Bidding Behavior in a Repeated Procurement Auction

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Abstract: This paper formulates and estimates a dynamic auction game under the presence of capacity contraints. The estimation strategy is computationally simple as it does not require solving for the equilibrium of the game. It uses a two stage approach. In the first stage the beliefs of bidders are estimated using data on bids. In the second stage, an expression of the expected sum of future profits based on the beliefs is obtained, and costs are inferred based on the first order condition of optimal bids.

We apply the estimation method to repeated highway construction procurement auctions in the state of California between May 1996 and May 1999. In this market, previously won uncompleted contracts reduce the probability of winning further contracts. We quantify the effect of intertemporal constraints on bidders' costs and on bids.

Due to the intertemporal effect and also due to bidder asymmetry, the auction can be inefficient. Based on the estimates of costs, we calculate the cost minimizing allocation of contracts and we quantify efficiency losses.

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1. INTRODUCTION

Most of the literature on empirical estimation of auctions assumes a static auction setting. Parsch (1992), Laffont and Vuong (1995), Guerre, Perrigne and Vuong (2000), and others develop an empirical approach to quantify informational uncertainty in static auction games. On the contrary, there is little empirical work on dynamic auction games[2] or dynamic oligopoly games.[3]

This paper develops and illustrates an estimation method for dynamic oligopoly games. The method is computationally simple as it does not require solving for the equilibrium of the game. We apply the method to estimate a dynamic auction game under the presence of capacity contraints.

The estimation strategy is based on the first order condition of optimal bids. Our approach builds partially on the two stage approach that Elyakime, Laffont, Loisel and Vuong (1994), and Guerre, Perrigne and Vuong (2000) use for static models. These papers estimate the distribution of equilibrium actions based on the data. The estimate of the distribution function is then used to infer bidders' valuations based on the first order condition of optimal actions. We extend the estimation method to dynamic games. Our crucial idea is that the expected discounted sum of future profits, which enter the first order condition, can be written looking forward and depending entirely on the distribution of bidders bid choices.[4] Our method is computationally simple as it does not require

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[2] Laffont and Robert (1999) and Donald, Paarsch and Robert (1997) analyze finitely repeated auctions. Laffont and Robert consider a sequence of auctions in which, at each stage, an identical object is sold. Their model generates complex intra-day dynamics which are applied to data on eggplant auctions. Donald, Paarsch and Robert consider a model in which a finite number of objects are sold in a sequence of ascending-price auctions. They estimate the model using data on the sales of Siberian timber-export permits.

[3] Pakes (1994) summarizes the literature on estimation in dynamic games.

[4] A related estimation strategy of the value function is employed by Hotz and Miller (1993). They approximate the value function with discrete choices using estimates of choice probabilities. Their framework differs from ours in a number of ways: First, they consider a single agent dynamic decision problem. Second, they restrict their attention to discrete actions. Finally, they do not model informational constraints.

calculating the equilibrium of the game. We show that the value function is characterized by a linear difference equation which can be easily solved numerically.

In work in progress, Berry and Pakes (2000) consider a related estimation strategy for dynamic games. The distinguishing feature of their approach is to consider an alternative representation of the value function in which the expected sum of future profits is replaced with a sequence of future profit realizations. This representation is less attractive in dynamic auction games, because profits are not observed and cannot be expressed indirectly, from observed bids, without knowing the bidding equilibrium.

We apply the method to repeated procurement auctions for highway paving contracts. In this setting, previously won uncompleted contracts may affect the ability to win further contracts. Two distinct effects may arise: First, since the duration of highway paving contracts is a number of months, winning a large contract may commit some of the bidder's machines and paving resources for the duration of the contract. Although rental of additional equipment is available, this may increase total cost. Second, an experience effect may arise, since supplying services on a large contract may give a bidder the necessary expertise to conduct further services. The expertise effect may lower the cost for future contracts.

The features of our dynamic auction model are the following: Bidders learn their costs every period anew. Costs are drawn from a distribution that depends on the bidders state. We assume that the state is determined by the backlog, which measures the dollar value of previously won uncompleted contracts, the distance to the job, and the number of plants within the region. We do not specify the nature in which the state affects the distribution of costs. Instead, we let the data decide whether there is any relationship, and if so, how the state affects costs. In examining the equilibria of the game, we restrict our attention to markovian strategies that depend on payoff relevant variables. Specifically, we assume that bidding strategies are only a function of the state variables.

The results of our estimation using highway procurement data show that capacity constraints do affect firms' bidding strategies. In particular, the cost of taking on an additional contract is increasing in backlog. The increase in costs resulting from a larger than average backlog seems to cancel out any cost-reducing learning effects, if they exist. Based on the estimates of costs, we calculate the cost minimizing outcome and we quantify the efficiency losses.

The paper is organized as follows: Section 2 describes the industry and the data. We examine data on highway procurement contracts in California. The descriptive data analysis suggests the presence of capacity constraints. Estimates of the probability of submitting a bid reveal that bidders with low backlog levels are about twice as likely to submit a bid than bidders with high backlog levels.

Section 3 provides a simple theoretical example that illustrates the expected effect under the presence of capacity constraints. The example illustrates that, on average, constrained bidders bid less aggressively than unconstrained bidders. On the other hand, the expected future discounted payoff for constrained bidders is lower than for unconstrained bidders. The distributional assumptions in the example are a special case of the more general econometric model that we wish to take to the data.

Section 4 discusses the econometric method. The first order condition of optimal bids in the dynamic bidding game requires an expression for the expected sum of future profits. We establish that the expected future discounted payoff of bidders can be calculated based on estimates of bidders' beliefs. The method involves the numeric calculation of the future expected profits based on estimates of the distribution of bids.

Section 5 presents the estimates of the bidding model. The distribution of costs exhibits the expected properties of capacity constraints. To illustrate the estimates, we evaluate the estimated bid and cost distributions at average sample characteristics. The distribution of bids and costs at low backlog values stochastically dominates (in the first order sense) the distribution at high backlog values. Moreover, increasing the backlog appears to monotonically decrease the discounted sum of future profits.

Section 6 compares the observed outcome to the cost minimizing allocation of contracts. We find that inefficiencies arise on at least 20% of the contracts.

Section 7 provides some discussion and concludes.

2. THE DATA AND INDUSTRY

In this section, we describe some characteristics of the highway construction industry with emphasis on California. We present our data and describe the awarding process for contracts. In addition, we report evidence on the effect of previously won and uncompleted contracts on bid submission and bid level decisions.

2.1. THE CALIFORNIA MARKET

According to the 1992 US Census of Construction Industries[5] a total of $35.3 billion were spent during 1992 on highway and street construction activities. Transportation costs play an important role in this industry and we consider California as a market.[6]

Our data consist of California Department of Transportation (Caltrans) contract awards for highway and street construction made between December 1988 and May 1999.[7] Information on bids is available from May 1st, 1996 through May 31st, 1999. During the latter period, Caltrans advertised 2,566 projects from which 2,207 were finally awarded, 343 cancelled or postponed and 16 received no bids.[8]

The bid data contain the following information on every project awarded: Bid Opening Date; Contract Number; Location; Reservation Price; Number of Working Days and

[5] U.S. Department of Commerce, Economics and Statistics Administration, Bureau of Census.

[6] According to the 1992 census, 93% of the $2.7 billion California highway construction work was done by 896 establishments located in California.

[7] We obtained our data from the California Department of Transportation. The Office of Engineers publishes the data on the web: http : //tresc.dot.ca.gov/office/engineer

[8] According to the Federal Acquisition Regulation, part 14, a contract might be cancelled before opening if either the project is no longer needed or if the advertised contract characteristics become obsolete or inadequate and have to be revised. Other reasons to cancel are that all bids are either unreasonable or collusive or both. Cancellation can also occur if all reasonable bids belong to bidders that can not prove to be responsible. Additionally, the awarding agency might postpone the opening bids if it believes that a large fraction of bids have been delayed in the mail or other disruptive circumstances interfered in the regular reception of bids.

the Engineers' Estimate. Additionally, the data provides the Name, the Address, the Amount of the Bid and the Rank of the Bid for each of the bidding firms. In order to obtain a measure of past performance and maximum capacity of the firms active in our period of analysis, we complement the bid data with the Caltrans Contract Performance database. This source contains information on contracts awarded between December 1988 to May 1999. It provides the actual dollar amount received for the contract, the contract duration and the identity of the contractor.

Contracts are awarded by the California Department of Transportation subject to Federal Acquisition Regulations[9] and, therefore, is very similar to other states' procedures[10]. The process can be described in three steps: First, the Caltrans' Headquarters Office Engineer announces a project that is going to be let and the invitation to submit bids starts. This period is called the Advertising period and its length ranges between 4 and 10 weeks, depending on the size or complexity of the project. Occasionally, the Advertising period will be reduced to expedite project scheduling. Second, potential bidders may collect bid proposals that explain the plans and specifications of the work required.[11] Based on the proposal, bidders may submit a sealed bid. Bidders do not know who else submits a bid. For each bid, Caltrans checks that the bidding firm is a among the firms that are qualified to do business with Caltrans.[12] Third, on the letting day, the bids re-

[9] See the Project Submission and Estimate Guide at the Caltrans Office Engineer site, the Federal Acquisition Regulations and the Transport Acquisition Manual at the Department of Transportation Site.

[10] See Porter R.H. and Zona J.D. for a detailed explanation of New York State Department of Transportation, for instance.

[11] I.e., the project's characteristics, terms and identification number.

[12] Prior to the bidding, potential bidders have to qualify for contractual work for the Department of Transportation. In addition, firms are required deposit a predetermined amount of funds that have to be available. Receipt of funds clearance, permit issuance and local agencies approvals are needed for the bid of a firm to be accepted. A submitted bid can be rejected if it fails to conform to the essential requirements of the invitation for bids; does not conform to the applicable specifications without having been authorized to do so; fails to conform to the delivery schedule or permissible alternates stated in the invitation.

ceived are unsealed and ranked. The project is awarded to the lowest bidder provided it is below the reserve price and that the required responsibility criteria are fulfilled.[13] After each letting, a list of all bids and their rankings is announced and made accessible to the public.

The highway paving industry has already been studied by a number of authors. Porter and Zona (1993) and Feinstein, Block and Nold (1985) study issues of bidder collusion. Bajari (1997) studies asymmetry between bidders. He estimates a static bidding model based on a numerical calculation of equilibrium bid functions.

2.2. THE DATA

Between May 1st of 1996 and May 31st of 1999, the Caltrans awarded 2,207 contracts. The total value of the contracts was $4,661.73 million. Contracts are offered for sale on a regular basis with several letting dates per week. According to our data, the average duration between sales equals 2.96 days.

(TABLE 1A and TABLE 1B about here)

According to Table 1A, on average, there were 4.63 bidders per contract, ranging from 0 to 19 bidders across contracts. A total of 10,289 bids were received for these contracts and 16 contracts received no bids.[14] Table 1B reveals that a total of 96 contracts received one bid, 285 contracts received two, 393 contracts received three bidders and so on.

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[13] The bid is accepted if all computations and cost imputations are considered correct. The reserve price consists of a fixed non-random dollar amount which is assigned prior to the bidding. The winning firm is awarded the project no more than 30 days after the letting date.

[14] A total of 1,466 submitted bids, or 12% of all bid observations, violate the reserve price requirements. We exclude these bids from the analysis. These bids may have been submitted erroneously. Alternatively, bidders may have expected that the reserve price rule would not be enforced. According to conversations with Caltrans, it is indeed possible that the reserve price is altered ex post. Nevertheless, our data do not include information on bids below the reserve price being rejected, or bids above the reserve price being accepted. The lack of data points that fall into either of these two categories, suggests that the probability of these events is low. In our analysis we assume that the reserve price rule is binding.

Table 1B illustrates that, in highway procurement, competition and informational asymmetries may be important. As the number of bidders increases, the relative difference between the low bid and the Caltrans estimate falls. The low bid is 11% above the estimate when there is one bidder, and the low bid falls to 14% below the estimate when there are nine or more bidders.

The difference between the low and second lowest bid measures the money left on the table. As expected, the difference declines as the number of bidders increases. However, it does not approach zero. When there are nine or more bidders, the money left on the table is about 6% of the low bid, which suggests that the magnitude of informational asymmetries may be quite large.

In total, more than 500 bidders submit a bid at least once. Most of these bidders submit a bid only once, or only on a few occasions. For these bidders, the number of bid observations are too few to make inference about their behavior in a repeated game setting. We classify these bidders as fringe bidders. On the other hand, there is a small number of bidders that submit bids regularly and win a substantial fraction of contracts. With "regular" bidders we denote the set of the largest 10 firms in dollar value won that submit a bid at least 80 times during the sample period. These 10 firms win 25% of the total dollar value awarded and 17% of all contracts. We supplement the data with information on the locations of plants for these 10 selected firms. For each firm, we create a variable called distance that measures the distance between the contract location and the closest plant of a bidder.

2.3. THE EFFECT OF BACKLOG

During our sample period, the average contract duration is 156 days. We define Backlog as the amount of work measured in dollars that is left to do from previously won projects. The backlog variable is constructed in the following way: For every contract previously won, we calculate the amount of work measured in dollars that is left to do by taking the initial size of the contract and multiplying it by the fraction of time that is left until the project's completion date. For contracts that finished prior to the end of the sample period, we use the actual completion date of the contract, while for contracts that did not finish by the end of the sample period, we use the planned completion date. Based on this calculation, we determine the total amount of work measured in dollars that is left to do at any given point in time. We standardize the backlog variable by subtracting the bidder specific mean (calculated using daily observations) and dividing this difference by the bidder specific standard deviation. The resulting backlog variable is a number that is comparable across bidders.[15] There is substantial variation in the backlog variable. On average about 10% of the regular bidders have no capacity committed at the letting day while about 5% of the firms are about two standard deviations above their average backlog.

Under the presence of capacity constraints, bidders with above average backlog levels, or constrained bidders, may have, at least on average, a higher cost than unconstrained bidders. We may expect that constrained bidders bid less frequently and higher than unconstrained bidders. Alternatively, there may be benefits to performing several contracts simultaneously. This expertise effect may lower the cost of additional projects. We asses the presence of these intertemporal effects by using a reduced form analysis.

(Table 2)

Table 2 reports six columns of estimation results. The first and second columns report Probit estimates of the decision to submit a bid. The third and fourth column report Tobit estimates of the bid level decision. The fifth and sixth column report Heckman estimates of the bid level decision. We observe bids only if they are below the reserve price. To apply the Tobit and Heckman analysis we consider a transformation of the bid. The dependent

[15] We experimented with different definitions of the backlog variable. In particular, we also used a variable that measures the total backlog from previously won uncompleted contracts divided by the maximum dollar amount won during the sample period. The estimation results were very similar. We prefer the described specification because we do not have an accurate estimate of the maximum capacity. We also experimented with regional backlog variables. The regional effects appear less important perhaps because capacity and resources can be moved. Therefore, we report the results of the analysis when only the aggregate backlog level is used.

variable equals the reserve price minus the bid and is divided by the engineers' estimate. The dependent variable is negative if the bid is not observed, and it is positive if the bid is below the reserve price. Explanatory variables include contract specific characteristics, such as the estimate and the number of working days, and bidder specific characteristics such as the firm's size, measured as the number of plants in the region, the distance of the bidder's closest plant to the project location, and backlog. For each regression, we report two sets of estimates: with and without a set of firm specific dummy variables.

Backlog has a significant effect in all specifications. The sign of the coefficients suggests the presence of capacity constraints. The magnitude of the effect is substantial. An examination of the coefficients reveals that, on average, a constrained bidder is 50% less likely to submit a bid than an unconstrained bidder. An increase of the backlog from -1 to 1 increases the bid level between 2.5% and 7.6%.[16]

Firm heterogeneity that is not accounted for by the variables included in Table 2 may explain part of the backlog effect. To examine this hypothesis, we include estimates in columns two, four and six that include a set of firm specific dummy variables. We can test whether the coefficient of the backlog variables changes as we introduce firm specific fixed effects. As is evident in the Table, the coefficients do not change significantly. Unobserved firm heterogeneity does not explain the backlog effect.

In addition to capacity effects, asymmetries between bidders due to location and size are also important. Distance to the project decreases the probability of submitting a bid and increases the bid level. The size of the bidder, measured by the number of plants within the region, increases the probability of submitting a bid. The effect of size on the bid level decision is negative in the Tobit model and not significant in the Heckman model.

The estimates in Table 2 are suggestive for the presence for capacity constraints. Next section considers a theoretical example that illustrates the expected effect that capacity

[16] Note that if a firm's backlog changes from -1 to 1, its committed capacity increases from one standard deviation below its average to one standard deviation above its average value.

constraints may have on bidding behavior.

3. AN EXAMPLE OF THE EFFECT OF CAPACITY CONSTRAINTS

This section provides an example of the effect of capacity constraints on bidding behavior. The example requires that costs are drawn from an exponential distribution. We show that the resulting equilibrium distribution of bids is also an exponential distribution. The example serves two purposes: First, we illustrate the expected effects of capacity constraints. Second, the example provides guidance on how to interpret the estimates presented in section 5, since the distributional assumptions are a special case of the econometric model considered.

The model has the following features: A buyer offers an identical project for sale in every period, $t = 0, 1, \ldots, \infty$. The project is sold in a first price auction in which the winner is the seller with the lowest bid and a price equal to his bid.

Sellers (or bidders) draw a cost for the project every period. Bidders are one of two types: constrained bidders have a state of κ, while unconstrained bidders have a state of u. The bidder is constrained for exactly one period. The assumption can be formulated by defining a state variable, s_i, with $s_i^{t+1} = \kappa$ if bidder i wins in period t, and $s_i^{t+1} = u$ otherwise. The cost of unconstrained bidders is drawn from the distribution function, $F_u(c) = 1 - \exp^{-\alpha_u c}$, with $0 < c < \infty$. The cost of a constrained bidder is drawn from the distribution function, $F_\kappa(c) = 1 - \exp^{-\alpha_\kappa (c-\alpha)}$, with $0 < \alpha < c < \infty$, $\alpha_u > \alpha_\kappa > 0$ and $\alpha = (\alpha_u - \alpha_\kappa)/\{[(n-2)\alpha_u + \alpha_\kappa](n-1)\alpha_u\}$. Bidders are risk neutral. The expected payoff of bidder i in period t is given by his bid minus the cost times the probability of winning. Bidders discount future payoffs with a discount factor β. With $V_i(s_1, \ldots, s_n)$ we denote the expected discounted sum of future payoffs.

We consider subgame perfect equilibria and restrict attention to markovian strategies. We assume that bids are a function of the cost and whether the bidder is constrained or not. The following Proposition summarizes the equilibrium bid functions. All proofs are given in the Appendix.

Proposition 1. *The equilibrium bidding function of the unconstrained bidder, $b_u(c)$, and the bidding function of a constrained bidder, $b_\kappa(c)$ are given by:*

$$b_u(c) = c + \frac{1}{(n-2)\alpha_u + \alpha_\kappa} + \beta\alpha,$$

$$b_\kappa(c) = c + \frac{1}{(n-1)\alpha_u} + \beta\alpha.$$

The equilibrium bid prices in Proposition 1 are of a simple form. They equal to marginal cost plus a mark-up. The mark-up consists of two terms: The first mark-up reflects the level of competition in the current period. Intuitively, constrained bidders charge a higher mark-up because they face tougher competitors than unconstrained bidders. The second mark-up measures the discounted cost of being constrained in the future. It equals the difference in the expected discounted future sum of profits for an unconstrained and a constrained bidder.

The following Corollary summarizes a number of properties induced by the equilibrium bid functions.

Corollary. *The following properties are satisfied in equilibrium:*

(i) $b_\kappa(c) < b_u(c)$ *for a given cost draw* c.

(ii) Let $G(b|s_i, s_{-i})$ *denote the distribution function of bids by bidder* i:

$$G(b|s_i = \kappa, s_{-i}) < G(b|s_i = u, s_{-i}).$$

(iii) $V_i(s_1, \ldots, s_i = u, \ldots, s_n) > V_i(s_1, \ldots, s_i = \kappa, \ldots, s_n)$.

(iv) $b_\kappa(c)$ *and* $b_u(c)$ *are increasing in the discount factor* β.

The first property in the Corollary states that, for a given cost, the constrained bidder bids more aggressively, or lower, than an unconstrained bidder. The intuition is that, conditional on the cost draw, constrained bidders expect tougher competition than unconstrained bidders. The property implies that first price auctions can be inefficient in the sense that the winner need not be the firm with the lowest cost. The second property states that, ex ante, constrained bidders are less likely to submit a low bid than unconstrained bidders. More precisely, the distribution of bids by an unconstrained bidder is stochastically dominated by the distribution of bids by a constrained bidder. The third property states that unconstrained bidders have a higher future discounted expected profit than constrained bidders. The fourth property states that the bids are increasing in the discount factor. In other words, the opportunity cost of being constrained in the future causes bidders to bid less aggressively and to shade their bids up.

It can be shown that properties (i), (ii) and (iii) hold more generally for arbitrary cost functions provided the following condition on the shape of cost distribution functions is satisfied: For any cost realization, the hazard function of costs of the constrained bidder is less than the hazard function of costs of the unconstrained bidder. Properties (i) and (ii) in the Corollary arise also in static asymmetric auctions. Maskin and Riley (1998) establish them for two bidder auctions. Qualitatively similar implications have been obtained in static models of collusive bidding, in Pesendorfer (1998), and mergers among bidders, in Waehrer (1999).

Table 2 provides some evidence on the shape of the bid distribution function. The evidence appears to confirm property (ii). However, to shed more light on properties (i) to (iv) and to quantify effects, we take the bidding model to the data. The next section describes our econometric model. The econometric model is more general than the model considered in this example.

4. ECONOMETRIC MODEL

This section describes the model and the estimation strategy. Subsection 4.1. explains the model assumption. Subsection 4.2. describes the procedure to infer costs. Subsection 4.3. describes the parametric specification of the bid distribution functions.

The bidding model that we take to the data has the following features: There are an infinite number of periods, $t \in \{0, 1, \ldots, \infty\}$. In every period, the buyer offers a single contract for sale. The contract characteristics, s_0, are revealed at the beginning of each period and are not previously known.[17] There are two types of bidders: regular and fringe bidders. Fringe bidders have a short life and exit in the period they entered.[18] Regular bidders stay in the game forever. The set of regular bidders is denoted by $\{1, \ldots, n\}$ and the set of fringe bidders is denoted by $\{n+1, \ldots, n_F\}$.

Each period, bidder i learns her cost for the contract. The cost is privately known and denoted by c_i^t. The priors of all bidders about the cost of a regular bidder c_i^t are identical and represented by the distribution function $F(c|s_0, s_i^t)$. The variable s_0 measures contract characteristics and s_i^t is a vector of variables that summarizes bidder characteristics including the backlog of bidder i. We assume that both s_0 and s_i^t are observable to all bidders. The distribution of costs has a continuous density function $f(c|s_0, s_i^t)$ and support $[0, C]$. Similarly, the cost of a fringe bidder is drawn from a continuous distribution function $F(c|s_0)$ with associated density function $f(c|s_0)$ and support $[0, C_F(s_0)]$. The bidders may submit a bid for each contract which is the price at which they are willing to provide the service. The bidder with the lowest bid wins the contract and receives her bid. All agents are risk neutral. Ties are resolved by the flip of a coin. The buyer imposes a fixed (non random) reserve price $R(s_0)$, meaning that bids above $R(s_0)$ are rejected. To

[17] In highway procurement auctions, future and upcoming projects are known to bidders only shortly in advance. In particular, project descriptions become available 10 weeks prior to the letting date for projects with estimated values exceeding $50 million. The length of the announcement period decreases as the size of the project becomes smaller. Projects with an estimated value of 1 million or less are announced 4 weeks prior to the letting date.

[18] In the data, we observe a number of bidders that submit a bid only once, or a small number of times. On the other hand, we observe a number of bidders that submit bids frequently. To account for this difference, we classify firms into two groups: Regular bidders, which are the largest 10 firms in $ value won and with at least 80 bids submitted; and Fringe bidders which are the remaining firms.

simplify notation we write $R(s_0)$ as R. We assume that $C_F = R$. We introduce a common discount factor parameter, $\beta \in (0,1)$, that measures firms' patience with regard to future profits.

We consider subgame perfect equilibria and restrict attention to markovian strategies. A strategy for bidder i is a function of the state vector and bidder i's cost at time period t. The strategy can be written as $b(c_i^t, s_0, s_i, s_{-i})$ where s_{-i} denotes the vector of the states of the remaining regular bidders $(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n)$. We sometimes use the symbol s to denote the vector of all bidders' specific state variables $(s_1, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_n)$.

Each period, payoffs are bounded by the reserve price. The boundedness ensures that the discounted expected future payoff for regular bidder i exists. It can be written as,

$$V_i(s_i, s_{-i}) = E_{s_0}\Big[\int \max\nolimits_b \Big\{[b - c] Prob(i \;\text{ wins } | b, s_0, s)$$

$$+\beta[\sum_{j=1}^{n_F} Prob(j \;\text{ wins}| b, s_0, s) V_i(s' | s_0, s, \text{j wins})\Big\} f(c | s_0, s_i) dc\Big], \qquad (4.1.)$$

where E denotes the expectation operator with respect to contract characteristics and s' denotes next period's state. Notice that next period's state is determined by s_0, s and the identity of the winning bidder. In addition, the state variables are not binary variables, as in the example in section 3, but continuous variables.

For a fringe bidder, the ex ante payoff equals to:

$$E_{s_0}\Big[\int \max\nolimits_b \Big\{[b - c] Prob(i \;\text{ wins } | b, s_0, s)]\Big\} f(c | s_0) dc\Big].$$

We assume that a markovian equilibrium with strict monotone bidding strategies $b_i(c|s_0, s)$ exists. The existence of an equilibrium in asymmetric static games is established by Maskin and Riley (1996).[19]

[19] Maskin and Riley consider payoff functions of the form $U_i(b, c) \cdot Prob(\text{bidder } \; i \; \text{ wins})$.

In subsequent sections, we use the following notation: We denote the distribution function of equilibrium bids of bidder i by $G(b|s_0, s_i, s_{-i})$, and the associated density function by $g(b|s_0, s_i, s_{-i})$. The probability that a bid b wins the contract for bidder i can be written as $\prod_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})]$.

4.2. ESTIMATION METHOD

Our estimation method is computationally simple as it does not require calculation of the equilibrium.[20] It is based on the necessary first order condition of optimal bids. Estimation methods based on the first order condition in static games are well known in the literature. In the context of static first price auctions, Elyakime, Laffont, Loisel and Vuong (1994) make the observation that beliefs about the equilibrium play of agents can be estimated based on data on bids. The cost realization is then inferred from the first order condition.[21] We extend the estimation method to dynamic oligopoly models.

We describe the method beginning with the necessary first order condition for equilibrium bids. Let $\phi(.)$ denote the unobserved cost associated with a bid. It is a function of the bid, b and the state, s. Let $\tau(b|s_0, s) = \frac{g(b|s_0,s)}{1-G(b|s_0,s)}$ denote the hazard function of

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Our model is contained in this class of payoff functions. To see this, notice that we can write the payoff in period t to bidder i with contract realization s_0 as: $\big[b - c_i^t +$

$\beta V_i(s'|s_0, s, \text{i wins}) + \beta \sum_{j \neq i} \frac{Prob(j \;\; \text{wins}|b,s_0,s)}{Prob(i \;\; \text{wins}\;|b,s_0,s)} V_i(s'|s_0, s, \text{j wins})\big] \cdot Prob(i \;\; \text{wins}\;|b, s_0, s) +$

constant. Maskin and Riley show, in this class of static games, that an equilibrium exists

if two conditions are satisfied: $\frac{\partial U_i}{\partial c} < 0$ and $\frac{\partial^2 U_i}{\partial c \partial b} \leq 0$. In our model these conditions are satisfied, which implies that there exists an equilibrium for any state s and for any continuation value $V_i(s)$. Due to the markovian assumption, this implies that there exists an equilibrium in the repeated game.

[20] In dynamic games, current computing methods do not permit estimation based on a comparison of numerical calculations of equilibrium outcomes and observed outcomes.

[21] An attractive feature of the static auction model is that the residual of the first order condition is the cost realization, providing a link between the theoretical and econometric model. As we describe below, the same property extends to a dynamic auction model.

bids. The first order condition for optimal bids yields the following equation[22] for privately known costs, ϕ:

$$\phi = b - \frac{1}{\sum_{j \neq i} \tau(b|s_0, s_j, s_{-j})}$$

$$+\beta \sum_{j \neq i} \frac{\tau(b|s_0, s_j, s_{-j})}{\sum_{l \neq i} \tau(b|s_0, s_l, s_{-l})} [V_i(s'|s_0, s, \text{i wins}) - V_i(s'|s_0, s, \text{j wins})] \qquad (4.2.)$$

Equation (4.2.) provides an explicit expression of privately known costs that involves the hazard function of bids, τ, and the value function, V_i. Equation (4.2.) has the same interpretation as the bid functions in Proposition 1 which are derived under a number of additional assumptions. Specifically, equation (4.2.) states that the cost equals the bid minus a mark-down. The mark-down has two parts: The first part accounts for the level of competition in the current period. The second part accounts for the incremental effect on the future discounted profit if firm i wins the contract instead of another firm.

The inference of costs based on equation (4.2.) requires an estimator of the bids hazard function and an estimator for the value function. An estimator of the hazard function can be directly obtained from the data on bids and state variables. An expression of the value function is given in equation (4.1.). However, equation (4.1.) involves cost variables that are unobserved and decisions by multiple agents which are endogenous.

The crucial idea of our method is that the distribution of bids, which determines the law of motion of the state variables, also determines the expected future profits. Thus, we can write the value function as a recursive equation that is determined by the distribution of bids. The following Proposition states this result.

Proposition 2. *In equilibrium, the expected future discounted sum of profits for a*

[22] See Appendix for a more detailed description.

regular bidder is given by the following expression:

$$V_i(s_i, s_{-i}) = E_{s_0}\{\mathrm{E}_{b_i}[\frac{1}{\sum_{j\neq i}\tau(b|s_0, s_j, s_{-j})}]+$$

$$+\beta\sum_{j\neq i}\mathrm{E}_{b_j}[1+\frac{\tau(b|s_0, s_i, s_{-i})}{\sum_{l\neq i}\tau(b|s_0, s_l, s_{-l})}]V_i(s'|s_0, s, \text{j wins})\} \qquad (4.3.)$$

where $\mathrm{E}_{b_i}[\alpha(b)] = \int_{\underline{b}(s)}^{R}[\prod_{j\neq i}\alpha(b)[1-G(b|s_0, s_j, s_{-j})]g(b|s_0, s_i, s_{-i})]db$*, for any given function* $\alpha(b)$.

Proposition 2 shows that the beliefs of bidders characterize the equilibrium payoffs of the dynamic game. To illustrate the intuition for the result, we point out the implications that knowing the distribution of bids has: First, the distribution of bids by other bidders reduce the dynamic game to a single agent dynamic decision problem. I.e., bidder i maximizes the discounted sum of future payoffs taking into account only the distribution of bids by other bidders. Second, the distribution of bids determines the equilibrium law of motion of the state variables, which determines the weight assigned to future payoff realizations. Finally, the first order condition of optimal bids provides us with an explicit expression of the unknown costs in terms of equilibrium bids and the distribution of bids. If we make use of this expression in the value function, then we obtain an expression of the value function that depends entirely on the distribution and the density of equilibrium bids.

Proposition 2 provides the basis for our empirical analysis. It says that if an estimator of the distribution and density of bids is available, then, the value function is given implicitly by equation (4.3.). Our empirical strategy is the following: We infer costs based on the first order condition of optimal bids (4.2.). To make the inference, we adopt a two stage approach: First, we estimate the equilibrium distribution function of bids from the bidding data. Second, we use equation (4.3.) to write the equilibrium value function in terms of bids and to infer costs based on equation (4.2.).

In the example in section 3 we were able to solve equation (4.3.) analytically. In general, numerical methods can be used to approximate the value function based on equation (4.3.). The assumption of a markovian strategy space reduces the dimensionality of the value function approximation drastically. In particular, the dimensionality does not increase as the number of bidders increases. Markovian strategies require that bidders with the same state follow the same bidding strategy. Thus, we can exchange the state of two competitor bidders without affecting the value of bidder i's expected future payoffs. As been shown by Pakes (1994) exchangeability restricts the functional form of the value function. In particular, for a Jth order polynomial approximation of the value function the polynomial coefficients associated with each state variable s_j are identical. Thus, a Jth order polynomial approximation of the value function involves $2 \cdot J$ variables given by $s_i, \sum_{j \neq i} s_j, s_i^2, \sum_{j \neq i} s_j^2, \ldots, s_i^J, \sum_{j \neq i} s_j^J$.

In the empirical section, we approximate the value function with a three dimensional state vector for each bidder: The first dimension of the state variables is backlog. The second dimension accounts for distance. Observe that, when the value function is evaluated, the location of future contracts is not yet known. Thus, the state variable has to summarize the distribution of the bidder's distances to possible contract locations. We use the average of the distances between the bidder's closest plant and the observed contract locations as a measure that summarizes the bidder's distribution of distances. The third dimension of the state variable accounts for the firm's size. When we evaluate the value function, the region of the contract is not yet known. The third state variable has to summarize the distribution of the firm's plants across regions. Similar to distance, we use the bidder's average number of plants per region across all observed contracts, as a measure to summarize this distribution.

Due to the exchangeability property, the value function for bidder i, involving three state variables per bidder, can be reduced to a four dimensional problem. The dimensions are: The backlog of bidder i, and three state variables per competitor. By approximating a value function for each bidder separately, we can drop the location and size measures of the bidder under consideration without affecting the approximation.

Numerical methods to approximate the value function are discussed in more detail in Judd (1998). We briefly summarize the method we use: We select a grid of state vectors $S = (s^1, \ldots, s^m)$. For every point $s \in S$ the expectations on the right hand side of equation (4.3.) in Proposition 2 can be calculated. Specifically, we numerically evaluate the expected values $A(s) = E_{s_0}\{\mathrm{E}_{b_i}[\frac{1}{\sum_{j\neq i} \tau(b|s_0,s_j,s_{-j})}]\}$ and $B_j(s) = E_{s_0}\{\mathrm{E}_{b_j}[1 + \frac{\tau(b|s_0,s_i,s_{-i})}{\sum_{l\neq i} \tau(b|s_0,s_l,s_{-l})}]\}$. The first expectation is with respect to contract characteristics, which we model as a uniform draw from the discrete set of observed contract characteristics. The second expectation is determined by the estimates of the bid distribution which we describe in the next section. The value function is given by the equation, $V_i(s) = A(s) + \beta \sum B_j(s) V_i(s^j)$. Observe that s^j is contained in the grid S and, since A and B are known, we can rewrite the value function in matrix notation as: $[I - \beta B]V_i = A$, where V_i denotes the vector $(V_i(s_1), \ldots, V_i(s_m))$, I denotes the m-dimensional identity matrix and B denotes the transition matrix obtained based on the coefficients $B_j(s)$. The value function can be expressed as $V_i = [I - \beta B]^{-1} A$. To evaluate the value function for all points, possibly outside the grid S, we approximate the function with a quadratic polynomial.

A nice feature of the auction model is that, based on the estimates of the bid distribution and associated value function, the cost realizations can be inferred as the residual from the first order condition of optimal bids. The distribution of costs are the only unknown parameter in the model. Of course, our estimation method extends to models in which the per period payoff function may depend on additional parameters. For example the Hotz and Miller (1993) estimator can be applied.[23]

[23] Hotz and Miller define an iterative procedure for single agent dynamic decision problems in which the value function is calculated for a given parameter vector at every step of the iteration. A methods of moments estimator is defined which compares the predicted and observed actions based on the first order condition. Although Hotz and Miller consider discrete choices, it can also be applied to continuous choices.

In the following section, we discuss our estimators of bid distribution functions. In general there are a number of methods to estimate the model based on equations (4.2.) and (4.3.). We choose a parametric approach to permit a richer specification of the state variables.[24]

The density function of bids by fringe bidders is specified as a beta density function. In our specification, the parameters of the density are a function of the state variables. By suppressing the parameters' dependence of the state variable, we can write the density function as:

$$g_f(b|s_0, s) = (\frac{b-\theta_3}{R})^{\theta_4 - 1}(\frac{R-b}{R})^{\theta_5 - 1}\frac{1}{B(\theta_4, \theta_5)}$$

where the distributional parameters, $\theta_3, \theta_4, \theta_5$, depend on the state variables as will be described below, R denote the reserve price, the support of fringe bids is $[\theta_3, R]$ and the function $B(\theta_4, \theta_5)$ denotes the beta function.[25] This specification reflects our assumption that bidders know whether a fringe bidder is present at the auction.

We experimented with different specifications for the distribution function of bids by regular bidders. We finally decided on a mixture of two distribution functions in order to permit a flexible functional form. The distribution function of bids by regular bidder i is the product of a power function distribution and a Weibull distribution function. We may write the density function of bids in the following way:

$$g(b|s_0, s_i, s_{-i}) =$$

[24] At an earlier stage of the research, we used non-parametric Kernel methods to estimate the distribution of bids. The results are discussed and summarized in Jofre-Bonet and Pesendorfer (2000). A shortcoming of this approach is that the number of covariates has to be small, which limits the ability of the model to capture the richness of the data.

[25] $B(v, w) = \int_0^1 u^{v-1}(1-u)^{w-1}du.$

$$\begin{cases} \frac{\theta_1}{\theta_2}(\frac{b-\theta_3}{\theta_2})^{\theta_1-1}\exp^{-(\frac{b-\theta_3}{\theta_2})^{\theta_1}} & \text{if } b > R \\ \theta_6\frac{(b-\theta_3)^{\theta_6-1}}{(R-\theta_3)^{\theta_6}} \cdot [1-\exp^{-(\frac{b-\theta_3}{\theta_2})^{\theta_1}}] + (\frac{b-\theta_3}{R-\theta_3})^{\theta_6}\frac{\theta_1}{\theta_2}(\frac{b-\theta_3}{\theta_2})^{\theta_1-1}\exp^{-(\frac{b-\theta_3}{\theta_2})^{\theta_1}} & \text{if } b \leq R. \end{cases}$$

The support of bids for regular bidders is $[\theta_3, \infty)$.[26] The parameters of the density function are $\theta_1, \theta_2, \theta_3, \theta_6$ and they depend on the state variables as will be explained below.

As mentioned, the distributional parameters are a function of state variables. Nevertheless, there are a number of restrictions on the functional form of the dependence of the parameters on the state variables. First, there are the restrictions that $\theta_1, \theta_4, \theta_6 \geq 1$ and that $\theta_2, \theta_5 > 0$. These restrictions ensure that the conditions for a probability density function are satisfied and that the monotonicity of the bid hazard functions holds, which is a required condition from the bidding model.[27] To impose these restrictions, we define, $\theta_j = 1 + \exp^{\theta_{0j}}$ for $j = 1, 4$ and $\theta_j = \exp^{\theta_{0j}}$ for $j = 2, 5$.

The second restriction is that the lower bound of bids, $\theta_3(s)$, is a function of the state vector in which the identity of bidders does not matter. The reason is that the markovian strategies require that bidders with the same state follow the same bidding strategy. Similarly, for the parameters entering the bid distribution of fringe firms, θ_{04}, θ_{05}, the order of elements in the vector s does not matter, as the order of elements in the vector s_{-i} does not matter for parameters θ_{01}, θ_{02} either. We consider the following specification which incorporates the described restrictions:

[26] In principle, it is possible to estimate different supports of bid distributions for individual bidders. However, we restrict the supports to be identical. The main reason is that, empirically, with a small data sample, it is difficult to determine whether bidders have indeed different supports or not. Waehrer (1999) shows that the identical support assumption arises in a static first price auction, when costs are drawn from distributions with identical supports.

[27] For $\theta_1, \theta_6, \theta_4 < 1$ the hazard of bids can be decreasing which would violate the condition that equilibrium bids are monotone increasing.

$$\theta_{0j}(s) = \gamma_{j,0} + \gamma_{j,1}s_0 + \gamma_{j,2}s_i + \gamma_{j,3} \cdot \sum_{l=1}^{n} s_l \qquad \text{for } j = 1, 2$$

$$\theta_{0j}(s) = \gamma_{j,0} + \gamma_{j,1}s_0 + \gamma_{j,2} \cdot \sum_{l=1}^{n} s_l \qquad \text{for } j = 4, 5$$

The state variable, s_i, denotes a vector of bidder characteristics, and the state variable, s_0, is a vector of contract characteristics.[28]

There is a large literature on the estimation of parameters for the beta, the power and the Weibull distribution function. Smith (1985) considers the estimation problem with a one dimensional lower bound. He establishes that the maximum likelihood estimates of the parameters are consistent provided that $\theta_1, \theta_6, \theta_4 \geq 1$.[29] Smith (1985) shows that estimation of the lower bound and the other (scale and shape) parameters are asymptotically independent. He establishes that the asymptotic distribution of the scale and shape parameters is normal. Harter and Moore (1965) and Smith (1985) describe an estimation procedure which involves two stages: In the first stage the lower bound is estimated using the sample minimum. In the second stage the observation involving the sample minimum is dropped and the shape and scale parameters are estimated using maximum likelihood. This method yields consistent estimates.[30] Smith shows that the shape and scale parameters are asymptotically normally distributed and also asymptotic efficient provided $1 \leq \theta_1, \theta_6, \theta_4 \leq 2$. In our case the parameters $\theta_1, \theta_6, \theta_4$ take on different values depending on the state variables and we are not able to apply the efficiency result.

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[28] The state variables for bidders include the standardized backlog from previously won uncompleted projects, the distance between the closest plant of the bidder to the contract location and a dummy variable for each bidder. The contract characteristics include the estimated cost of the project, the number of working days and the number of fringe bids.

[29] For $\theta_1, \theta_6, \theta_4 < 1$ the maximum likelihood estimators may be inconsistent.

[30] Here, the parameters are consistently estimated even when $\theta_1, \theta_6, \theta_4 < 1$. Thus, this restriction can be omitted in the estimation and used as a test of the auction model.

We follow the later two stage estimation procedure. In our case, the lower bound of bids is not a single parameter, but depends on contract characteristics. To accommodate this dependence, we consider a projection of bids onto state variables, $\ln(b) = a_0 s_0 + a_1 \sum s_i + \varepsilon$, to estimate the lower bound. The estimator of the lower bounds of bids, $\hat{\theta}_3$, is defined as $\exp(\hat{a}_0 s_0 + \hat{a}_1 \sum s_i + \min_j \hat{\varepsilon}_j)$, where $\min_j \hat{\varepsilon}_j$ denotes the lowest residual from the projection. In the second stage, the bid observation with the lowest residual $\min_j \hat{\varepsilon}_j$ is dropped and the parameters $\theta_1, \theta_2, \theta_4, \theta_5, \theta_6$ are estimated using maximum likelihood.

Finally, in order to estimate the density functions described, we have to take into account that the bid data for regular bidders are censored. We only observe bids that are below the reserve price, R. Let o_i^t be a dummy variable that equals one if we observe a bid by bidder i on contract t, and zero otherwise. In an abuse of notation, we abbreviate the dependence of parameters on the state vector with superscripts, e.g. we write $\theta_j(s)$ as θ_j^t. We denote with n_t the set of fringe bidders on auction t. Doing so, we may write the likelihood of bids by:

$$L = \prod_i \prod_t [\frac{\theta_6}{R^t - \theta_3^t} (\frac{b_i^t - \theta_3^t}{R^t - \theta_3^t})^{\theta_6 - 1} \cdot [1 - \exp^{-(\frac{b_i^t - \theta_3^t}{\theta_2^{it}})^{\theta_1^{it}}}] +$$

$$+ \ (\frac{b_i^t - \theta_3^t}{R^t - \theta_3^t})^{\theta_6} \frac{\theta_1^{it}}{\theta_2^{it}} (\frac{b_i^t - \theta_3^t}{\theta_2^{it}})^{\theta_1^{it} - 1} \exp^{-(\frac{b_i^t - \theta_3^t}{\theta_2^{it}})^{\theta_1^{it}}}]^{o_i^t} \cdot [\exp^{-\left(\frac{R^t - \theta_3^t}{\theta_2^{it}}\right)^{\theta_1^{it}}}]^{1 - o_i^t} \quad .$$

$$\cdot \prod_t \prod_{j \in n_t} (\frac{b_j^t - \theta_3^t}{R^t})^{\theta_4^t - 1} (\frac{R^t - b_j^t}{R^t})^{\theta_5^t - 1} \frac{1}{B(\theta_4^t, \theta_5^t)},$$

where b_i^t is the bid by regular bidder i in contract t, b_j^t is the bid by fringe bidder j in contract t, and superscript i in parameters θ_1^{it} and θ_2^{it} accounts for the possibility that parameters θ_1 and θ_2 may differ between bidders.

We maximize the logarithm of the likelihood. The next section reports the estimation results.

5. ESTIMATION RESULTS

This section discusses the estimates. We report how well the estimates predict the data. We then illustrate the predicted effect of selected variables. The illustrations suggest that the effect of the backlog variable is in accordance with the expected effect under the presence of capacity constraints. Moreover, the effect is substantial, suggesting that capacity constraints play an important role in highway bidding.

The parameter estimates for five sets of parameters are reported in Tables 3. The parameter vectors θ_1, θ_2, and θ_6 characterize the distribution function of regular bidders, the parameter vector θ_3 characterizes the lower bound of bids and θ_4, θ_5 characterize the distribution of fringe bids.

(Table 3)

The backlog variable enters directly in θ_1, θ_2. To test whether backlog variables have jointly no significant effects on regular bids, we construct a likelihood ratio test. The data reject the null hypothesis of no significant effects.

As a measure of the goodness of fit of the model, we randomly draw bids from the estimated distribution of bids and compare them to the observed bids. To account for contract heterogeneity, we normalize bids by dividing them by the reserve price.

The estimates predict well the observed distribution of fringe bids. We draw 10,000 fringe bids. On average, the predicted fringe bid equals 77.04% of the reserve price with a standard deviation of 13.05%. The observed fringe bid equals 76.97% of the reserve price with a standard deviation of 12.42%. The difference between the two means is not significant.

The predicted probability of observing a regular bid equals 4.89%. In the data, the probability of observing a regular bid equals 4.80%. The difference between the two numbers is not significant. Conditional on observing a regular bid, the mean bid equals

80.63% of the reserve price with a standard deviation of 14.14%. The observed regular bid equals 78.90% with a standard deviation of 11.09%. Conditional on observing a regular bid, the predicted bid is, on average, higher and has a higher standard deviation.

A closer inspection of the distribution of regular bids conditional on observing a regular bid, reveals that the difference between the predicted and observed distribution is attributable to a small fall in the number of bid observations close to the reserve price. This decline in the number of observations implies a decrease in the hazard rate at points close to the reserve price. The decrease in observations at a given point violates the assumption of a monotone increasing hazard rate in the bidding model, which guarantees that the bid function is invertible. Although outliers in the data may cause the fall, there are two alternative explanations on why the decline in number of points close to the reserve price might be occurring: First, bidders may expect a secret reserve price in addition to the announced reserve price. Second, bidders may not be fully aware of the reserve price rule. Unfortunately, our data are not rich enough to permit us to distinguish the alternative explanations. We do not observe bids below the reserve price that were rejected and the data do not provide information explaining the beliefs of bidders about the reserve price rule. For these reasons, we decided to impose the monotonicity of the hazard function in the estimation. The monotonicity requirement accounts for the weaker fit at points close to the reserve price.[31]

To assess the effect of individual variables, we evaluate their effect on the probability of submitting a bid at sample average values of explanatory variables. In general the predicted effect confirms the intuition: The probability of submitting a bid decreases

[31] The restriction to a parametric class of distribution functions does not explain the weak fit. Indeed, a closer fit is obtained when we estimate a product of a Beta and a Weibull distribution function. Conditional on observing a regular bid, the resulting predicted mean bid equals 79.17% of the reserve price with a standard deviation of 9.81%. The reason for the improvement is that the estimated parameters for the beta density permit a fall in density close to the reserve price, which improves the fit, but violates the monotonicity assumption of the hazard function.

monotonically in backlog which is consistent with the notion of capacity constraints. An increase in the number of competing fringe firms has a negative effect on the bid submission decision although the effect is small in magnitude. Distance affects the probability of bid submission negatively, and the number of wins in the region has a positive effect.

(Figure 1)

Figure 1 illustrates the effect of the backlog variable on the bid distribution of regular bidders. It shows the distribution function between the lower bound of bids and the reserve price and evaluated at sample average values of state variables. Two distribution functions are reported. The solid function assumes a backlog equal to -2 and the dashed function assumes a backlog equal to 2. The dotted lines represent 90% confidence intervals. The confidence interval in Figure 1 (and all subsequent standard errors of estimates) are calculated using the delta method. The Figure illustrates that the distribution of constrained bids stochastically dominates, in the first order sense, the distribution of unconstrained bids. On average, unconstrained bidders are about twice as likely to submit a bid than constrained bidders. This finding is in accordance with property (ii) in the Corollary in section 3. The evidence suggests the presence of capacity constraints.

(Figure 2)

Figure 2 presents the quadratic approximation of the value function for bidder 3. We arbitrarily select one bidder, bidder 3, and depict estimates fixing the state variables at the sample average values for bidder 3. The plot illustrates the discounted expected future profit of bidder 3 by varying the backlog variable of bidder 3 between -1.6 and 1.6. We assume an annual discount factor of 0.90 in the calculation of the value function. The dotted lines depict the 90% confidence interval, again calculated using the delta method. The derivative of the value function with respect to the parameter vector is obtained by taking the derivative in equation (4.3) and solving the resulting recursive equation numerically in the same way as the value function.

Figure 2 illustrates that the value function is decreasing in the level of backlog, which is in accordance with the expected effect in (iii) of the Corollary in section 3. In Figure

2, backlog reduces the value function in total by about 30%. Value function estimates for other bidders are of different magnitude, but in general similar shape. An exception is bidder 5 for which the value function increases initially and then decreases.

(Figure 3)

Figure 3 illustrates the equilibrium bid function for bidder 3 which is estimated using equation (4.2.). The bid function is plotted by fixing the state variables at sample average values for bidder 3 and varying the cost. In addition to the bid function, the 45 degree line is reported. The mark-up denotes the difference between the bid and the cost of a bidder. In the Figure, the mark-up is the distance between the bid and the 45 degree line. An examination of all observed bids by bidder 3 reveals that the median estimated mark-up for this bidder equals 15.3% of the bid. The mean mark-up is higher and equals 36.4% of the bid. The estimated mark-up differs across bidders. The median mark-up across all observed regular bids equals 27.7% and the mean mark-up equals 38.4%. Although the magnitude of the mark-up may appear large, it appears in accordance with descriptive evidence in Table 1a and Table 1b. The difference between the lowest and second lowest bid is, on average, 10% of the value of the bid.

A substantial portion of the mark-up of regular bidders is attributable to the loss in future discounted value due to limited capacity. This loss reflects the cost of winning today versus winning later. We can measure this loss based equation (4.2.) which decomposes the mark-up into two parts: The first part reflects contemporaneous competition. The second part measures the loss in value of winning today versus winning later. For bidder 3, on average, across all observed bids 48.6% of the mark-up is attributable to the second part, which is the option value of winning today versus winning later. The number varies across bidders. Across all regular bidders 50.4% of the mark-up is attributable to the second part.

(Figure 4)

Figure 4 depicts the distribution function of costs for bidder 3. Distribution functions are reported for two values of backlog and holding other state variables at sample average values. The backlog values are -2 and 2. The dotted lines represent 90% confidence intervals. The estimated cost distribution functions are truncated due to the truncation of the bid distribution functions and reported for a common range of costs. Figure 4 documents that the cost distribution of the constrained bidder stochastically dominates in the first order sense the cost distribution of the un-constrained bidder. On average, the probability that the cost is below a certain threshold is more than twice as high when the bidder is un-constrained than when the bidders is constrained.

6. THE EFFECT OF BACKLOG AND INEFFICIENCIES

This section reports two applications of the estimates: First, we illustrate the effect of backlog on the low regular bid. Second, we determine to what extent the auction rule does not select the low cost bidder. We quantify the magnitude of inefficiencies.

In order to assess the magnitude of the backlog effect on regular bids, we conduct the following experiment: We select the contract with an estimate equal to the sample average. For each regular firm, we randomly draw a bid from the bid distribution under the assumption that their backlog equals -2. Then, we calculate the low bid from this set of regular bids. Similarly, we randomly draw bids from the bid distributions of regular bidders under the assumption that their backlog equals 2. Then, we determine this draw's low bid. We repeat this sampling procedure to obtain 1,000 observations. Finally, we compare the low bids between both cases.

When backlog equals -2, the average of low bids equals $539,845. When backlog equals 2, the average of low bids increases to $657,016. The difference in the mean is significant and equals about 18% of the average at the -2 backlog level. In the data, we do observe cases where all regular bidders are two standard deviations above and two standard deviations below the mean backlog. However, it is not a common event. A more frequent event is that bidders are one standard deviation above or below the mean backlog level. We repeat the above calculations for backlog levels equal to -1 and 1. The difference in the average of low bids between backlogs of -1 and +1 is significant and equals about 12%

of the average low bid at a backlog of -1. Thus, the low bid of unconstrained bidders is substantially lower than the low bid of constrained bidders.

Next, we assess the magnitude of the inefficiencies. Notice, that due to the presence of intertemporal effects and due to bidder heterogeneity, a first-price auction need not select the efficient firm. The bidder with the lowest bid need not be the bidder with the lowest cost. The reason is that constrained (or smaller) bidders may bid more aggressively than unconstrained (or larger) bidders. This property is illustrated in item (i) of the Corollary in section 3. The strategic bid shading can imply that a constrained firm wins although it did not have the lowest cost.

To assess the magnitude of inefficiencies at auction t, we characterize a lower bound for the efficiency loss. The bound is calculated in the following way: We select the firm that minimizes costs at auction t and we take as given that the first-price auction is used at auction $t+1$ and onwards.[32] To assess which firm to select, we take into account contemporary and future costs. Contemporary costs are those implied by the observed bids. Future costs, V^c, are approximated using the estimates reported in section 5. Specifically, $V^c(s) = \int \phi(b; s) dF_1(b) + \beta \sum_{j=1}^{n} \text{Prob}(\ j \ \text{wins}) V^c(s'|s,\ j \ \text{wins})$, where F_1 denotes the distribution function of the winning bid. We evaluate V^c numerically in the same way as the value function. The low cost firm is the firm the firm j that minimizes $c_j + \beta V^c(s'|s, \text{j wins})$.

(Table 4)

Table 4 reports the frequency and amount of inefficiencies associated with the observed bids. Inefficiencies are reported as a fraction of the initial engineers' estimate. On 20% of all contracts an inefficient bidder is selected. The average efficiency loss per contract amounts to 15% of the engineers' estimate. In dollar value, this amounts to $352,550 per contract, on average. In addition to the overall results, Table 4 reports efficiency losses for a range of selected engineers' estimate values. In general, inefficiencies arise for small and

[32] The full cost minimizing problem is a dynamic decision problem involving ten state variables, one backlog variable for each regular bidder. This problem is too complex for current computing techniques.

large contracts. Nevertheless, inefficiencies appear more likely and of larger magnitude for larger contracts.

Table 4 also reports efficiency losses for two subsets of the data: Contracts won by regular bidders and contracts won by fringe bidders. Table 4 illustrates that the probability of inefficiencies is higher, if a contract is won by a regular bidder than when it is won by a fringe bidder. The average efficiency loss is similar between the two groups of contracts at 13% and 15% of the estimate, respectively. The efficiency loss for contracts won by regular bidders is smaller on larger contracts than on smaller contracts. The efficiency loss in percent of the estimate declines from 19% to about 6% as the contract size increases from below 400,000 to above 5 million. For contracts won by fringe bidders, there is no clear trend evident. If anything larger contracts have larger efficiency losses, measured in percent of the engineers' estimate.

7. CONCLUSIONS

This paper examines bidding behavior in highway procurement auctions in California. We consider a dynamic bidding model that takes into account the presence of intertemporal effects and bidder asymmetry. We take the model to the data and estimate the parameters of the model. Based on the estimates, we characterize costs as a function of state variables and illustrate the equilibrium bid functions.

The data suggests the presence of capacity constraints. Bidders that have a large fraction of their capacity committed have, on average, higher costs than bidders with little capacity committed. We find that when all bidders are capacity constrained, the resulting low regular bid is about 18% higher than when all regular bidders are unconstrained. There are at least two implications from our analysis:

First, scheduling and timing of contracts offered for sale influences the final price. Preventing that bidders operate close to the capacity constraint may save costs.

Second, due to intertemporal constraints and bidder heterogeneity an inefficient firm may be chosen. The data indicate that inefficiencies arise on about 20% of all contracts and they amount to 15% of the expected contract size. Although our estimates may be crude, they do suggest that auction rules that cope better with bidder asymmetry could be a cost saving alternative.

Appendix

Proof of Proposition 1: We have to verify that the strategies constitute an equilibrium. Let V_u denote the expected sum of future payoffs of an unconstrained bidder. Let V_κ denote the expected sum of future payoffs of a constrained bidder. The equilibrium bidding function together with the cost distribution functions yields an expression for the distribution of equilibrium bids. For constrained bidders the bid distribution function equals: $1 - \exp^{-\alpha_\kappa(b-\underline{b})}$. For unconstrained bidders this distribution function equals: $1 - \exp^{-\alpha_u(b-\underline{b})}$, with $\underline{b} = \frac{1}{(n-2)\alpha_u+\alpha_\kappa} + \beta\alpha$. Thus, we can calculate the probability that a bid by a constrained bidder wins. It is the probability that the bid is lower than $n-1$ unconstrained bids. Thus, the probability that a bid by a constrained bidder wins is given by: $\exp^{-(n-1)\alpha_u(b-\underline{b})}$. Using this probability, we can write the expected sum of discounted payoffs for a constrained bidder with cost c as:

$$(b - c + \beta V_\kappa)\exp^{-(n-1)\alpha_u(b-\underline{b})} + \beta V_u[1 - \exp^{-(n-1)\alpha_u(b-\underline{b})}].$$

The necessary first order condition for an optimal bid yields,

$$\exp^{-(n-1)\alpha_u(b-\underline{b})}[1 - (n-1)\alpha_u(b-c) - \beta(n-1)\alpha_u(V_\kappa - V_u)] = 0$$

Observe that the second order condition for a maximum is satisfied. Rewriting the first order condition yields:

$$b_\kappa - c = \frac{1}{(n-1)\alpha_u} + \beta(V_u - V_\kappa) \tag{A3.2}$$

Consider next an unconstrained bidder. An unconstrained bidder wins the auction if his bid is lower than $n-2$ unconstrained bids and 1 constrained bid. The winning probability is given by: $\exp^{-(n-2)\alpha_u(b-\underline{b})-\alpha_\kappa(b-\underline{b})}$. We can write the expected sum of payoffs of an unconstrained bidder with cost c as:

$$(b - c + \beta V_\kappa)\exp^{-(n-2)\alpha_u(b-\underline{b})-\alpha_\kappa(b-\underline{b})} + \beta V_u[1 - \exp^{-(n-2)\alpha_u(b-\underline{b})-\alpha_\kappa(b-\underline{b})}]$$

The necessary first order condition for optimal bids yields,

$$\exp^{-(n-2)\alpha_u(b-\underline{b})-\alpha_\kappa(b-\underline{b})}[1-((n-2)\alpha_u+\alpha_\kappa)(b-c)-\beta((n-2)\alpha_u+\alpha_\kappa)(V_\kappa-V_u)]=0$$

Observe that the second order condition is satisfied. Rewriting the first order condition yields,

$$b_u-c=\frac{1}{(n-2)\alpha_u+\alpha_\kappa}+\beta(V_u-V_\kappa) \qquad \text{(A3.3)}$$

In order to obtain an expression for V_u and V_κ, we can substitute the equilibrium bids in the payoff function and evaluate the *ex ante* expected payoff. For constrained bidders the payoff equals,

$$V_\kappa=\int_{\underline{b}}^{\infty}\alpha_\kappa[b-c+\beta(V_\kappa-V_u)]\exp^{-[(n-1)\alpha_u+\alpha_\kappa](b-\underline{b})}db+\beta V_u\int_{\underline{b}}^{\infty}\alpha_\kappa\exp^{-\alpha_\kappa(b-\underline{b})}db$$

Substituting (A3.2), yields:

$$=\frac{\alpha_\kappa}{(n-1)\alpha_u}\frac{1}{(n-1)\alpha_u+\alpha_\kappa}+\beta V_u$$

Similarly for unconstrained bidders the *ex ante* expected discounted sum of profits equals,

$$V_u=\int_{\underline{b}}^{\infty}\alpha_u[b-c+\beta(V_\kappa-V_u)]\exp^{-[(n-1)\alpha_u+\alpha_\kappa](b-\underline{b})}db+\beta V_u\int_{\underline{b}}^{\infty}\alpha_u\exp^{-\alpha_\kappa(b-\underline{b})}db$$

Substituting (A3.3), yields:

$$=\frac{\alpha_u}{(n-2)\alpha_u+\alpha_\kappa}\frac{1}{(n-1)\alpha_u+\alpha_\kappa}+\beta V_u$$

The last expression is a geometric sum. Solving the sum yields,

$$V_u = \frac{\alpha_u}{(1-\beta)[(n-2)\alpha_u + \alpha_\kappa][(n-1)\alpha_u + \alpha_\kappa]}$$

The expression, $V_u - V_\kappa$, is thus given by:

$$V_u - V_\kappa = \frac{\alpha_u - \alpha_\kappa}{[(n-2)\alpha_u + \alpha_\kappa](n-1)\alpha_u} = \alpha.$$

Substituting this expression into the bidding functions (A3.2) and (A3.3) yields the stated result.

Q.E.D.

Subsection 4.2.: Equation (4.2.) The probability that bidder i assigns to the event that bidder j wins the contract when bidder i bids b, can be written as $\int_{\underline{b}}^{b} g(x|s_0, s_j, s_{-j}) \prod_{l \neq i,j}[1 - G(x|s_0, s_l, s_{-l})]dx$. The first order condition of equilibrium bids by a regular bidder is given by:

$$[b-c] \cdot \sum_{j \neq i} \prod_{l \neq i,j} [1 - G(b|s_0, s_l, s_{-l})][-g(b|s_0, s_j, s_{-j})] + \prod_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})] +$$

$$+\beta V_i(s'|s_0, s, \text{i wins}) \sum_{j \neq i} \prod_{l \neq i,j} [1 - G(b|s_0, s_l, s_{-l})][-g(b|s_0, s_j, s_{-j})] +$$

$$+\beta \sum_{j \neq i} \Big[g(b|s_0, s_j, s_{-j}) \prod_{l \neq i,j} [1 - G(b|s_0, s_l, s_{-l})] \cdot V_i(s'|s_0, s, \text{j wins})\Big] = 0 \qquad \text{(A4.1)}$$

We can rearrange this expression by dividing by $\prod_{l \neq i}[1 - G(b|s_0, s_l, s_{-l})]$. This yields,

$$[b-c] \cdot \sum_{j \neq i} \frac{-g(b|s_0, s_j, s_{-j})}{1 - G(b|s_0, s_j, s_{-j})} + 1 + \beta V_i(s'|s_0, s, \text{i wins}) \sum_{j \neq i} \frac{-g(b|s_0, s_j, s_{-j})}{1 - G(b|s_0, s_j, s_{-j})} +$$

$$+\beta \sum_{j \neq i} \frac{g(b|s_0, s_j, s_{-j})}{1 - G(b|s_0, s_j, s_{-j})} \cdot V_i(s'|s_0, s, \text{j wins}) = 0 \qquad \text{(A4.2)}$$

The hazard function, $\tau(b|s_0, s_j, s_{-j}) = g(b|s_0, s_j, s_{-j})/[1 - G(b|s_0, s_j, s_{-j})]$ can be substituted into this equation. This substitution yields equation (4.2.).

The first order condition for optimal bids by a fringe bidder is obtained in analogous way. In particular evaluating (A4.2) at $V_i(s') = 0$, gives the first order condition of the fringe bidder.

Proof of Proposition 2: The probability that bidder i assigns to the event that bidder j wins the contract when bidder i bids b, can be written as: $\int_{\underline{b}}^{b} g(x|s_0, s_j, s_{-j}) \prod_{l \neq i,j} [1 - G(x|s_0, s_l, s_{-l})]dx$. From equation (4.1.) the value function is given by:

$$V_i(s_i, s_{-i}) = E_{s_0}\{\int \max_b \{[b - c]Prob(i \text{ wins } |b, s_0, s) +$$

$$+\beta \sum_{j=0}^{n} Prob(j \text{ wins}|b, s_0, s) V_i(s'|s_0, s, \text{j wins})\} f(c|s_0, s_i) dc\}.$$

The first order condition for equilibrium bids, equation (4.2.), is given by:

$$c = b - \frac{1 - \beta \sum_{j \neq i} \tau(b|s_0, s_j, s_{-j})[V_i(s'|s_0, s, \text{i wins}) - V_i(s'|s_0, s, \text{j wins})]}{\sum_{j \neq i} \tau(b|s_0, s_j, s_{-j})}$$

We denote by $b(c)$ the equilibrium bid by bidder i. Substituting the first order condition into the value function yields:

$$V_i(s_i, s_{-i}) =$$

$$E_{s_0}\{\int [\frac{1 - \beta \sum_{j \neq i} \tau(b(c)|s_0, s_j, s_{-j})[V_i(s'|s_0, s, \text{i wins}) - [V_i(s'|s_0, s, \text{j wins})]}{\sum_{j \neq i} \tau(b(c)|s_0, s_j, s_{-j})}]$$

$$\cdot Prob(i \text{ wins } |b(c), s) + \beta[\sum_{j=0}^{n} Prob(j \text{ wins}|b(c), s) V_i(s'|s_0, s, \text{j wins})\} f(c|s_0, s_i) dc\}$$

The expression $\int \beta[\sum_{j=0}^{n} Prob(j \text{ wins}|b(c), s) V_i(s'|s_0, s, \text{j wins}) f(c|s_0, s) dc$ equals the ex ante expected value which is given by $\beta \sum_{j=0}^{n} Prob(j \text{ wins}|b, s_0, s) V_i(s'|s_0, s, \text{j wins})$.

The expression $\frac{\sum_{j \neq i} \tau(b(c)|s_0, s_j, s_{-j}) V_i(s'|s_0, s, \text{j wins})}{\sum_{j \neq i} \tau(b(c)|s_0, s_j, s_{-j})}$ reduces to $V_i(s'|s_0, s, \text{j wins})$ and cancels with the second term involving $V_i(s'|s_0, s, \text{j wins})$. Making these changes we can write the value function as:

$$V_i(s_i, s_{-i}) = E_{s_0}\{\int [\frac{1 + \beta \sum_{j \neq i} \tau(b(c)|s_0, s_j, s_{-j}) V_i(s'|s_0, s, \text{j wins})}{\sum_{j \neq i} \tau(b(c)|s_0, s_j, s_{-j})}]$$

$$\cdot Prob(i \text{ wins } |b(c), s_0, s) \cdot f(c|s_0, s_i) dc + \beta[\sum_{j \neq i} Prob(j \text{ wins}|b, s_0, s) V_i(s'|s_0, s, \text{j wins})\}$$

Next consider a change of variable of integration from c to b. Notice that $db = \frac{\partial b(c)}{\partial c} dc$. Let b^{-1} denote the inverse function of the equilibrium bid function. By assumption the inverse bidding function exists. The inverse bidding function allows us to write the distribution function of cost in terms of the distribution functions of bids. Specifically, $F(b^{-1}(b)|s_0, s_i) = G(b|s_0, s_i, s_{-i})$. Taking the partial derivative yields a relationship between the density of costs and bids: $f(b^{-1}(b)|s_0, s_i) \cdot \frac{\partial b^{-1}(b)}{\partial b} = g(b|s_0, s_i, s_{-i})$. Also notice that $\frac{\partial b_i^{-1}(b,s)}{\partial b} = \frac{1}{\frac{\partial b_i(c,s)}{\partial c}}$. Finally the probability that bidder i wins can be written as, $Prob(i \text{ wins } |b, s_0, s) = \prod_{j \neq i}[1 - G(b|s_0, s_j, s_{-j})]$. Applying the change of variables in the above equation yields:

$$V_i(s_i, s_{-i}) = E_{s_0} \int_{\underline{b}(s)}^{R} \frac{\prod_{k \neq i}[1 - G(b|s_0, s_k, s_{-k})}{\sum_{j \neq i} \tau(b|s_0, s_j, s_{-j})} g(b|s_0, s_i, s_{-i}) db$$

$$+\beta\int_{\underline{b}(s)}^{R}\prod_{k\neq i}[1-G(b|s_0,s_k,s_{-k})]\cdot\sum_{j\neq i}\frac{\tau(b|s_0,s_j,s_{-j})}{\sum_{l\neq i}\tau(b|s_0,s_l,s_{-l})}\cdot V_i(s'|s_0,s,\text{j wins})\cdot g(b|s_0,s_i,s_{-i})db$$

$$+\beta[\sum_{j\neq i}\int_{\underline{b}(s)}^{R}\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]g(b|s_0,s_j,s_{-j})db\cdot V_i(s'|s_0,s,\text{j wins})$$

Observe that the expression in the second line of the value function can be rewritten in the following way:

$$+\beta\int_{\underline{b}(s)}^{R}\prod_{k\neq i}[1-G(b|s_0,s_k,s_{-k})]\cdot\sum_{j\neq i}\frac{\tau(b|s_0,s_j,s_{-j})}{\sum_{l\neq i}\tau(b|s_0,s_l,s_{-l})}\cdot V_i(s'|s_0,s,\text{j wins})\cdot g(b|s_0,s_i,s_{-i})db$$

$$=\beta\sum_{j\neq i}\int_{\underline{b}(s)}^{R}\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]\cdot\frac{1-G(b|s_0,s_j,s_{-j})}{1-G(b|s_0,s_i,s_{-i})}\cdot\frac{\tau(b|s_0,s_j,s_{-j})}{\sum_{l\neq i}\tau(b|s_0,s_l,s_{-l})}$$

$$\cdot V_i(s'|s_0,s,\text{j wins})\cdot\frac{g(b|s_0,s_i,s_{-i})}{g(b|s_0,s_j,s_{-j})}\cdot g(b|s_0,s_j,s_{-j})db$$

$$=\beta\sum_{j\neq i}\int_{\underline{b}(s)}^{R}\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]\cdot\frac{\tau(b|s_0,s_i,s_{-i})}{\sum_{l\neq i}\tau(b|s_0,s_l,s_{-l})}\cdot V_i(s'|s_0,s,\text{j wins})\cdot g(b|s_0,s_j,s_{-j})db$$

The first equality follows by taking the sum $\sum_{j\neq i}$ outside, augmenting the expression by $\frac{1-G(b|s_0,s_i,s_{-i})}{1-G(b|s_0,s_i,s_{-i})}$ and $\frac{g(b|s_0,s_j,s_{-j})}{g(b|s_0,s_j,s_{-j})}$ and rearranging terms. The second equality is obtained by rewriting $\frac{g(b|s_0,s_i,s_{-i})}{1-G(b|s_0,s_i,s_{-i})}$ as $\tau(b|s_0,s_i,s_{-i})$ and canceling.

Thus, the value function is given by:

$$V_i(s_i,s_{-i})=E_{s_0}\int_{\underline{b}(s)}^{R}\frac{\prod_{k\neq i}[1-G(b|s_0,s_k,s_{-k})}{\sum_{j\neq i}\tau(b|s_0,s_j,s_{-j})}g(b|s_0,s_i,s_{-i})db$$

$$+\beta\sum_{j\neq i}\int_{\underline{b}(s)}^{R}\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]\cdot\frac{\tau(b|s_0,s_i,s_{-i})}{\sum_{l\neq i}\tau(b|s_0,s_l,s_{-l})}\cdot V_i(s'|s_0,s,\text{j wins})\cdot g(b|s_0,s_j,s_{-j})db$$

$$+\beta[\sum_{j\neq i}\int_{\underline{b}(s)}^{R}\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]g(b|s_0,s_j,s_{-j})db\cdot V_i(s'|s_0,s,\text{j wins})$$

Observe that the lines two and three of the value function have a number of terms in common. Simplifying terms yields the following expression for the value function:

$$V_i(s_i,s_{-i})=E_{s_0}\int_{\underline{b}(s)}^{R}\frac{1}{\sum_{j\neq i}\tau(b|s_0,s_j,s_{-j})}\prod_{k\neq i}[1-G(b|s_0,s_k,s_{-k})g(b|s_0,s_i,s_{-i})db$$

$$+\beta[\sum_{j\neq i}\int_{\underline{b}(s)}^{R}[\frac{\tau(b|s_0,s_i,s_{-i})}{\sum_{l\neq i}\tau(b|s_0,s_l,s_{-l})}+1]\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]g(b|s_0,s_j,s_{-j})db$$

$$\cdot V_i(s'|s_0,s,\text{j wins})$$

Observe that $\prod_{l\neq j}[1-G(b|s_0,s_l,s_{-l})]g(b|s_0,s_j,s_{-j})$ is the probability density function that a bid of bidder j is the low bid which yields the expression (4.3.) in the Proposition.

QED

REFERENCES

Bajari, Patrick, "Econometrics of the First Price Auction with Asymmetric Bidders." mimeo Stanford (1997)

Berry, Steve and Pakes, Ariel, "Using First Order Conditions to Estimate Dynamic Oligopoly Models," mimeo, Yale University, May 2000.

Donald, Stephen G., Paarsch Harry J. and Robert, Jacques "Identification, Estimation, and Testing in Empirical Models of Sequential, Ascending-Price Auctions with Multi-Unit Demand: An Application to Siberian Timber-Export Permits." mimeo, University of Iowa, (1997).

Elyakime, Bernard, Laffont Jean Jacques, Loisel Patrice and Vuong, Quang, "First-Price Sealed-Bid Auctions with Secret Reservation Prices." Annales d'Economie et de Statistique, 34, (1994).

Feinstein, Jonathan S., Block, Michael K. and Nold, Frederick C., " Asymmetric Information and Collusive Behavior in Auction Markets." American Economic Review (June 1985).

Guerre Emmanuel, Perrigne Isabelle and Vuong, Quang, "Optimal Nonparametric Estimation of First-Price Auctions." forthcoming in Econometrica, (2000).

Harter, H.L. and Moore, A.H., "Maximum Likelihood Estimation of the Parameters of Gamma and Weibull Populations from Complete and from Censored Samples," Technometrics, 7, 639-43.

Jofre-Bonet, Mireia, and Pesendorfer, Martin, "Bidding Behavior in a Repeated Procurement Auction: A Summary", (1999), Forthcoming in the European Economic Review, Papers and Proceedings of the Fourteenth Annual Congress of the European Economic Association, Santiago, Spain.

Judd, Kenneth, "Numerical Methods in Economics." MIT Press, (1998).

Laffont, Jean Jacques, Ossard, H. and Vuong, Quang, "Econometrics of First-Price Auctions." Econometrica, 63, (1995).

Laffont, Jean Jacques and Robert, Jacques, "Intra-Day Dynamics in Sequential Auctions: Theory." mimeo, Toulouse, (1999).

Maskin, Eric and Riley, John, "Equilibrium in Sealed High Bid Auctions." mimeo Harvard University and UCLA, (December 1996).

Paarsch, Harry, "Deciding between the Common and Private Value Paradigms in Empirical Models of Auctions," Journal of Econometrics, 51 (1992), 191-215.

Pesendorfer, Martin, "A Study of Collusion in First-Price Auctions." (1998), forthcoming in the Review of Economic Studies.

Pakes, Ariel, "Dynamic Structural Models, Problems and Prospects: Mixed Continuous Discrete Controls and Market Interactions." in C. Shims and J. Laffont, Advances in Econometrics: Proceedings of the 1990 Meetings of the Econometric Society, 1994.

Porter, Robert H. and Zona, J. Douglas, "Detection of Bid Rigging in Procurement Auctions." Journal of Political Economy 101 (1993), pp. 518-538.

Smith, Richard, "Maximum Likelihood Estimation in a Class of Nonregular Cases," Biometrika, 72(1), 1985, 67-90.

Waehrer, Keith, "Asymmetric Private Values Auctions with Application to Joint Bidding and Mergers." International Journal of Industrial Organization 17(3), 1999, 437-52.

Table 1A: Descriptive Statistics of Selected Variables

	Number of Observations	Mean	Standard Deviation	Minimum	Maximum
Number of Bidders	2223	4.63	2.46	0.00	19.00
Estimate*	2223	13.41	1.35	9.47	18.31
(Ranked1-Estimate)/Estimate**	2207	-0.04	0.22	-0.79	3.07
(Ranked2-Ranked1)/Ranked1**	2111	0.09	0.12	0.00	2.62
Backlog***	22230	0.00	1.00	-3.24	2.97

*Logarithm of the engineers' estimate.
**Ranked1 and Ranked2 are the winning bid and the bid ranked in second position, respectively.
***Backlog measures the $ value of previously won uncompleted contracts. It is standardized by subtracting the bidder specific mean and dividing by the bidder specific standard deviation.

Table 1B: Descriptive Statistics of Selected Variables by Number of Bidders

	Number of bidders:	All	0	1	2	3	4	5	6	7-8	9-19
	# Observations:	2223	16	96	285	393	432	356	237	251	157
Estimate*											
	Mean	13.41		13.14	13.47	13.49	13.32	13.42	13.55	13.48	13.12
	Standard Deviation	1.35		1.04	1.27	1.33	1.29	1.35	1.59	1.47	1.19
(Ranked1-Estimate)/Estimate**											
	Mean	-0.04		0.11	0.03	-0.01	-0.04	-0.06	-0.09	-0.10	-0.14
	Standard Deviation	0.22		0.36	0.29	0.21	0.20	0.19	0.16	0.20	0.16
(Ranked2-Ranked1)/Ranked1**											
	Mean	0.09			0.14	0.11	0.09	0.08	0.06	0.07	0.06
	Standard Deviation	0.12			0.11	0.19	0.10	0.09	0.06	0.07	0.07

*Logarithm of the engineers' estimate.
**Ranked1 and Ranked2 are the winning bid and the bid ranked in second position, respectively.

Table 2: Bid Subsmission and Bid Level Decisions

Estimation Method:	Probit		Tobit		Heckman	
Dependent Variable:	Bid Submission		(R-Bid)/Estimate*		(R-Bid)/Estimate*	
Number of observations:	22230	22230	22230	22230	22230	22230
Chi^2:	1605.65	1984.17	1518.99	1883.42	420.41	444.64
Degrees of freedom:	6	15	6	15	6	15
Log Likelihood:	-4281.26	-4092.39	-3765.05	-3582.84	-3404.46	-3394.35
Variable						
Constant	-2.8485	-(3.169)	-1.1654	-1.2511	0.3093	0.2734
	(0.173)	(0.184)	(0.089)	(0.091)	(0.072)	(0.076)
Estimate	0.2905	0.3024	0.1235	0.1220	0.0040	0.0038
	(0.015)	(0.016)	(0.008)	(0.008)	(0.006)	(0.007)
Working Days	-0.3176	-0.3234	-0.1498	-0.1446	-0.0533	-0.0540
	(0.022)	(0.023)	(0.011)	(0.011)	(0.008)	(0.009)
Nbid-Fringe	-0.1835	-0.1913	-0.0882	-0.0875	-0.0613	-0.0599
	(0.027)	(0.027)	(0.013)	(0.013)	(0.007)	(0.008)
Distance	-0.5193	-0.4805	-0.2536	-0.2240	-0.1196	-0.0978
	(0.023)	(0.024)	(0.012)	(0.012)	(0.008)	(0.009)
# Plants within Region	0.1807	0.0513	0.0638	0.0078	-0.0051	-0.0193
	(0.051)	(0.054)	(0.025)	(0.024)	(0.014)	(0.015)
Backlog	-0.0835	-0.0856	-0.0383	-0.0372	-0.0127	-0.0127
	(0.015)	(0.015)	(0.007)	(0.007)	(0.004)	(0.005)
Firm_2		0.6784		0.2985		0.1204
		(0.061)		(0.029)		(0.019)
Firm_3		-0.0338		-0.0081		-0.0223
		(0.073)		(0.034)		(0.024)
Firm_4		0.1499		0.0649		0.0011
		(0.074)		(0.034)		(0.022)
Firm_5		-0.0325		-0.0133		-0.0097
		(0.073)		(0.033)		(0.023)
Firm_6		-0.1885		-0.0976		-0.0458
		(0.076)		(0.035)		(0.023)
Firm_7		0.2011		0.0969		0.0054
		(0.072)		(0.033)		(0.022)
Firm_8		-0.0515		-0.0212		-0.0357
		(0.073)		(0.034)		(0.023)
Firm_9		-0.2070		-0.0893		-0.0372
		(0.077)		(0.035)		(0.023)
Firm_10		0.2277		0.1297		0.0742
		(0.069)		(0.031)		(0.021)
Mills Ratio					0.2342	0.2233
					(0.011)	(0.011)

All variables except Backlog are in logarithm. The numbers in parenthesis are standard deviations.

*(R-Bid)/Estimate denotes the logarithm of the variable (Reserve price minus the Bid) over the engineers' Estimate plus one.

Table 3: Parameter Estimates of the Bid Distributions

Data:	Regular Bids		All Bids	Fringe Bids	
Number of Observations:	22,230		10,289	8,941	
R2:			0.973		
Log Likelihood:	3,140.92			-4,605.30	
Variables	θ_1	θ_2	θ_3	θ_4	θ_5
Constant	7.7525	0.1107	0.4361	-1.7444	-0.0363
	(4.029)	(0.284)	(0.017)	(0.089)	(0.048)
Ln Estimate	-1.6249	5.2721	1.0246	0.2450	0.1594
	(0.583)	(0.528)	(0.002)	(0.007)	(0.003)
Ln Working Days	2.1394	-0.3537	0.0035	0.0210	0.0174
	(0.803)	(0.063)	(0.002)	(0.008)	(0.014)
Estimate/Reserve_Price	0.1107	-0.0467	-0.9867	-0.0649	-2.6591
	(0.284)	(0.065)	(0.009)	(0.060)	(0.123)
Nbid-Fringe	-0.1231	0.2680	-0.0004	0.0029	0.0088
	(0.055)	(0.083)	(0.001)	(0.004)	(0.006)
Distance	0.0167	0.0676			
	(0.021)	(0.015)			
# of Plants within the Region	-2.2533	0.1062			
	(1.087)	(0.006)			
Backlog	0.5242	-0.4509			
	(0.370)	(0.080)			
Sum_Distance	0.0043	0.1000	0.0000	0.0009	0.0014
	(0.010)	(0.027)	(0.000)	(0.000)	(0.001)
Sum_# of Plants within the Region	0.7557	0.0016	-0.0007	-0.0136	-0.0151
	(0.223)	(0.001)	(0.001)	(0.007)	(0.010)
Sum_Backlog	-0.1977	0.1190	-0.0039	-0.0133	0.0027
	(0.151)	(0.044)	(0.001)	(0.007)	(0.009)
θ_6	1.3461				
	(0.039)				

Table 4: Estimates of Efficiency Losses

Variable	E < 100,000	100,000 E < 400,000	400,000 E < 1Mio	1Mio E < 5Mio	E > 5 Mio	Overall
	Range of Engineers' Estimate					
All Contracts:						
Number of Contracts	53	900	525	545	184	2207
Prob of an Inefficiency	0.08	0.13	0.20	0.30	0.23	0.20
Average Efficiency Loss*	0.07	0.12	0.15	0.20	0.15	0.15
Contract Won by a Regular Bidder:						
Number of Contracts	1	96	91	147	42	377
Prob of an Inefficiency	1.00	0.26	0.27	0.31	0.17	0.27
Average Efficiency Loss *	0.46	0.19	0.14	0.10	0.06	0.13
Contract Won by a Fringe Bidder:						
Number of Contracts	52	804	434	398	142	1830
Prob of an Inefficiency	0.06	0.12	0.19	0.30	0.25	0.18
Average Efficiency Loss*	0.06	0.11	0.15	0.23	0.18	0.15

*Efficiency losses are reported as a fraction of the engineers' estimate.

FIGURE 1: BID DISTRIBUTION FUNCTION

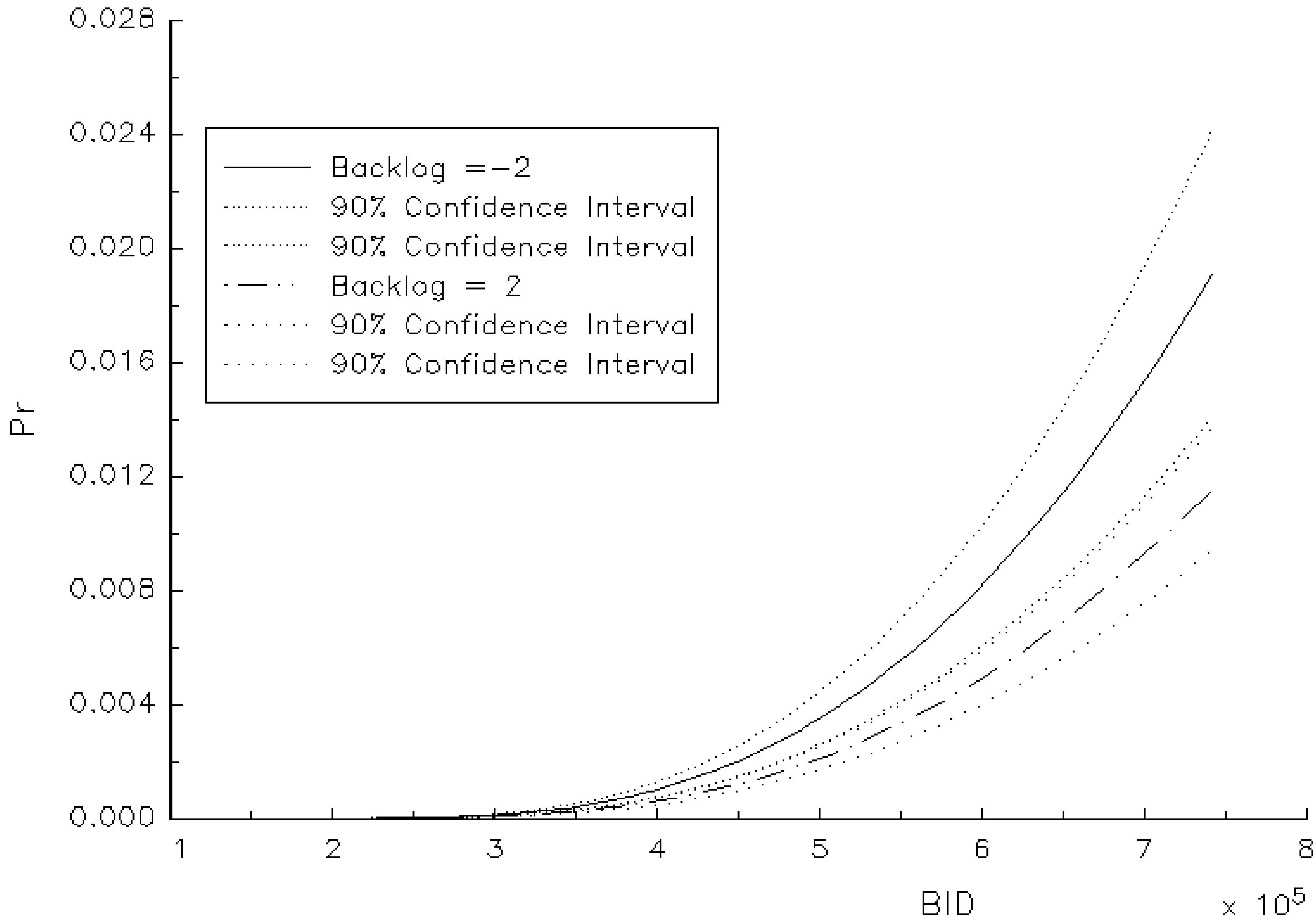

FIGURE 2: VALUE FUNCTION

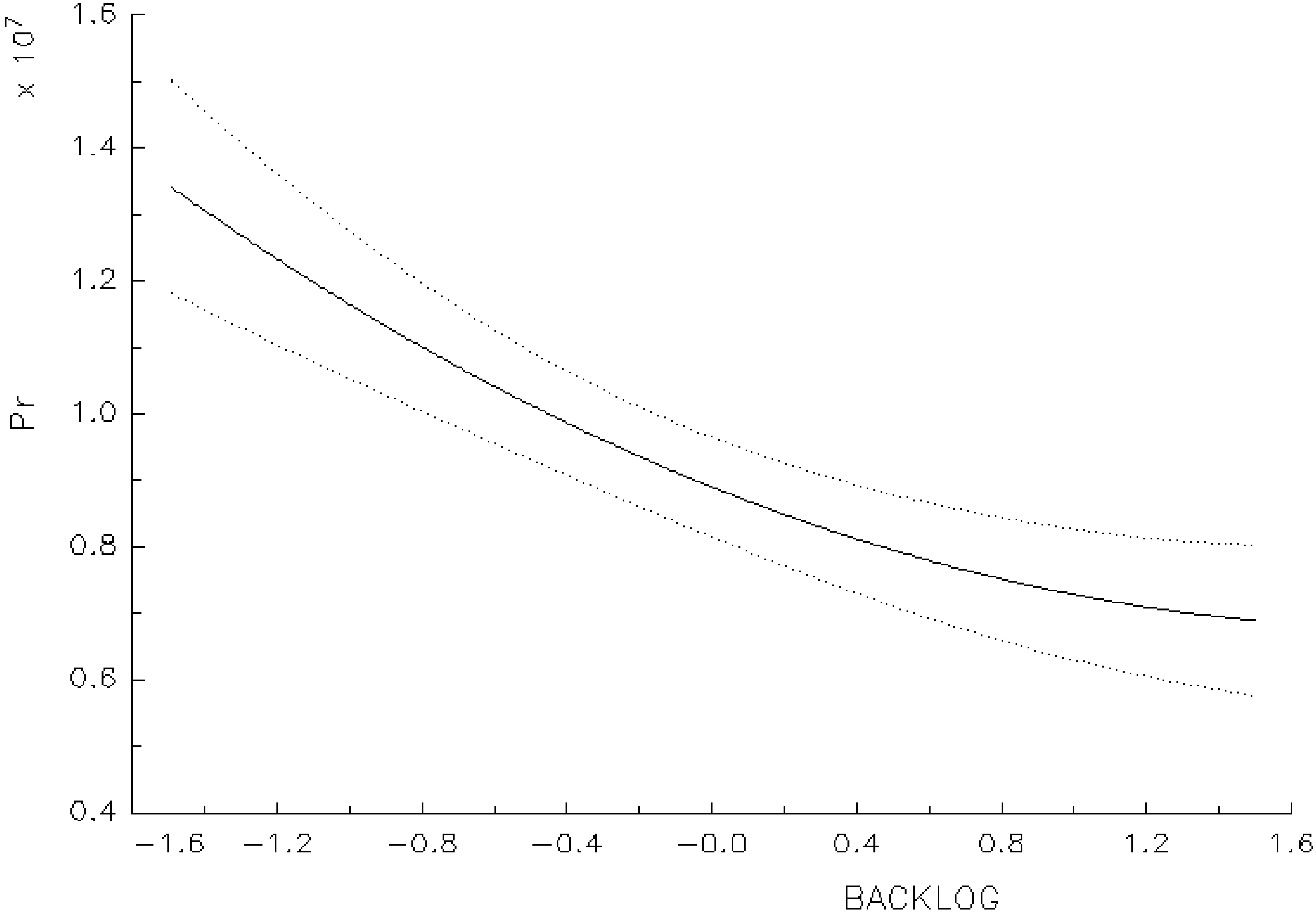

FIGURE 3: BID FUNCTION

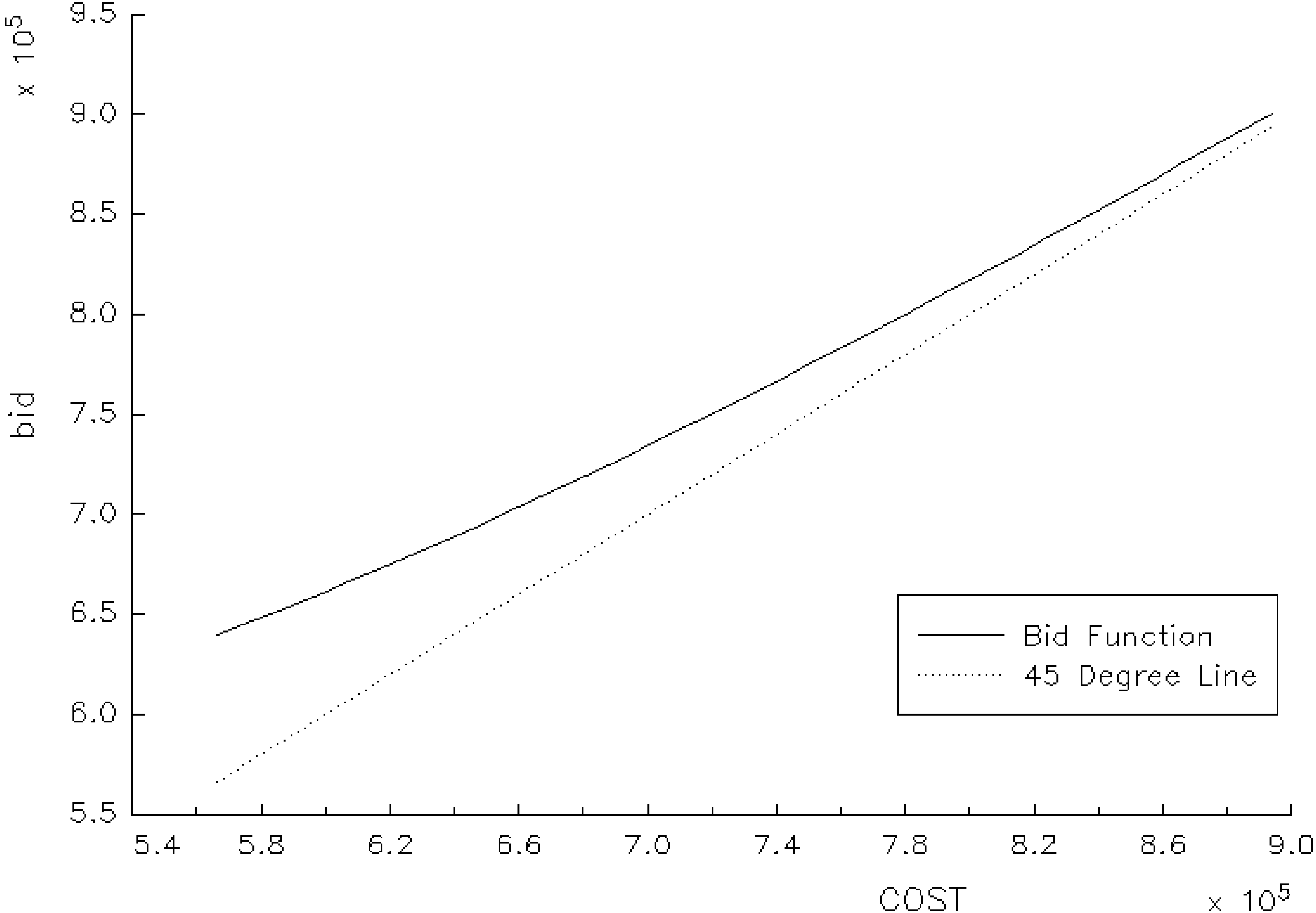

FIGURE 4: COST DISTRIBUTION FUNCTION