$arg\,max\ U_2 g(U_2) = arg\,max\ (-X/2, Y)U_2^2 + XU_2\ \ with\ \ FOC\ \ (X/Y)U_2 + X = 0$

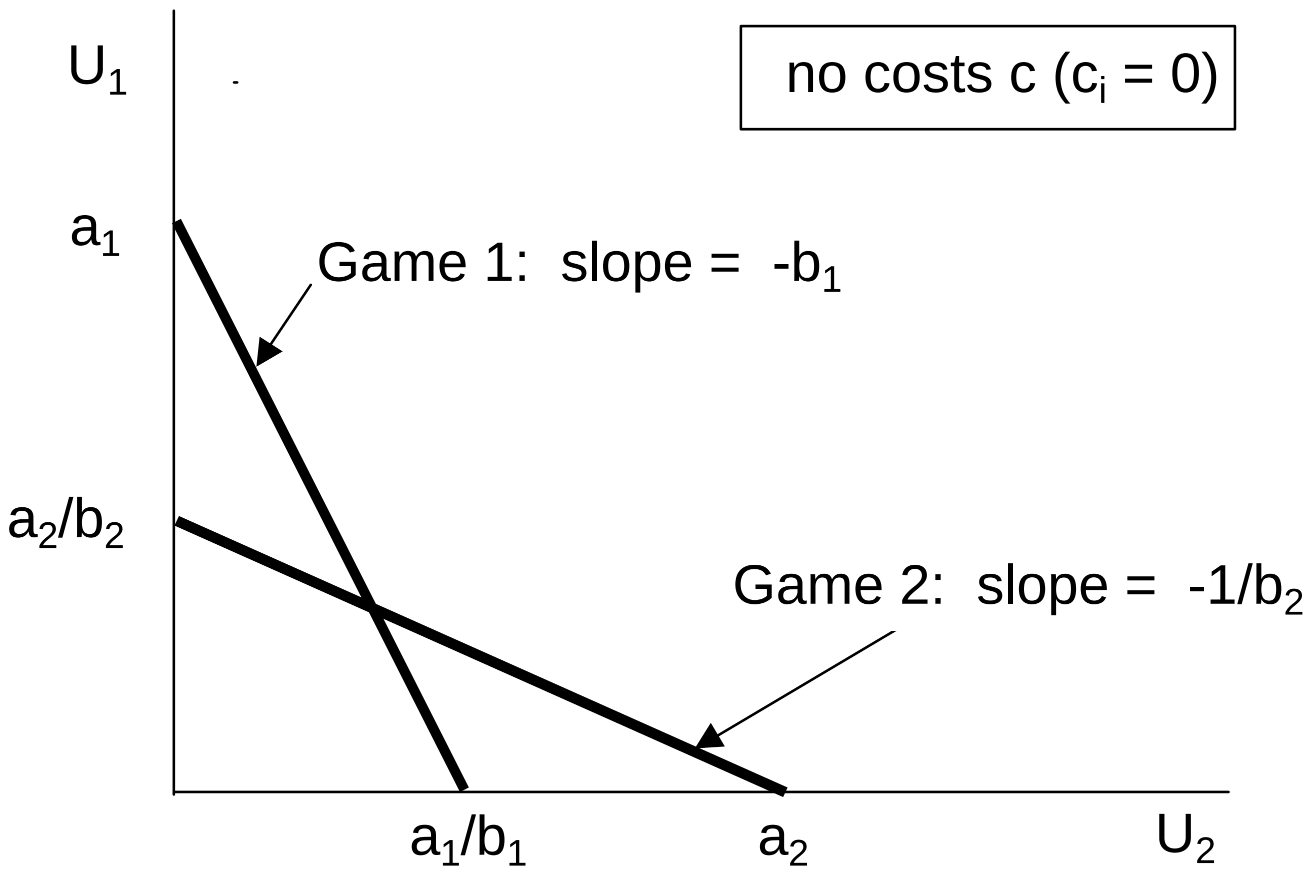
Hence $(X/2)Y \geq U_2 g(U_2)$ with equality only at $U_2 = Y$.

Since f( ) is a decreasing and concave function, we know that $U_2 > Y \Rightarrow g(U_2) \geq f(U_2)$

Putting the two inequalities together, we get that $U_2 > Y$ implies that

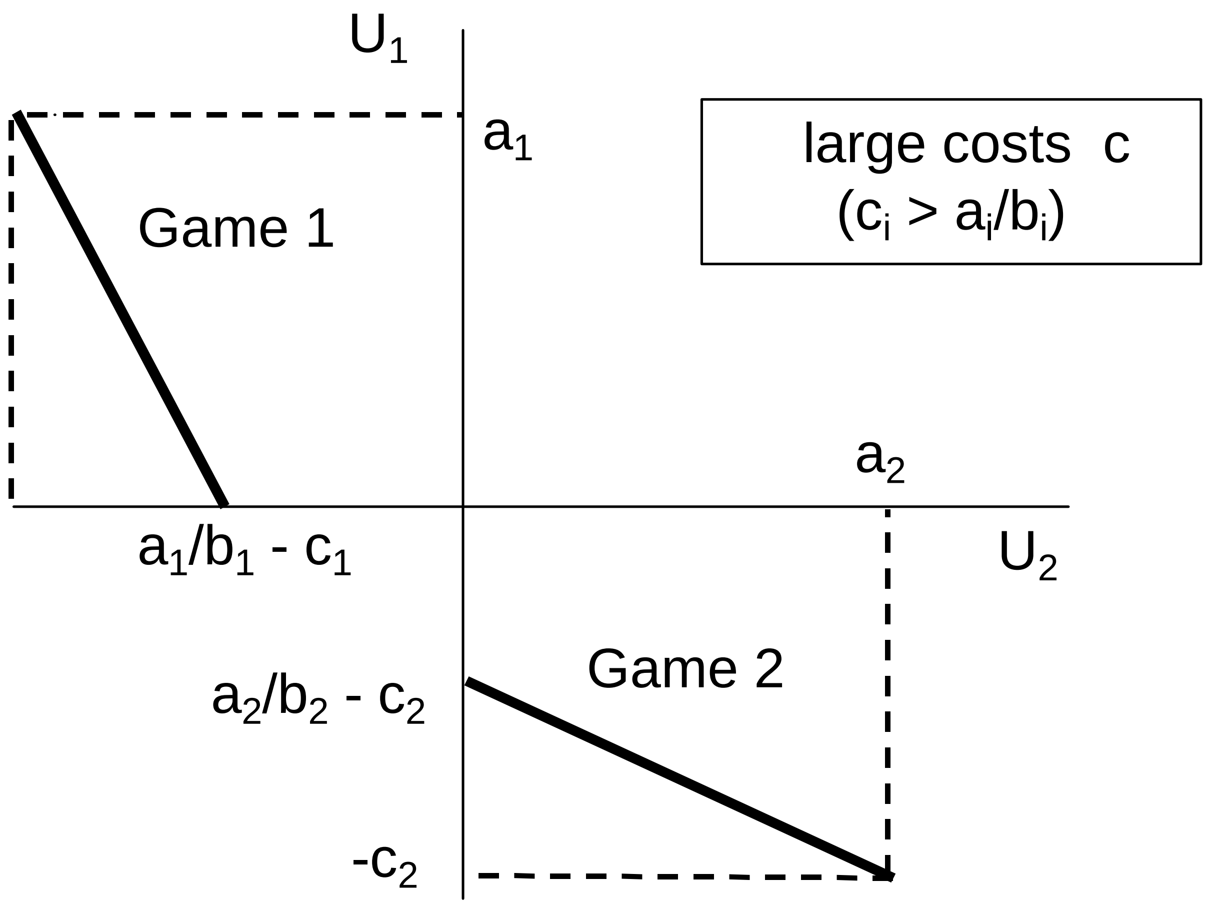
$U_2 f(U_2) \leq U_2 g(U_2) < (X/2)Y\ \ therefore\ \ U_2^* \leq Y\ \ and\ \ U_1^* \geq X/2$

Figure 1A: Two symmetric games; player i has a comparative interest in game i



Notation: $U_i = a_i - b_i U_j$ games 1,2

Figure 1B: Two symmetric games; player i has a comparative interest in game i



Notation: $U_i = a_i - b_i(U_j - c_i)$

Figure 2: Two symmetric games: player i receives strictly positive payoffs in both games

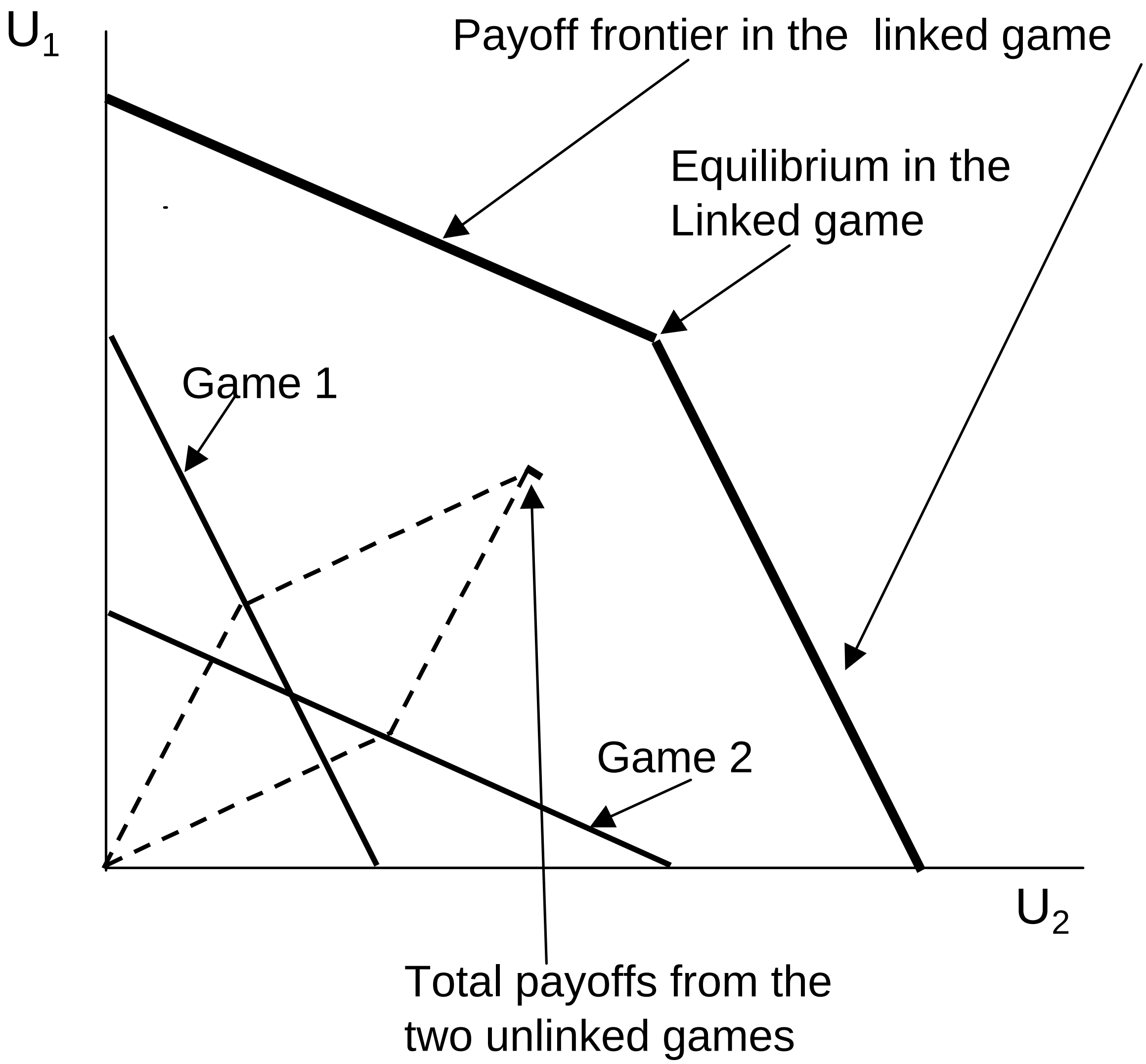
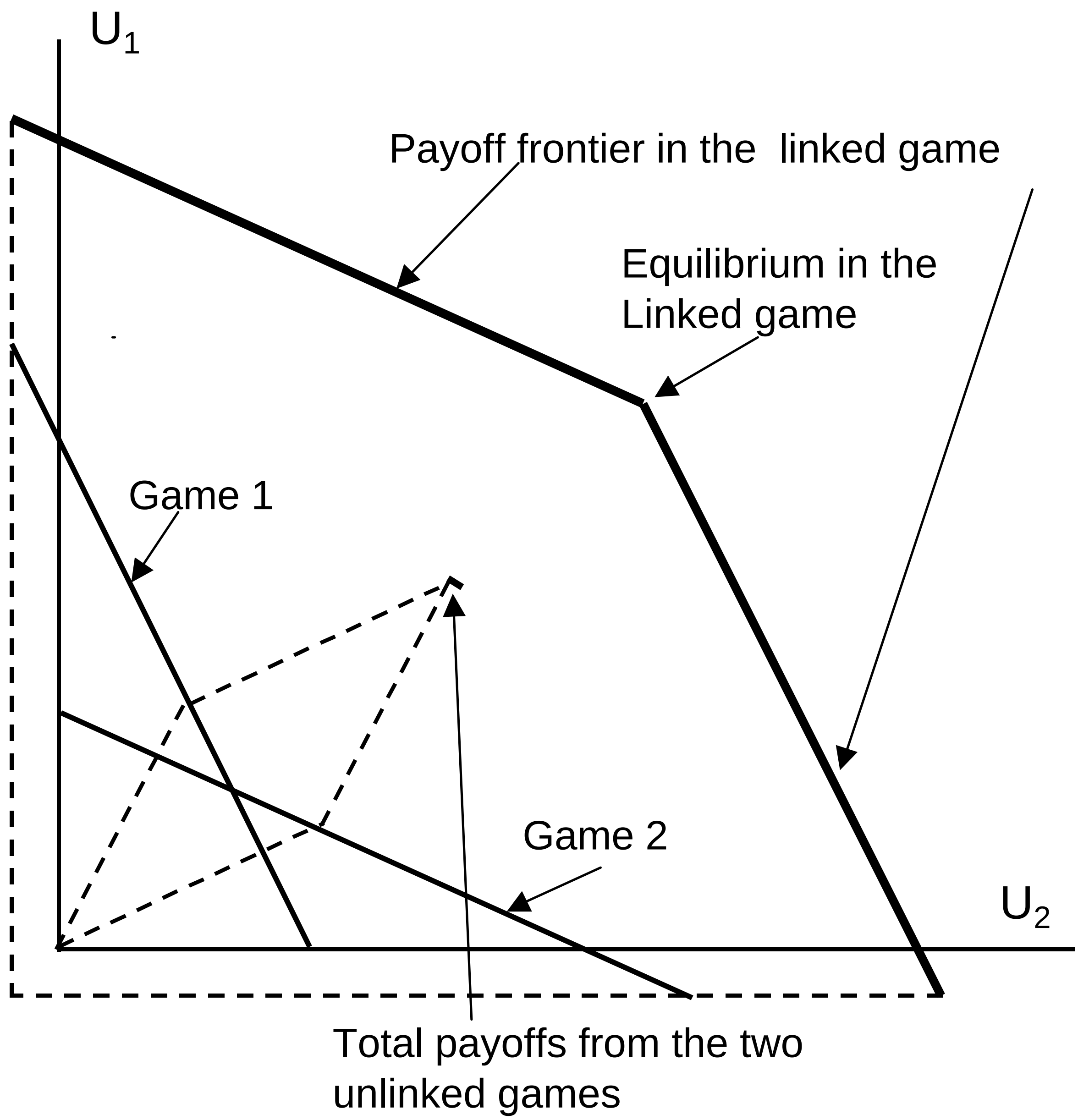


Figure 3: Two symmetric games: linking can be Pareto improving even if a player receives a negative payoff on one issue



N.B. Compared to Figure 2, the total payoffs from the two unliked games is the same, but the payoffs in the linked game are *larger* . Here it pays to take a negative payoff from one issue in order to gain more on your comparative-interest issue

Figure 4: Two symmetric games; player i cannot receive a positive payoff in game j

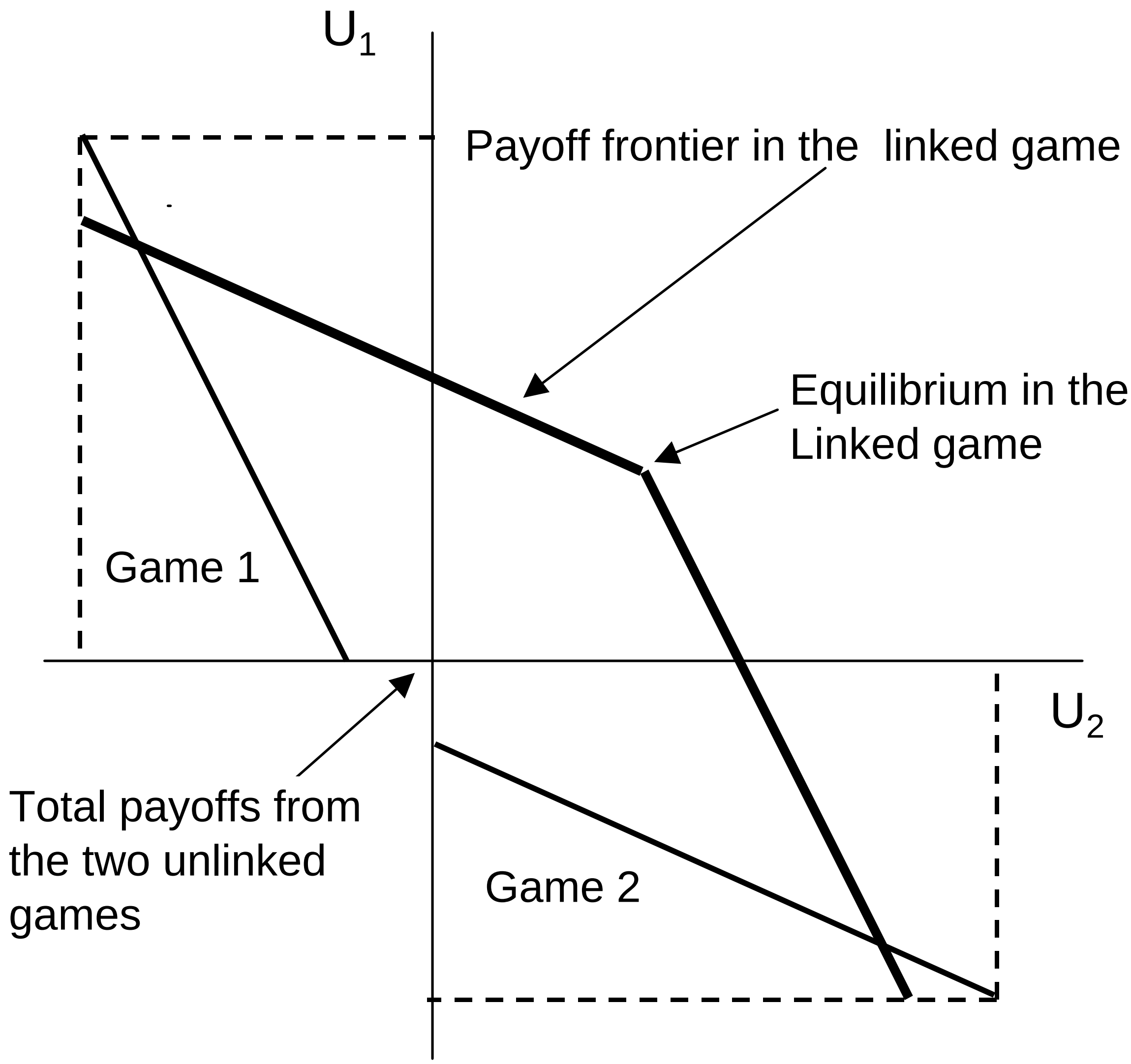
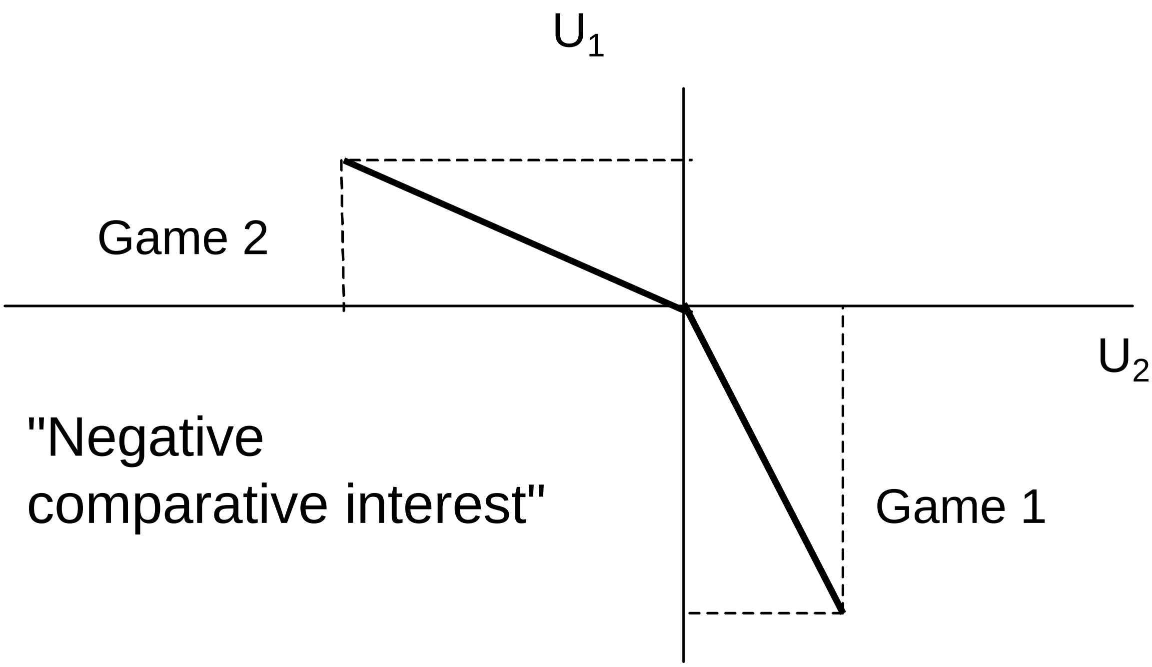


Figure 5: When is linking not Pareto improving?



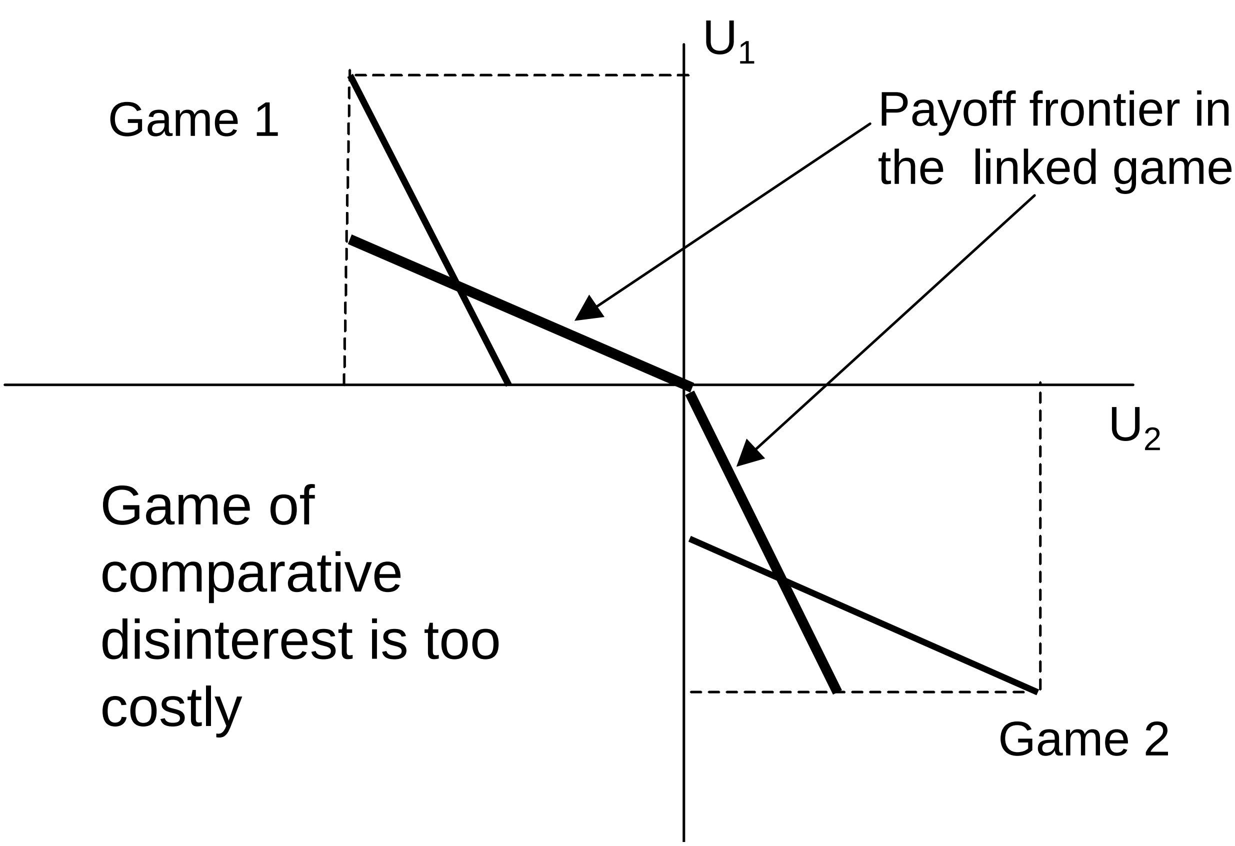
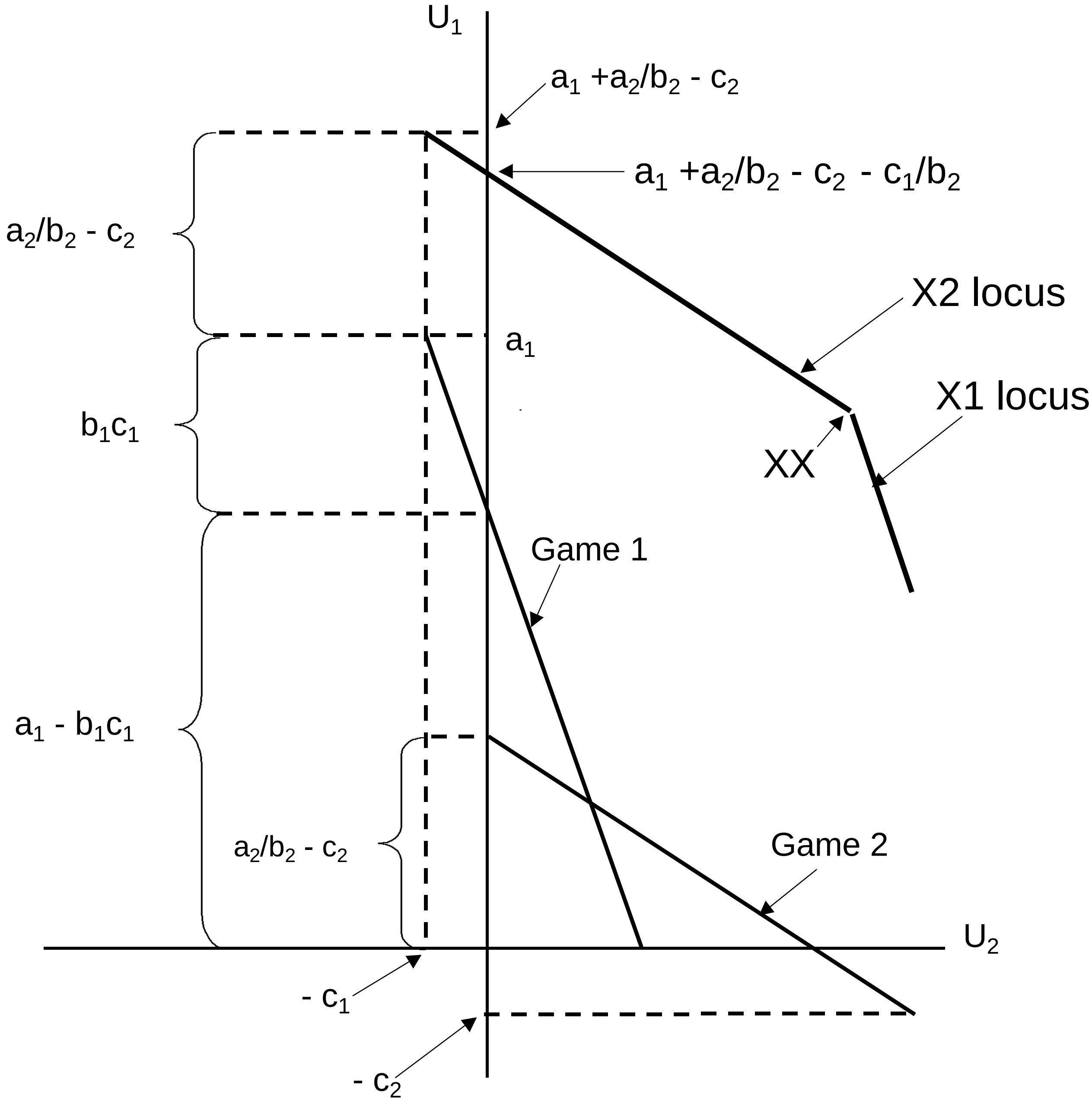


Figure 6: Geometric interpretation and proof of Proposition 1

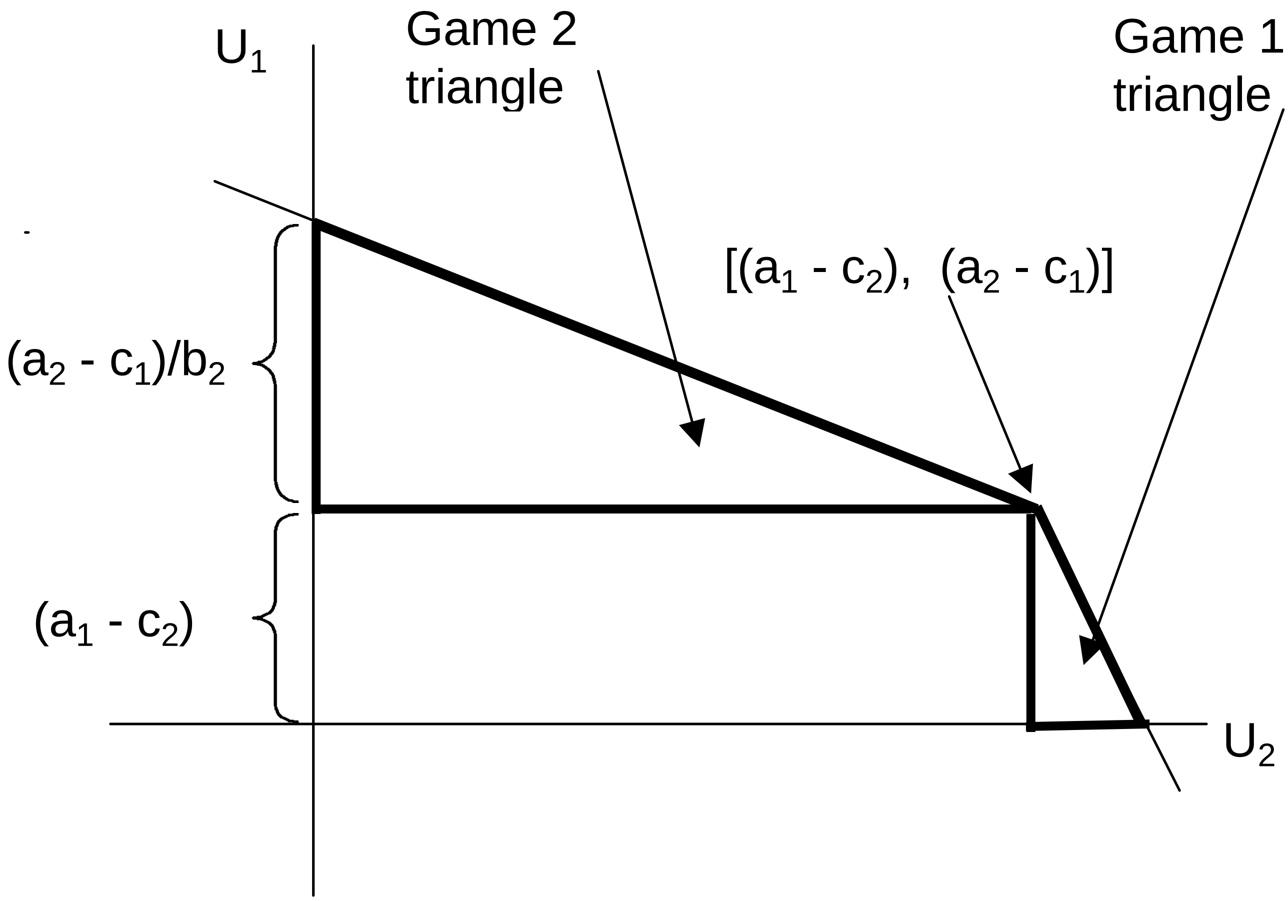


Minimum payoff to player 1 in the linked game: $U_1^* = [a_1 + a_2/b_2 - c_2 - c_1/b_2]/2$

Sum of payoffs to player 1 in the two unlinked games: $U_1' = [a_1 + a_2/b_2 - c_2 - c_1 b_1]/2$

Minimum Difference: $U_1^* - U_1' = c_1(b_1 - 1/b_2)/2 >= 0$

Figure 7: Game 1 is "small" if the game 1 triangle fits inside the game 2 triangle



"smallness" is satisfied for game 1 here if:
$(a_1 - c_2) < (a_2 - c_1)/b_2$

Figure 8A: Game 1 is small. All gains from linkage captured by player 2, "knife-edged" case

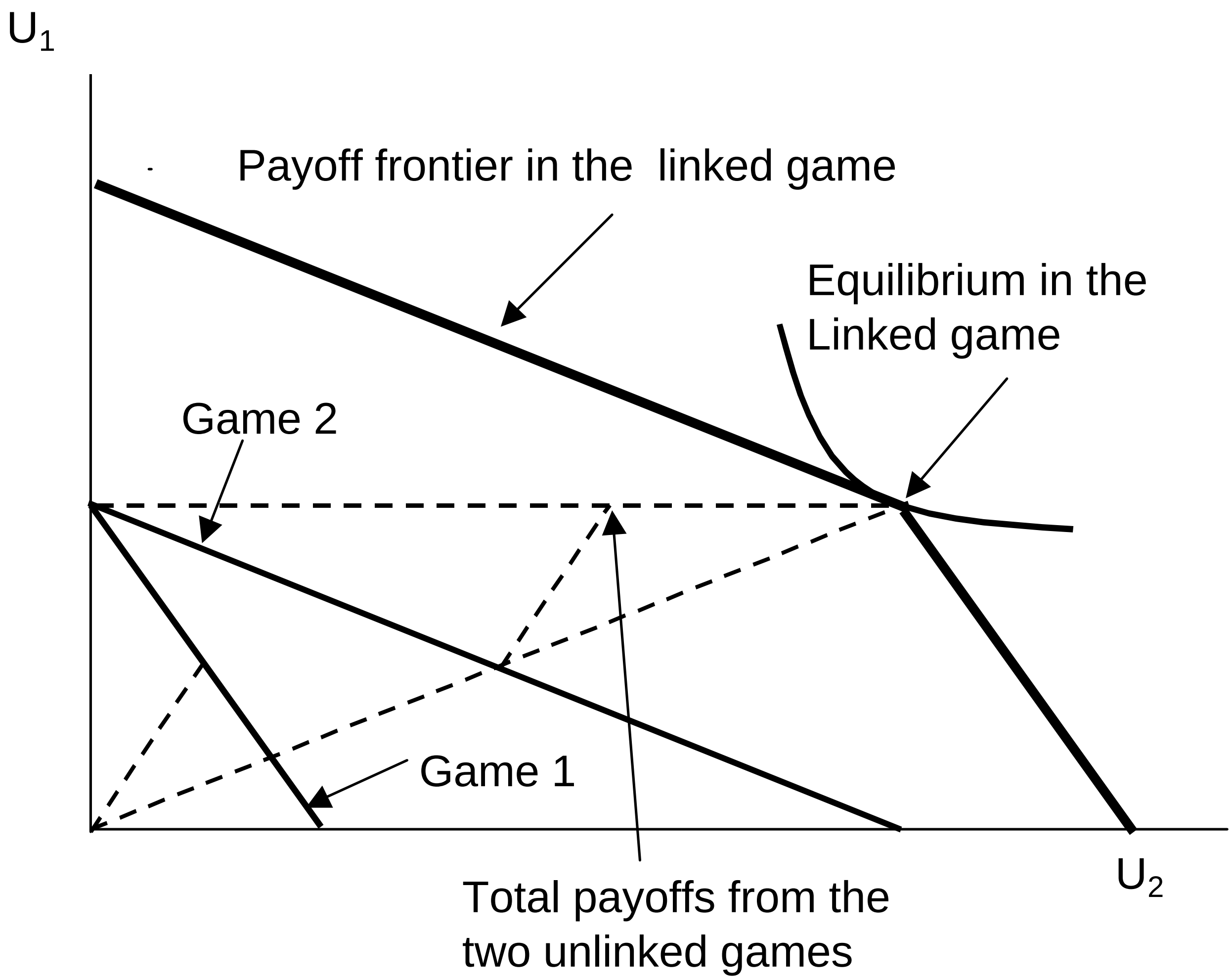


Figure 8B: Game 1 is small. All gains from linkage captured by player 2, "interior" case

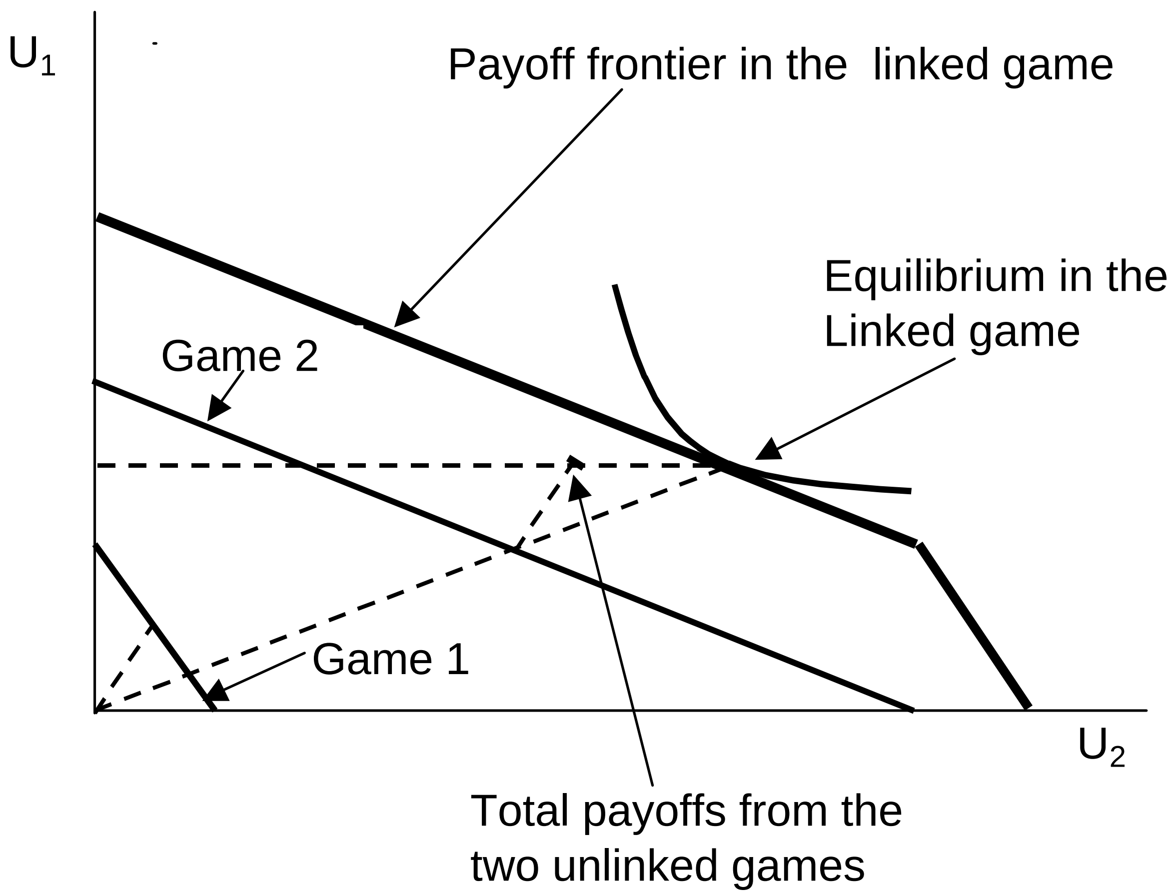
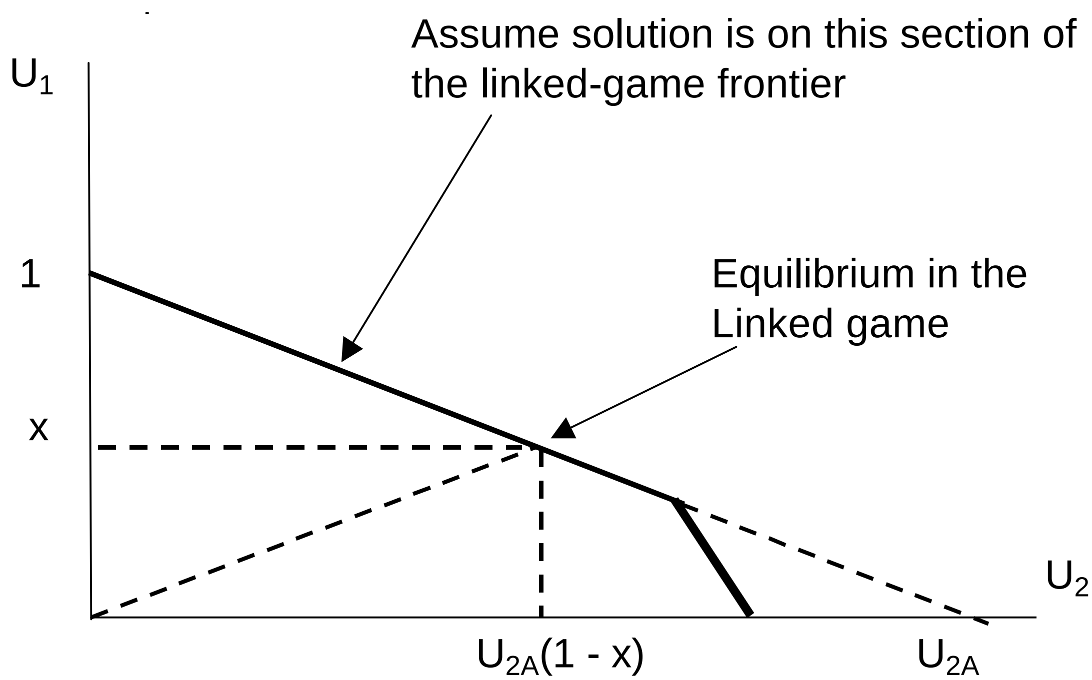


Figure 8C: (Attempt at) intuition why the player with the comparative interest in the big game gets the surplus



Rubenstein Solution (δ = discount factor):

Assume game ends at T, player 1 makes last offer = x

T: Player 1 demands (and receives) x

T - 1: Player 2 offers y

T - 1: $y = \delta x$ (2's best offer)

T - 2: $U_{2A}(1 - x) = \delta U_{2A}(1 - y)$ (1's best offer)

Solution: $x = 1/(1+\delta)$, $\Rightarrow$ $x = 1/2$ as $\delta \Rightarrow 1$

X and therefore U_1 do NOT depend on the size of U_{2A}

Figure 9A: Game 2 has no positive payoff for player 1; linking is Pareto improving, but all gains may go to player 2

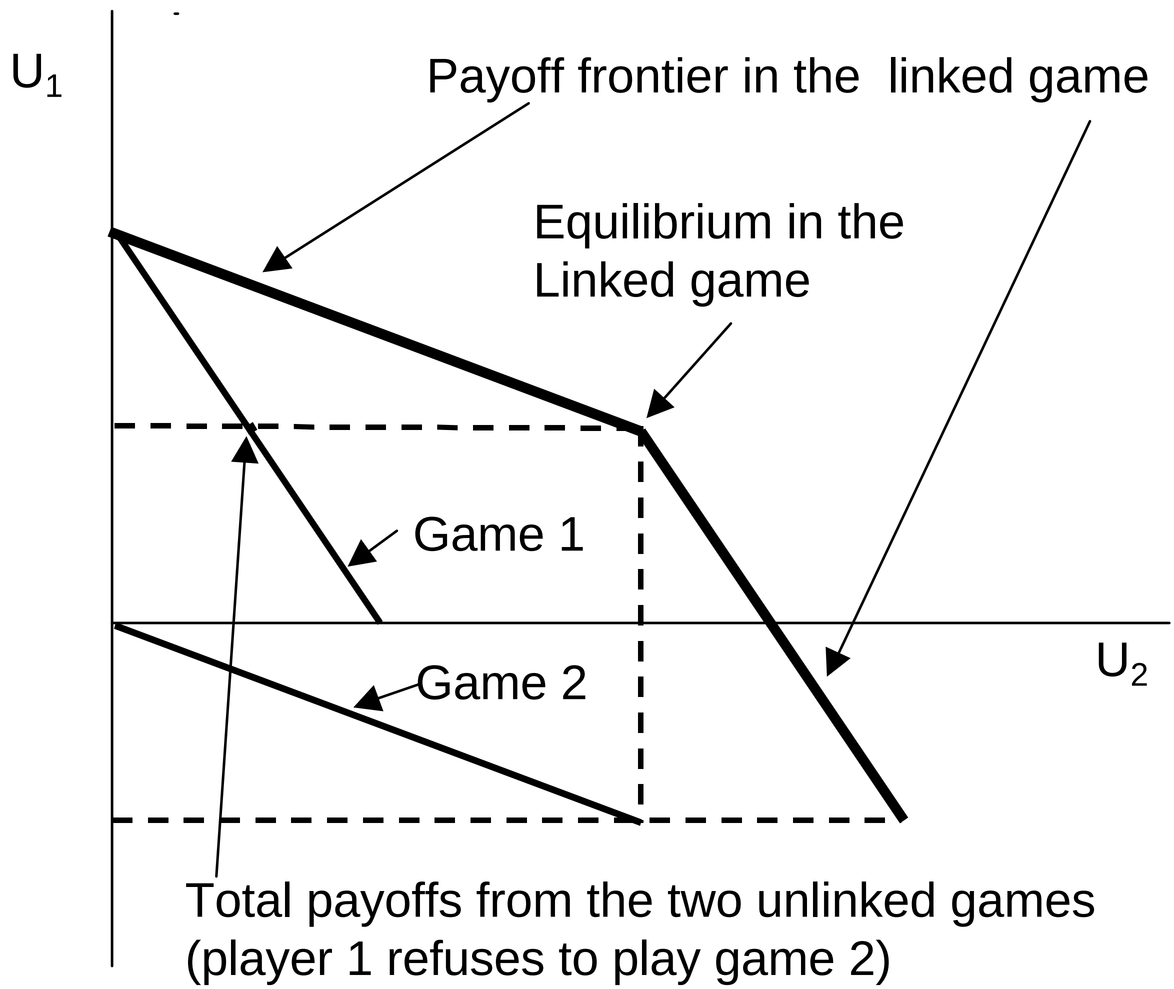


Figure 9B: A strictly negative payoff for U_1 in game 2 is not sufficient for linking worsen 1's welfare

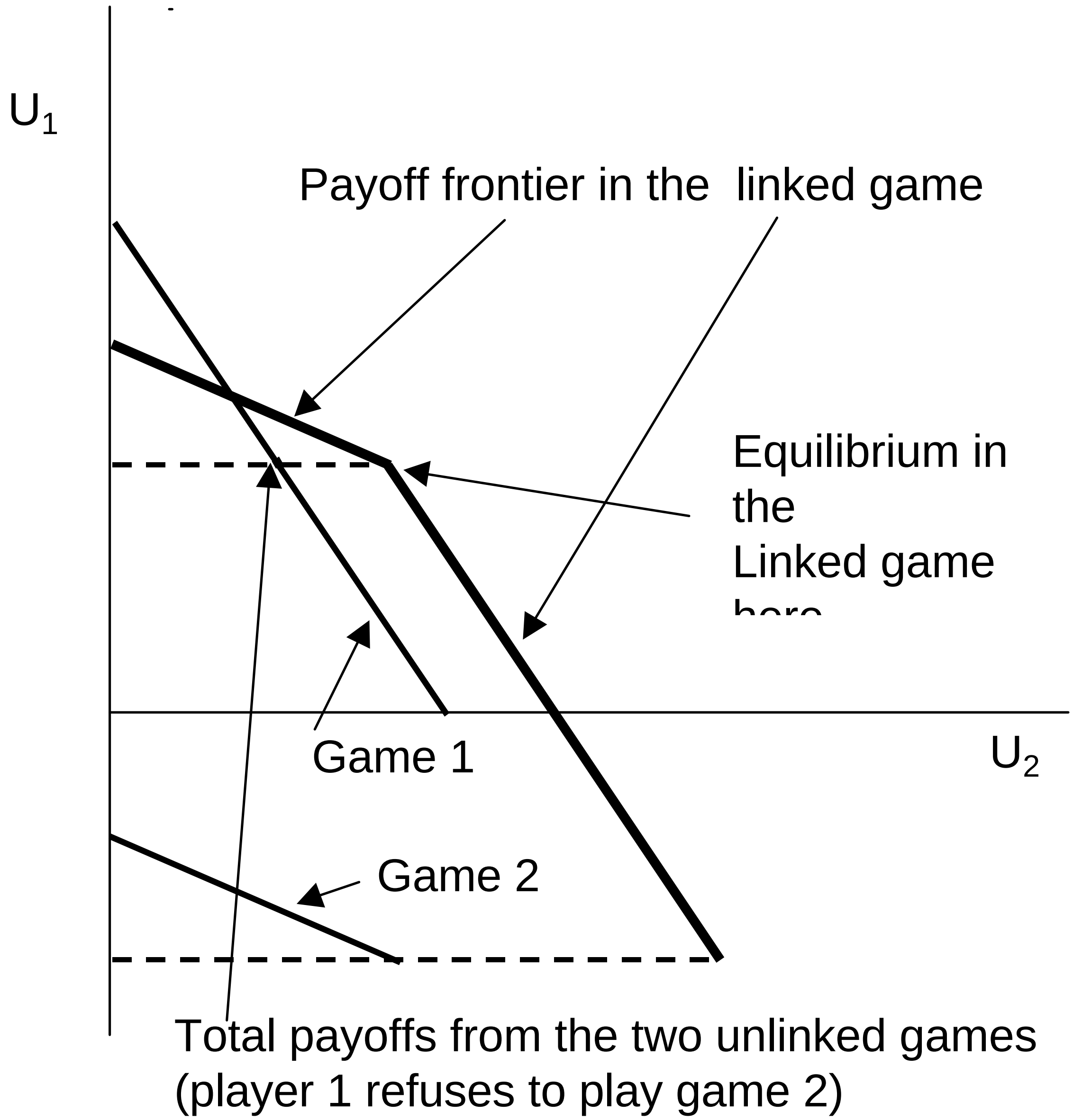


Figure 10A: A case where player 1 gains from linking even though game 2 has no positive payoff for player 1

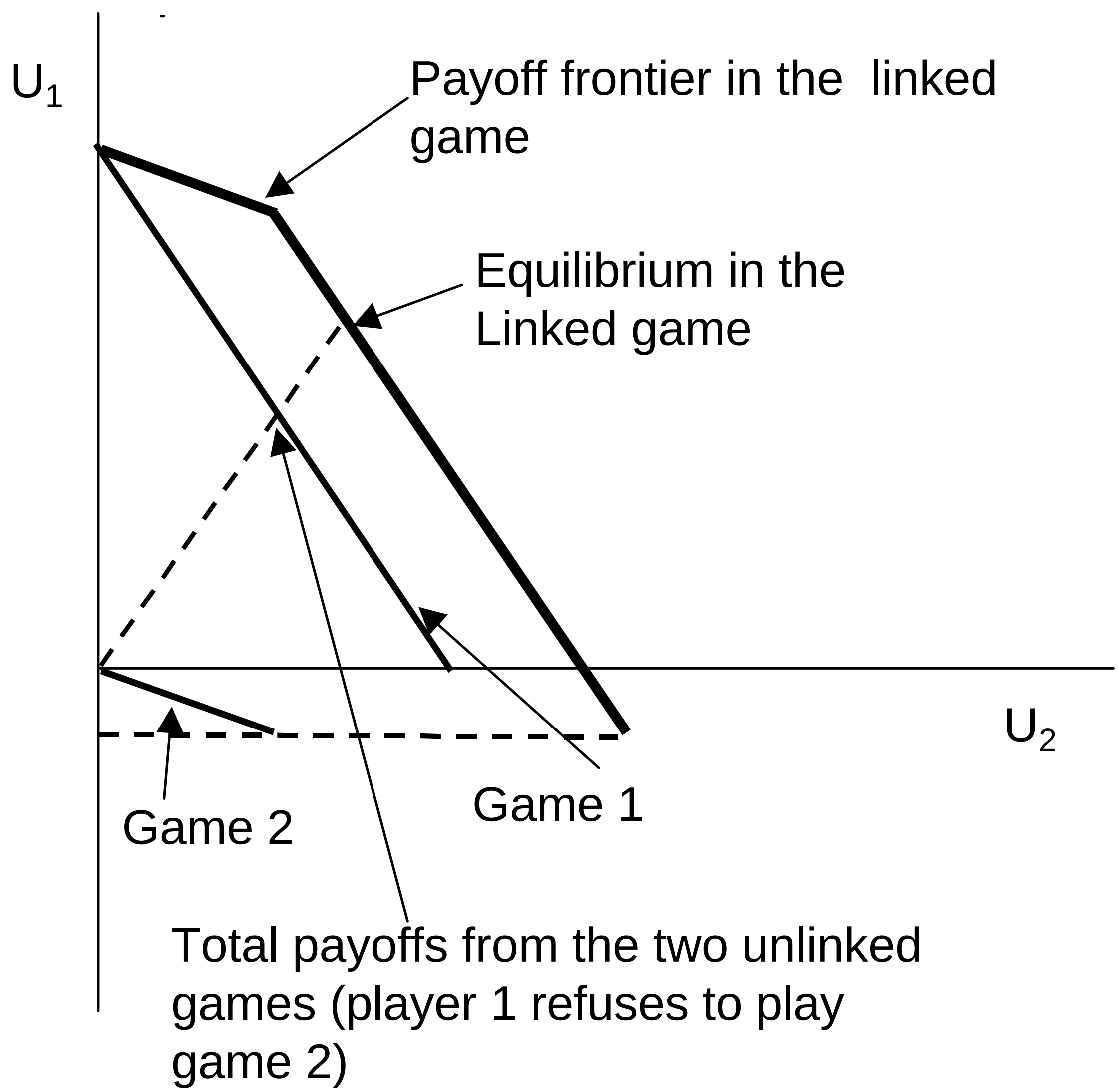
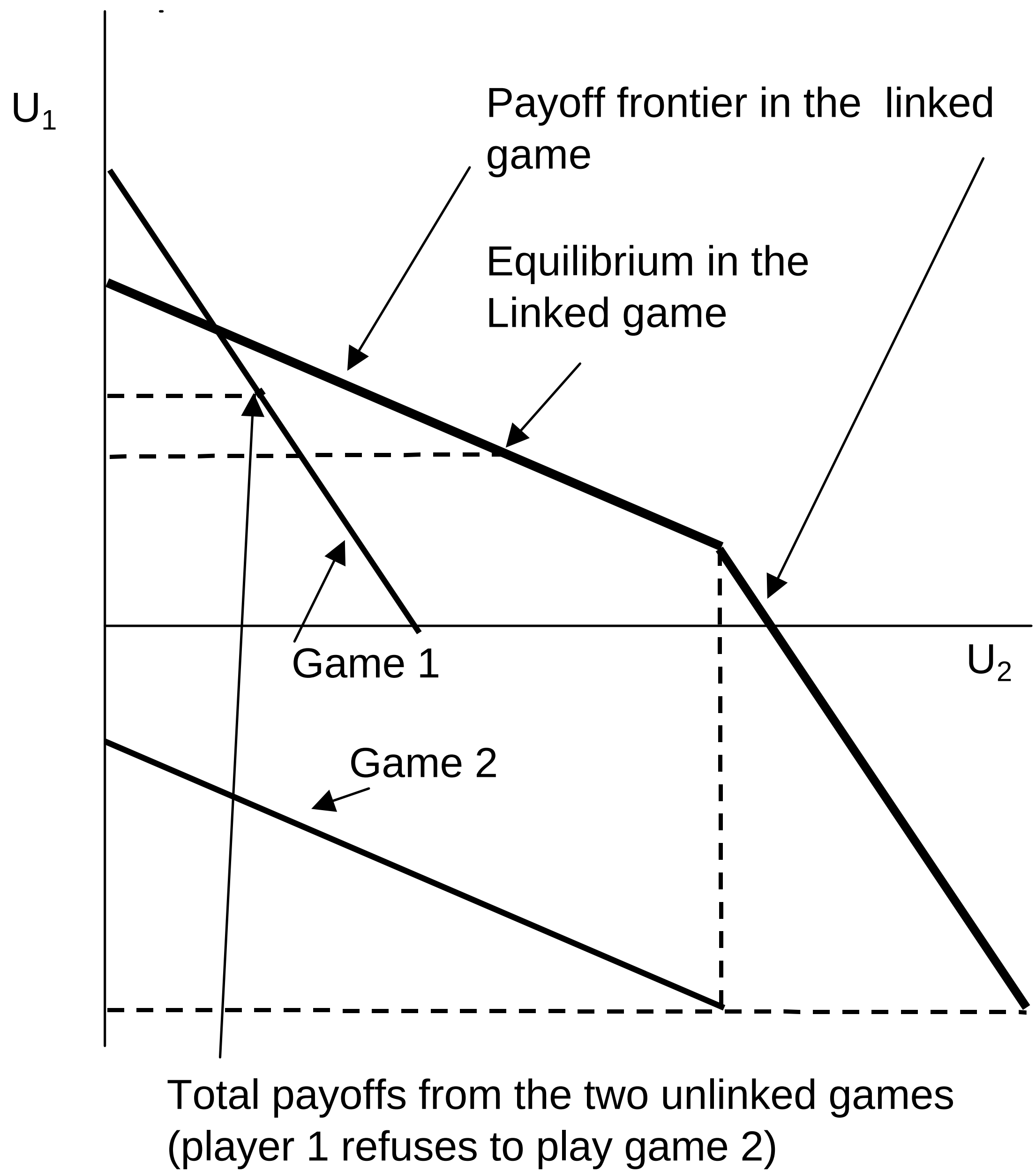


Figure 10B: Linking worsens player 1's welfare (a strictly negative payoff for U_1 in game 2 seems to be a necessary condition)



1. Preliminaries

Consider a situation in which there are n goods, $X_1, X_2, ..., X_n$, to be divided between two agents, 1 and 2. Agent i's preferences are described by a standard utility function given by $U_i = u_i(x_{1,i}, x_{2,i}, ..., x_{n,i})$ where $x_{j,i}$ is agent i's share of good X_j. Suppose that the allocation of goods is determined by an offer-counteroffer bargaining process where an offer by agent i is required to be an allocation, $(x_{1,i}, x_{2,i}, ..., x_{n,i})$, of all n goods.[1] Agents must either accept or reject the entire offer and an allocation of all n goods is made once agreement has been reached (an offer has been accepted). In this world, an offer $(x_{1,1}, x_{2,1}, ..., x_{n,1})$ is equivalent to and offer (U_1, U_2) with $U_1 = u_1(x_{1,1}, x_{2,1}, ..., x_{n,1})$ and $U_2 = u_2(X_1 - x_{1,2}, X_2 - x_{2,2}, ...X_n - x_{n,2})$. That is, we can map the game of offers over allocations of X into one of offers over utility pairs (U_1, U_2). Let $\mathcal{U}$ be the utility possibility set and define $U_1 = g(U_2)$ as the utility possibility frontier, given by $\max_{x_{1,1}, x_{2,1}, ..., x_{n,1}} u_1(x_{1,1}, x_{2,1}, ..., x_{n,1})$ subject to $U_2 = \overline{U}_2$. Then an offer in the game is a choice from the set $\mathcal{U}$. An equilibrium offer is a utility pair supportable by a pair of subgame perfect Nash equilibrium strategies. Assume that if no agreement is ever reached that each agent obtains utility of zero. Bargaining is

[1] Note that it is assumed here that if agent i offers j the allocation $(x_{1,i}, x_{2,i}, ..., x_{n,i})$, then i receives the allocation $(X_1 - x_{1,j}, X_2 - x_{2,j}, ...X_n - x_{n,j})$.

assumed to begin with agent 1; agents alternate offers subsequently. Both agents discount the future at a common rate, δ.

Following Shaked and Sutton, SPE offers can be defined as follows: Let $\overline{M}$ be the present value of utility for agent 1, discounted to period 3, in the SPE that yields agent 1 maximum utility. In period 2, agent 1 will then accept any offer that yields him utility of at least $\delta\overline{M}$. Since agent 2 has no incentive to make 1 strictly prefer to accept the offer, 2's offer in period 2 is $U_1 = \delta\overline{M}$. Further, since 1 accepts the offer $U_1 = \delta\overline{M}$ regardless of how it is constituted, 2 should choose the allocation of X such that his offer of $U_1 = \delta\overline{M}$ is on the utility possibility frontier. This means that the offer that 2 makes in period 2 is $(U_1 = \delta\overline{M}, U_2 = g^{-1}(\delta\overline{M}))$. This offer for 2 is the least that 2 obtains in any SPE.

Proceeding symmetrically, under the strategy yielding $\overline{M}$, 2 accepts any offer from 1 in period 1 that yields utility greater than $\delta g^{-1}(\delta\overline{M})$. Thus if 1 offers $U_2 = \delta g^{-1}(\delta\overline{M})$ and chooses the allocation of X such that U_1 is on the utility possibility frontier ($U_1 = g(\delta g^{-1}(\delta\overline{M}))$, this yields 1 the most utility in any SPE (since $\delta g^{-1}(\delta\overline{M})$ is the least that 2 obtains in any SPE). We have, then, that $\overline{M} = g(\delta g^{-1}(\delta\overline{M}))$.

We can do the same with the minimum payoff for 1 in any SPE, $\underline{M}$, and get that $\underline{M} = g(\delta g^{-1}(\delta \underline{M}))$. If $\underline{M} = \overline{M}$, then the SPE is unique and is defined by the point on the utility possibility frontier, U_1, solving $M = g(\delta g^{-1}(\delta M))$. If the function $g(\cdot)$ is concave, then the map $M - g(\delta g^{-1}(\delta M))$ is monotonic and so has a unique zero. This value of M is the unique equilibrium offer, U_1; the value of U_2 in the equilibrium is given by $U_2 = g^{-1}(M)$. In what follows, assume that $g(\cdot)$ is concave.

We now have that there is a unique equilibrium and it is a point on the utility possibility frontier. The equilibrium is defined by a pair of offers, one when it is agent 1's turn to offer (U_1^*, U_2^*) and one when it is agent 2's turn to offer (U_1^{**}, U_2^{**}). These offer pairs satisfy the equations

$$U_2^* = \delta U_2^{**} \tag{1.1}$$

$$g(U_2^{**}) = \delta g(U_2^*). \tag{1.2}$$

The first condition requires that 1's offer to 2, U_1^*, is such that 2 is just indifferent between accepting and rejecting and countering with U_2^{**}(recall above discussion). The second condition requires, similarly, that 2's offer U_2^{**} is such that 1 is just indifferent between accepting and rejecting and countering with U_2^*. The remaining conditions are, of course, that $U_1^* = g(U_2^*)$ and $U_1^{**} = g(U_2^{**})$ (offers are on utility possibility frontier).

For our purposes, it turns out that it is handy to rewrite conditions (1.1) and (1.2) in the following way:

$$U_2^* = U_2^{**} - (1-\delta)U_2^{**} \tag{1'}$$

$$g(U_2^{**}) = g(U_2^*) - (1-\delta)g(U_2^{**}). \tag{2'}$$

These expressions can be interpreted as follows: Agent 2 can accept the utility offer U_2^* or can reject and counter with the utility demand U_2^{**}. The cost of making this demand is $(1-\delta)U_2^{**}$. The condition for agent 1 has a similar interpretation. What this says is that, for agent 2, the cost of making a counteroffer is $(1-\delta)U_2^{**}$, while, for agent 1, the cost of a counteroffer is $(1-\delta)g(U_2^{**})$. This formulation is handy because we can now rewrite the equilibrium conditions (moving the utility counteroffers to the LHS and taking ratios) as

$$-\frac{g(U_2^*) - g(U_2^{**})}{U_2^* - U_2^{**}} = \frac{g(U_2^{**})}{U_2^{**}}. \tag{1.3}$$

If we take the limit of this expression as $\delta \to 1$, recognizing from (1.1) that $U_2^{**} \to U_2^*$ in this case, we get that the LHS of (1.3) is the slope of the utility possibility frontier. We have, then, that the equilibrium condition (1.3) coincides with the Nash bargaining solution as $\delta \to 1$; that is, the unique SPE of this game when $\delta \to 1$ is given by the Nash bargaining solution over the utility set $\mathcal{U}$.

How do we interpret all of this? The ratio of utilities in the Nash bargaining solution, U_1/U_2, gives the relative costs of the two agents for making a counteroffer. Basically, it's the cost to 1 of holding out for a better deal relative to that for 2. The slope of the utility possibility frontier gives the (technological) cost of turning 1's utility into utility for 2. If U_1/U_2 is large, then it's relatively costly for 1 to hold out for a better deal (1 is in a weak bargaining position). If $-g'(U_2)$ is small, then it's relatively cheap to turn U_1 into U_2. In this case, 2 can increase utility for himself at a low utility loss for 1; since it is costly for 1 to make a counteroffer relative to 2, 1 gives in to the utility swap rather than rejecting and countering. Thus if $-g'(U_2) < U_1/U_2$, we don't have an equilibrium; rather 2's utility should be increased and 1's decreased. Analogous arguments apply for the case of $-g'(U_2) > U_1/U_2$. Equilibrium is established where the relative cost of a counteroffer is just equal to the cost of swapping U_1 for U_2.

I note all of this partly because I think it's useful for generating intuition about why bargained outcomes differ across different procedures. Also, it's useful because we now have that the SPE is unique as long as the utility frontier is concave and that it converges to the Nash bargaining solution as $\delta \to 1$. These facts will be handy in comparing the linked and unlinked bargaining outcomes to follow.

2. Linked Bargaining

Assume that there are two goods, X and Y, to be allocated to two individuals. The amount of Y is normalized to 1 while the amount of X is assumed equal to s with $0 < s < \overline{S} > 1$. The utility functions for the two agents are

$$U_1 = asx + (k_1 - y) \tag{2.1}$$

$$U_2 = s(k_2 - x) + by, \tag{2.2}$$

with $a, b > 1$, $0 < k_1, k_2 \leq 1$ and $x \in [0, 1]$ giving agent 1's share of X, $y \in [0, 1]$ agent 2's share of Y. These utility functions capture the relevant features of the

Nash problem. Specifically, 1 prefers X to Y, with the degree of relative preference measured by a, while 2 prefers Y to X (with degree of relative preference given by b). In the separate bargain over X, the slope of the utility frontier is a, while in the bargain over Y the slope is $1/b$. (These correspond to b_1 and $1/b_2$ respectively in Proposition 1. The assumption that $a, b > 1$ gives Assumptions 1 and 2 of Proposition 1.) The values k_1 and k_2 give the costs of transferring utility in the good that agent i likes less. The interpretation is that, if agent 1 gets more than the fraction k_2 of good X, then agent 2 obtains negative utility from X; similarly, if agent 2 gets more than the fraction k_1 of good Y, then agent 1 gets negative utility from Y. Essentially, in transferring these goods, some of the good is lost. The k_i's produce the c_i's ($s(1 - k_1) = c_2, 1 - k_2 = c_1$) and the assumption that $0 < k_i$ gives assumption 3 in Proposition 1.

In the separate bargains over X and Y, the utility frontiers are linear (and so concave) and the above results apply. In the linked game, the utility frontier is piecewise linear with the slope increasing across the linear pieces (from $1/b$ to a) and so it this frontier is also concave. Again, the above results apply to the linked game as well. Thus, in all games there is a unique SPE that converges to the Nash bargaining solution as $\delta \to 1$. The SPE allocations are defined as above, as are the limiting allocations for the Nash bargaining solution. In the bargain over X, the limiting allocation (Nash bargaining allocations) is $x = k_2/2$; in the bargain over Y, the allocation is $y = k_1/2$. These allocations result in utilities of $U_1 = (ask_2 + k_1)/2$, $U_2 = (sk_2 + bk_1)/2$ for agents 1 and 2 respectively.

In the linked game, these utility allocations are strictly interior to the utility set, $\mathcal{U}_L$, for this game. To be on the utility frontier, at least one agent must obtain all of the good preferred by that agent (i.e., either agent 1 must at least have all of X or agent 2 at least have all of Y). Under the allocation from the separate bargaining games neither agent obtains all any good. Since, from above, the bargaining outcome from the linked game is on the utility frontier for $\mathcal{U}_L$, at least one agent must be made better off by linking. The only question that remains, then, is whether or not both are made better off. Certainly, there are allocations that make both better off. The question is whether the bargaining equilibrium picks one of these allocations. Doing so requires that the equilibrium in the linked game produces an outcome with $U_1 \geq (ask_2 + k_1)/2$ and $U_2 \geq (sk_2 + bk_1)/2$. If the point on the utility frontier for $\mathcal{U}_L$ associated with $U_2 = (sk_2 + bk_1)/2$ is such that $|\text{slope}| \leq U_1/U_2$ while the point on the utility frontier associated with $U_1 = (ask_2 + k_1)/2$ has $|\text{slope}| \geq U_1/U_2$, then we know from above that the limiting equilibrium must have $U_1 \geq (ask_2 + k_1)/2$ and $U_2 \geq (sk_2 + bk_1)/2$; that is, both agents are (weakly) better off.

The following Proposition provides conditions under which both are strictly better off. The proof of the proposition is in the Appendix.

Proposition 1. *If $k_1, k_2 \neq 1$, then, for all s, linking strictly increases the utilities of both agents.*

Several points about this result are worth noting. First, it's important for the result that $a, b \neq 1$; that is, it's important that the agents have comparative interests in different goods. Were $a, b = 1$, then the unlinked game would give a point on the utility frontier. Specifically, it would give the point $U_1 = U_2 = (k_1 + k_2)/2$. The slope of the utility frontier is 1 in this case and so we would have that the linked and unlinked games give the same outcome. That $a, b \neq 1$ (there are differing interests) means that the unlinked game puts the players interior to the utility space of the linked game. Since the linked game outcome must be on the utility frontier, linking produces utility gains for both agents by allowing an efficient allocation of the goods.

In a similar vein, it is important that there are costs of transfers ($k_1, k_2 \neq 1$). These costs mean that the unlinked game produces an additional misallocation due to agent i having to give j some of the good that i prefers just to induce j to participate at all in the unlinked bargaining. More concretely, consider the bargain over Y in the unlinked case. In this bargain, agent 1 has zero cost of making a counteroffer unless he obtains at least k_1 units of Y. This fact gives 1 extra bargaining power (relative to the case in which $k_1 = 1$) and so 1 extracts more Y (the good that 2 prefers). Linking allows 2 to compensate 1 for k_1 through X (and similarly allows 1 to compensate 2 for k_2 through Y) and so provides an additional efficiency gain.

To see the importance of positive costs of transfer, suppose that at least one of k_1, k_2 is 1. As the following proposition shows, in this case it's possible that only one of the agents is made strictly better off by linking (the other will be indifferent).

Proposition 2. *If $k_1 = 1$ and $k_2 < 2 - \frac{b}{s}$, then agent 1's utility strictly increases from linking while agent 2's utility remains unchanged; if $k_2 = 1$ and $k_1 < 2 - as$ then agent 2's utility strictly increases from linking while agent 1's utility remains unchanged.*

Note that, if s is small enough, then 2 extracts all of the gains from linking even when $k_1 = 1$; similarly, if s is large enough 1 extracts all of the gains from linking even when $k_2 = 1$. In the former case, the proposition says that if X is small enough, then even when there are no costs of transfers for either agent, agent 2 extracts all of the gains from linking. When X is large enough, agent 1 extracts all of the gains from linking. In essence, if one good/surplus is small enough relative to the other, the agent that prefers the small good obtains no surplus from linking. The reader can easily check that, for $s = 1$, both agents must gain from linking when $k_1 = k_2 = 1$.

What's the intuition for this result? Consider the case of $k_1 = k_2 = 1$. In this case, the unlinked game produces an inefficient outcome because it allocates some of both goods to both agents: an efficient allocation gives all of one good to one of the agents. In general, the unlinked game provides too little of X to agent 1 and too little of Y to agent 2. When s is small, this is not the case, though in the following sense: agent 1's utility in the unlinked game can be achieved through an efficient allocation in the linked game by giving 1 all of X and some share of Y (this is the essence of the condition on k_1 in the proposition). In terms of utility, then, agent 1 is not under allocated X in the unlinked game; only Y is under allocated to agent 2. As a result, the efficiency gains from linking are purely in terms of correcting the under allocation of Y to agent 2; effectively, agent 1 obtains no efficiency benefits from linking. For this reason, 2 obtains all of the gains from linking.

By contrast, when $s = 1$, the utilities in the unlinked game can only be achieved in the linked game if both agent 1 obtains less than all of X and agent 2 obtains less than all of Y. In this case, both goods are under allocated to both agents. As a consequence, both agents have efficiency benefits from linking and both agents' utilities increase in the linked game.

Another way to look at this is the following: 1) Efficient allocation of goods requires that agent 1 only be allocated a share of Y if 2 obtains none of X and that agent 2 only be allocated a share of X if 1 obtains none of Y. 2) Any interior point in the utility space $\mathcal{U}_L$ can be thought of as equivalent to an allocation in which this efficient rule is followed but some of either X or Y (or both) is not allocated. In particular, we can think of the utility levels in the unlinked game in this fashion. 3) The notion of small here is that, for agent 1 to obtain utility in the linked game equal to that in the unlinked game, 1 must obtain all of X and some share of Y (this is the essence of the condition on k_1 in the proposition). The implication of point 3 is that, when s is small, the unlinked bargain yields 1 a utility point in $\mathcal{U}_L$ that can only be equivalent to efficient rule allocations (point

2) in which all of X is allocated. Some of Y is not allocated only to achieve agent 2's utility outcome from the unlinked game. In this sense, the inefficiency in the unlinked game falls fully on agent 2 and so all of the efficiency gains from linking accrue to agent 2 through the efficient allocation of Y. Because of this fact, agent 2 captures all of the efficiency gains.

By contrast, when $k_1 = k_2 = s = 1$, the unlinked utility levels are equivalent to efficient rule allocations in which some of both X and Y are not allocated. As a result, both agents bear the efficiency costs of not linking and so both captures some of the efficiency gains from linking.

3. Bargaining Agenda and Implementation

In the standard bargaining over a single surplus, there is no decision about either the agenda (the order in which issues are bargained) or implementation procedures for agreements. Once there are multiple issues, both need to be considered. The games above assume particular agenda structures and implementation rules. What we are calling the unlinked game is an agenda in which the two issues are negotiated simultaneously but completely separately in the sense that there is no possibility for agent 1 to trade a concession on Y, say, for a concession by agent

2 on X. Implementation of agreements is also completely separate in the sense that a failure to reach agreement on one issue doesn't preclude implementation of agreement on the other issue. The linked game is an agenda in which the issues are negotiated simultaneously and jointly: an offer is an allocation of both goods and the entire offer must either be accepted or rejected. In this game, agents can trade a concession on one good for a concession on the other. Implementation occurs after an offer is accepted, so allocations only are made when both issues are settled.[2]

These two arrangements basically represent the two extremes of the set of possible bargaining procedures. In between are various schemes in which issues can be bargained sequentially and implementation of agreements on some issues linked in various way to whether or not agreement has been reached on other issues. In the case of only two goods, three basic procedures are possible: 1) The agents bargain only on X and once agreement has been reached on X bargain on Y. The agreement on X is binding (in the sense that it can't later be re-opened) and is implemented at a fixed date whether or not agreement is ever reached on Y. 2) The agents bargain only on Y and once agreement has been reached on Y

[2] Basically, the linked game is the multiple issue analogue of the standard Rubinstein bargaining framework.

bargain on X. The agreement on Y is binding (in the sense that it can't later be re-opened) and is implemented at a fixed date whether or not agreement is ever reached on X. 3) The agents bargain only on X (Y) and once agreement is reached on that good bargain on Y (X). The agreement on X (Y) is binding but is only implemented once agreement has been reached on Y (X). Fershtman (1990) has shown that, unless the two goods are of very different sizes, procedure 3 generates the same outcome as the linked game as $\delta \to 1$. In this case, then, there are basically two other agendas, each with a sequential (implement as agreement is reached) implementation rule. These two procedures essentially involve partial linkage.

How do the agents utilities under these two agendas compare to utilities under the linked an unlinked agendas? To illustrate the issues we assume that $k_1 = k_2 = s = 1$. Consider the first agenda X then Y. Once agreement is reached on X, the bargain on Y has no impact on the utility that agents receive from the X agreement: the agreement is binding and the allocation of X is made upon agreement. As a result, the bargain on Y is as in the unlinked game, with the limiting allocation being $y = 1/2$ and the limiting utilities from Y being $U_1 = 1/2, U_2 = b/2$.

The bargain on X is different from the unlinked game since bargaining is sequential rather than simultaneous: bargaining doesn't begin on Y until agreement is reached on X. The sequential structure of bargaining is important because failure to reach agreement on X delays agreement on (and consumption of) Y. Thus, both agents bear a utility cost from continued bargaining on X that reflects both their valuations of X and Y. In the unlinked game, because bargaining is simultaneous, the utility cost of continued bargaining on X reflects only the agents' valuations of X. Formally, letting (x_s^*, x_s^{**}) be the offers on X by agents 1 and 2 respectively in this sequential game, the conditions defining equilibrium are:[3]

$$1 - x_s^* = 1 - x_s^{**} - (1-\delta)(1 - x_s^{**} + \delta b) \tag{3.1}$$

$$a x_s^{**} = a x_s^* - (1-\delta)(a x_s^* + \delta). \tag{3.2}$$

[3]The structure of offers and counteroffers assumed here is that, if bargaining on X ends with agent 1 accepting an offer from agent 2, then agent 1 makes the first offer on Y. Similarly, if bargaining on X ends with 2 accepting an offer from 1, then 2 makes the first offer on Y. This means that the conditions for an equilibrium are:

$$\begin{aligned} 1 - x^* + \delta b/(1+\delta) &= \delta[1 - x^{**} + \delta^2 b/(1+\delta)] \\ a x^{**} + \delta/(1+\delta) &= \delta[a x^* + \delta^2/(1+\delta)], \end{aligned}$$

where we've used the fact that the offer 1 makes on Y gives 2 the share $\delta/(1+\delta)$ and the share demand that 2 makes gives 2 $1/(1+\delta)$.

Bargaining costs for this agenda are $(1-\delta)(1-x_s^{**}+\delta b)$ for agent 2 and $(1-\delta)(ax_s^*+\delta)$ for agent 1. By contrast, in the unlinked game, bargaining costs would be $(1-\delta)(1-x^{**})$ and $(1-\delta)ax^*$ for 2 and 1 respectively.

As before, the limiting equilibrium is given by an appropriately defined Nash bargaining solution. From the above, this solution is given by the condition

$$a \geq \frac{ax_s+1}{1-x_s+b}, \tag{3.3}$$

where the inequality allows for the fact that the solution may be a corner solution in which $x_s=1$. Indeed, if $b>1+\frac{1}{a}$, the outcome is a corner solution. In this case, the utility allocation is a point on the frontier of the utility possibility set, $\mathcal{U}_L$, with $U_1^{sx}=a+.5, U_2^{sx}=.5b$.

How does this outcome compare to that of the linked and unlinked games? For this case, the linked game outcome has agent 1 getting all of X and agent 2 all of Y, yielding utilities $U_1^l=a, U_2^l=b$. The unlinked game gives utilities $U_1^u=.5a+.5, U_2^u=.5b+.5$. Clearly, agent 1 prefers the sequential agenda to the linked agenda ($U_1^{sx}>U_1^l>U_1^u$) while agent 2 finds the sequential agenda worst of all ($U_2^{sx}<U_2^u<U_2^l$).

A similar analysis can be performed for the sequential agenda Y then X. As long as $a > 1 + \frac{1}{b}$, the outcome again will be on the utility frontier with agent 2 getting all of Y and half of X. The utilities for the two agents for this agenda are $U_1^{sy} = .5a, U_2^{sy} = b + .5$. In this case, agent 2 prefers this agenda to all others while agent 1 finds this agenda worse than even the unlinked game. So we have

Proposition 1. *If $k_1 = k_2 = s = 1$, then agent 1(2) prefers the sequential agenda X then Y (Y then X) to all other agendas. Agent 2(1) finds the sequential agenda X then Y (Y then X) worse than the unlinked game.*

The intuition for this result can be found in the way that the various agendas affect the agents' relative bargaining costs. In the linked game, agent 1 finds it costly to hold out for a positive share of Y since doing so delays agreement on (and consumption of) X, the good 1 prefers. Similarly, 2 finds it costly to hold out for a positive share of X since doing so delays agreement on (and consumption of) Y, the good 2 prefers. The result is that each agent obtains all of the good that that agent prefers and none of the other good.

In the sequential agenda X then Y, 1 has already obtained his allocation of X before bargaining on Y begins. As a result, it is now cheap for 1 to hold out for a share of Y since doing so doesn't delay consumption of X. In essence, 1's bargaining costs on Y are now low relative to 2's and so 1 obtains a positive share of Y. In the prior bargain over X, the agents' relative bargaining costs are not much changed from the linked bargain: it's relatively costly for 1 to concede some of X since this is the good 1 prefers and 2's holding out for a large share of X continues to delay agreement on Y. Overall, then, the sequential agenda X then Y lowers 1's bargaining costs relative to 2's and so puts one in a favorable bargaining position relative to the linked game. Two is damaged both relative to the linked game and the unlinked game since 2 continues to concede on X because not doing so delays agreement on Y (which is not so in the unlinked game). Analogous arguments explain 2's preference (and 1's dislike) for the agenda Y then X.

All of this raises the interesting question of how the agenda is determined. One possibility is that there is a pre-negotiation bargaining round that decides on the agenda. I would just note here that, if agent 1, say, believed that this bargaining would produce the sequential outcome Y then X with high probability, 1 might prefer some "custom" that required bargaining to be unlinked.

I should also add that the assumption $k_1 = k_2 = 1$ would seem to put the best face on the sequential agenda relative to the linked agenda. With the k_i's less than 1, the linked agenda will have efficiency benefits that the sequential one will not. I believe this doesn't change the fact that the agenda X then Y is worse for

2 than the unlinked game but it may mean that the linked game is preferred to the sequential agenda for some k_i. Also, as a, b are close to 1, then the sequential agendas will be interior to the utility space $\mathcal{U}_L$ which also makes this agenda less attractive relative to the linked game.

4. Appendix

Before proving the propositions, it is useful to provide the simple algebra for determining the points in $\mathcal{U}_L$ that yield utility equal to that from the unlinked games. So, suppose that we want to give agent 2 utility $U_2 = (sk_2 + bk_1)/2$ (his utility in the unlinked game). How much utility does 1 get? Suppose that we give agent 2 all of Y and agent 1 all of X. Then 2's utility is $U_2 = s(k_2 - 1) + b$. If $\frac{sk_2 + bk_1}{2} < s(k_2 - 1) + b$, then we can give 2 his unlinked utility by giving 1 all of X and some share of Y. This inequality is satisfied if $k_1 < 2(1 - \frac{s}{b}) + \frac{sk_2}{b}$. Agent 1's utility in this case can be found by solving for the value of y that yields agent 2 his unlinked utility. This value is defined by the equation $s(k_2 - 1) + by = \frac{sk_2 + bk_1}{2}$; or $y = \frac{s}{b} + \frac{k_1}{2} - \frac{sk_2}{2b}$. Substituting this value of y into agent 1's utility function ($U_1 = sa + k_1 - y$) yields $U_1 = as + \frac{k_1}{2} + \frac{sk_2}{2b} - \frac{s}{b}$. Since 1 has all of X and some Y, we are on that part of the utility frontier that is to the left of the kink. The slope there is $1/b$. The case in which $k_1 > 2(1 - \frac{s}{b}) + \frac{sk_2}{b}$ is determined analogously.

For for the case in which we want to fix agent 1's utility at his unlinked level, $U_1 = (sak_2 + k_1)/2$, we proceed similarly. In particular, suppose we give all Y to agent 2 and all of X to agent 1. Then 1's utility is $U_1 = sa + k_1 - 1$. If $\frac{sak_2 + k_1}{2} < sa + k_1 - 1$, then we can give 1 his unlinked utility by giving 2 all of Y and some share of X. This inequality is satisfied if $k_2 < 2(1 - \frac{1}{as}) + \frac{k_1}{as}$. Agent 2's utility in this case can be found by solving for the value of x that yields 1 his unlinked utility. This value is defined by the equation $\frac{sak_2 + k_1}{2} = sax + k_1 - 1$; or $x = \frac{2 + ask_2 - k_1}{2sa}$. Substituting this value of x into 2's utility function ($U_2 = s(k_2 - x) + b$) yields $U_2 = b + \frac{sk_2}{2} + \frac{k_1}{2a} - \frac{1}{a}$. Since 2 has all of Y and part of X, we are on the part of the utility frontier that is to the right of the kink. The slope there is a. The case in which $k_2 > 2(1 - \frac{1}{as}) + \frac{k_1}{as}$ is determined analogously.

Proof of Propositions 1 and 2:

Under separate negotiations, $U_1 = (ask_2 + k_1)/2$ and $U_2 = (sk_2 + bk_1)/2$. Fixing agent 2's utility at $U_2 = (sk_2 + bk_1)/2$, if $k_1 < 2(1 - \frac{s}{b}) + \frac{sk_2}{b}$, then agent 1 gets all of X and some share of Y. Agent 1's utility is given by $U_1 = as + \frac{k_1}{2} + \frac{sk_2}{2b} - \frac{s}{b}$. Also, the slope of the utility frontier at this point is $1/b$. From above, what we need to check is that $\frac{1}{b} < U_1/U_2 = \frac{2abs + bk_1 + sk_2 - 2s}{b(sk_2 + bk_1)}$. This inequality is satisfied if $2s(ab-1)>0$, which it is since $a,b>1$.

If $k_1 > 2(1-\frac{s}{b})+\frac{sk_2}{b}$, then agent 1 gets none of Y and only some share of X. Agent 1's utility is given by $U_1=(ab-1)+\frac{sak_2}{2}+k_1(1-\frac{ab}{2})$. Also, the slope of the utility frontier is a. Now we need to check that $a < U_1/U_2 = \frac{ab(2-k_1)+sak_2-2(1-k_1)}{(bk_1+sk_2)}$. This inequality is satisfied if $1-k_1 < ab(1-k_1)$. If $k_1 \neq 1$ then the inequality is satisfied strictly since $a,b>1$; if $k_1=1$ and $2-\frac{b}{s}>k_2$ (this is required to have $k_1 > 2(1-\frac{s}{b})+\frac{sk_2}{b}$) then it is satisfied as an equality and agent 1 gets all of the gains.

Now, fixing agent 1's utility at $U_1=(ask_2+k_1)/2$, if $k_2 < 2(1-\frac{1}{as})+\frac{k_1}{as}$, then agent 2 gets all of Y and some share of X.[4] Agent 2's utility is given by $U_2=b+\frac{sk_2}{2}+\frac{k_1}{2a}-\frac{1}{a}$. Also, the slope of the utility frontier at this point is a. From above, we need to check here that $a > U_1/U_2 = \frac{a(ask_2+k_1)}{2ab+k_1+ask_2-2}$. This inequality is satisfied if $2ab-2>0$, which it is since $a,b>1$.

If $k_2 > 2(1-\frac{1}{as})+\frac{k_1}{as}$, then agent 2 gets none of X and only some share of Y. In this case, 2's utility is given by $U_2=s(ab-1)+\frac{bk_1}{2}+sk_2(1-\frac{ab}{2})$. Also, the slope of the utility frontier at this point is $1/b$. From above, we need to check here again that $1/b > U_1/U_2 = \frac{ask_2+k_1}{abs(2-k_2)+2sk_2+bk_1-2s}$. This inequality is

[4]As with the case of $s=1$, if $k_1 > 2(1-\frac{s}{b})+\frac{sk_2}{b}$, then it must be that this inequality is satisfied. If $k_1 < 2(1-\frac{s}{b})+\frac{sk_2}{b}$, then either ineqaulity may be satisfied.

satisfied if $ab(1-k_2) > 1-k_2$. If $k_2 \neq 1$, then this inequality is satisfied strictly since $a, b > 1$; if $k_2 = 1$ and $2 - as > k_1$ (required so that $k_2 > 2(1 - \frac{1}{as}) + \frac{k_1}{as}$), then the condition holds as an equality and 2 gets all of the gains.