Turnover, Wage Determination, and the Formation of Human Capital

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Abstract

We analyse a model with human capital investments and turnover. We analyse under what conditions firms have the right incentives to train their employees themselves rather than raiding them from other firms. We show that if firms advertise and commit to a "life-time" wage schedule, the levels of on-the-job human capital acquisition, of turnover, and of unemployment are socially optimal. Without such commitment ability, there is too much turnover and too little training compared with the optimal allocation. Still, the number of training firms and the amount of human capital investments are second best optimal, and subsidising human capital investment will therefore reduce welfare. The paper also extends the competitive search equilibrium model by including on-the-job search.

1 Introduction

Most workers change employers several times during their careers and a substantial fraction of these changes are direct job-to-job movements. For the EU countries and Canada, Boeri (1999) reports that 45-70% of the hirings are job-to-job movements, and that the yearly job-to-job flow is 6-18%. In the U.S., 20 % of the hirings are job-to-job movements (Blanchard and Diamond (1989)), and the yearly turnover rate is around 30% (Layard et at (1991)). Furthermore, wages typically increase substantially with experience, indicating that workers become more productive throughout their career. Since wages generally increase when workers changes employers, at least part of this increase in productivity must reflect improved general skills (not only firm-specific skills).

These considerations indicate that investments in training are important. Furthermore, as firms may differ in their ability to train workers and to utilise their human capital, turnover is important in order to achieve efficiency. The extent to which the market provides the firms with incentives to invest in general and specific training rather than raiding other firms for qualified workers, and the extent to which the market induces turnover are therefore important determinants of economic welfare. Furthermore, as the turnover rate in the economy influence the incentives to invest in training, human capital acquisition and worker turnover are interrelated issues that must be analysed simultaneously

This paper analyses the conditions under which the outcome in the labour market is efficient in a setting with endogenous human capital formation and with endogenous turnover. To this end, we develop a search model with on-the-job search and endogenous search intensity, in which worker turnover is necessary in order to obtain an efficient allocation of resources. Within a competitive search equilibrium framework, we analyse to what extent firms have the correct incentives to train their workers themselves rather than to employ workers trained in other firms, and to which extent the firms that choose to train their own workers have the incentives to invest the socially optimal amount of human capital in their employees.

To be more specific, we study a sector in which a firm may enter the market as a training firm or as a raiding firm. By working in a training firm, a worker obtains an increased productivity in all firms in the sector as well as (possibly) some firm-specific skills. In contrast, raiding firms do not offer any training, general or specific, and only hire experienced workers. On the other hand, the productivity of an experienced worker is lower in a training firm than in a raiding firm. Thus, an experienced worker working in a training firm has an incentive to do on-the-job search in order to find a job in a raiding firm. One determinant of the amount of general human capital is the endogenous number of training firms in the market. Within this basic structure, we examine how wage determination affects the efficiency of the equilibrium outcome, focusing mostly on the mix of raiding and training firms and on the turnover rate in the economy.

Our first result is that the equilibrium outcome is efficient when training firms can advertise and commit to long-term wage contracts, so that the wage that a worker obtains as a novice (inexperienced and with low productivity) and when he eventually becomes experienced (which happens at a stochastic rate) is determined up-front before the employment relationship starts. Efficiency implies that for both raiding and training firms, the private and social gains from entry coincide, and the turnover rate is optimal. Moreover, training firms have the right incentives to provide general and firm-specific training. As in Moen (1997), efficiency is obtained despite the presence of search frictions.

This efficiency result is somewhat surprising, as it seems to be a consensus in the literature that the combination of turnover and frictions necessarily leads to underinvestments in training. This view is forcefully put forward by Acemoglu (1997). In his model, there is too little general training even though firms and workers can write binding long-term contracts. He attributes the inefficient outcome to the workers' inability to contract with future employers. Our efficiency result illustrates that contracts with future employers are not necessary to obtain efficiency, and that his inefficiency result arises because he let wages be determined by Nash bargaining.

The fact that the equilibrium that arises when firms may advertise (and stick to) long-term wage contracts yields an efficient allocation of resources serves as a convenient starting point when introducing other imperfections than search frictions into the model, and sharpens and simplify the intuition related to various kinds of inefficiencies that may arise. We first analyse the case where firms are unable to differentiate between wages paid to inexperienced and experienced workers. This is the situation studied in a well-known paper by Salop (1979). In this case, a (single) wage rate advertised by the training firms serve two purposes: to attract applicants in the first place and to economise on turnover. Wages are driven up, and we get too few training firms and too many raiding firms compared to the efficient solution. Our analysis sharpens the result obtained by Salop: in his model, turnover may influence wages only for parameter values within a certain range (which may be narrow), while in our model, wages are affected for all parameter values. Furthermore, in contrast to our model, his framework is not suitable for welfare comparisons, because it does not specify the search technology underlying the turnover process.

We show that although there are too few training firms entering the market compared to the socially optimal level, the number of training firms, and the amount they invest in each worker, is second-best optimal-Subsidising training firms will therefore actually reduce welfare.

We also consider the case where training firms at the hiring stage cannot commit to the wage that a worker will earn once he is experienced. Instead the wage for experienced workers is set so as to maximise the firm's *ex post* profit. Again we find that, compared with the socially optimal level, the equilibrium turnover rate is too high and there are too few training firms and too many raiding firms. However, it now turns out that investments in firm-specific training may be used as an efficient commitment device for the firm. When firms invest more in firm-specific training, the optimal wage for the firm to set *ex post,* when the worker has become experienced, increases. That is, firm-specific human capital is used strategically to reduce the workers' propensity to quit. Therefore, the productivity gains from firm-specific investment are lower than the investment costs. Nonetheless, these "over-investments" are welfare improving, because high firm-specific investment mitigates the adverse effects of excessive turnover.

The paper is organised as follows. Section 2 presents the basic model. Section 3 analyses the case where firms can commit to a "life-time" wage schedule. Section 4 considers the case where training firms cannot disentangle the wages of novices and experienced workers. Section 5 studies the case where training firms cannot commit to wages for experienced workers.

2 The model

The main features of the model is as follows:

- *Workers:* Workers are risk neutral and identical *ex ante*. They enter the labour market as unemployed, and leave the market at a constant, exogenous rate s. New workers enter the market at the same rate so that the total number of workers is constant.

- *Firms:* There are two types of firms: training firms and raider firms. In a training firm, the productivity of a novice is y_n^t and of an experienced worker y_e^t. In a raiding firm, the productivity of an experienced worker is y^r. We assume that $y_n^t < y_e^t < y^r$. A novice never starts working in a raiding firm. Each firm hires at most one worker.

- *Time structure:* The model is set in continuous time. A worker that is hired by a training firm will stay inexperienced for a period. The natural way to model a period of time within a continuous-time framework is to let the period length be stochastic: an inexperienced worker employed in a training firm becomes experienced at a rate γ.

- *Search technology:* The matching technology is described more fully in the next subsection together with the equilibrium concept. For now we just state that unemployed and employed workers search in different submarkets, with different wages, and do not cause congestion for each other. Let p^u and p^e denote the arrival rate of job offers to unemployed workers and to employed workers, respectively, searching with a search intensity equal to one. Employed workers choose a search intensity e, and the arrival rate of job-offers to these workers is thus ep^e, We assume that the search intensity for unemployed workers and firms are constant and normalised to one. For workers doing on-the-job search, the cost of search is given by a continuous, convex function $c(e)$, with $c(0) = 0$ and $c'(e) > 0$, $c''(e) > 0$ for $e > 0$.

Competitive search equilibrium

In the matching literature, there exist two ways to model wage competition between firms. In the Walrasian flavoured "competitive search equilibrium", the labour market is divided into submarkets with different wages, and workers and firms choose which submarket to enter (Shimer, Moen (1997), and Mortensen and Wright (1998)). In the wage advertisement literature, firms announce wages and workers respond strategically when sending off applications, and the equilibrium concept thus has a game-theoretic foundation. Peters (1994) shows that the two equilibrium concepts are equivalent in a one-shot matching game with an exponential matching function. In this paper, we choose to apply the competitive search equilibrium framework.

Define a concave and constant returns to scale matching function $x(u,v)$ that maps a certain number of workers and firms searching for each other into a a flow x of new matches (search-intensities will be introduced later). If p denotes the probability rate for an unemployed worker of finding a job, it folows that $p = x(u,v)/u = x(1,\theta) = p(\theta)$. If q is the probability rate that a firm with a vacancy finds a worker, it follows that $q = x(u,v)/v = x(1/\theta,1) = \tilde{q}(\theta)$. The matching technology can thus be summarised by a function $q = \tilde{q}(\theta) = \tilde{q}(p^{-1}(p)) = q(p)$. We introduce search intensity by assuming that the worker may choose the number e of efficiency units he puts into search, at a cost $c(e)$. The matching function is then given by $x(u\overline{e},v)$, where $\overline{e}$ denote the average number of efficiency units provided by the workers in the economy. If we let p denote the arrival rate of job offers to a worker that provides one unit of search effort, it still follows that $q = q(p)$. The arrival rate of job offers to a worker that searches with search intensity e is then pe.

We will now derive the competitive search equilibrium in a general matching market, without specifying whether the market in question is a job market for employed workers or for unemployed worker. Let Y denote the expected discounted joint income for a worker-firm pair that is matched. If the productivity of the firm is y, and the match dissolves at an exogenous rate s, it follows that $Y = y/(r+s)$, where r is the discount factor. Let w^s denote the income to searching workers while searching, and let W^s denote the expected discounted income for a searching worker. Similarly, let W^e denote the expected discounted income if the worker is employed. We assume for now that the search intensity is constant and equal to one. The asset value equation for the searching worker is then

$$(r+s)W^s = w^s + p(W^e - W^s) \tag{1}$$

The asset value equation for a vacancy is given by

$$rV = q(Y - W^e - V)$$

where V is the value of the vacancy. Since we assume free entry, the value of a vacancy is equal to the cost of creating a vacancy, which we denote by K. With free entry it thus follows that $W^e = Y - K\frac{r+q}{q}$, and thus that

$$(r+s)W^s = w^s + p(Y - K\frac{r+q(p)}{q(p)} - W^s) \tag{2}$$

In the competitive search equilibrium, all submarkets that attract searching workers must yield the workers exactly their equilibrium expected discounted income, which we refer to as $\overline{W}^s$. From (1) it follows that we can write $W^s = W^s(p, W^e)$, and it thus follows that in any submarket, $W^s(p, W^e) = \overline{W}^s$. Alternatively, we can write this as $q = q(p(W^e, \overline{W}^s))$. In competitive search equilibrium, firms with vacancies offer wages W^s so as to maximise the value of their vacancy given that $q = q(p)$ and that $W^s(p, W^e) = \overline{W}^s$. Define the maximum value of the vacancy as $V(\overline{W}^s)$. Equilibrium can then be written as (omitting the upper-bar for convenience)

$$V(W^s) = K \tag{3}$$

This equation uniquely determines W^s, and (2) then determines p and the fact that $q = q(p)$ then determines q. In Moen (1997), it is shown that the competitive search equilibrium also can be derived if each firm with a vacancy announces a wage W^s that maximises profit, and that the firms' beliefs (for all values of W, not just the equilibrium values) about the relationship between wages and the arrival rate of workers is given by $q = q(p(W^e, \overline{W}^s))$.

In this paper, we will characterise the competitive search model in a somewhat different way. Note that the competitive search equilibrium allocation is such that V is maximised given W^s, while free entry ensures that $V = K$. The dual problem to this maximisation problem is to maximise W^s given that $V = K$:

Lemma 1 In the competitive search equilibrium, W^s is maximised given that $V = K$

Let us now derive the optimal allocation of resources. Let N^s denote the number of searching worker, and let b denote an exogenous inflow of searching workers. Now suppose the productivity of the searching worker is equal to his wage w^s. Then the planner wants to maximise

$$R(N^s) = \int_0^\infty [pYN^s + w^s N^s - pN_e^t \frac{r + q(p)}{q(p)} K] e^{-rt} dt$$

with respect to p, given the constraint

$$\dot{N}^s = b - (s + p)N^s$$

The associated Bellman equation is given by

$$rR(N^s) = \max_p [pYN^s - N^s w^s + pN^s \frac{r + q}{q} K + R'(N^s)(b - (s + p)N^s)] \qquad (4)$$

It follows that $(r+s)R'(N^s) = w^s + p(Y - \frac{r+q}{q}K - R'(N^s))$, which is independent of N^s. Furthermore, by comparing (2) and (4) it follows that the expressions for R' and for W^s are equivalent. Now the maximisation problem in (4) can be written as

$$\max_p p(Y - \frac{r + q}{q} K - R')$$

which is equivalent to maximising R'. But it then follows that the planner maximises W^s given by (2), that is, maximises W^s given that $V = K$, just as in the competitive search equilibrium.

Proposition 1 The following holds:

a) The socially optimal allocation maximises W^s given that $V = K$.

b) The competitive search equilibrium allocation is socially efficient

c) In the competitive search equilibrium, the social and private value of an additional worker entering the search market coincide

3 Equilibrium with commitment

In this section, we derive and evaluate the equilibrium of the model where training firms, when advertising their vacancy, also advertise (and commit to) wages for the worker as a novice and as an experienced worker, denoted by w_n^t and w_e^t, respectively. We can then write the expected income of a worker employed in a firm offering a wage schedule (w_n^t, w_e^t) as $W_n^t(w_n^t, w_e^t)$, given by

$$(r+s)W_n^t = w_n^t + \gamma(W_e^t - W_n^t) \tag{5}$$

where W_e^t denotes the expected discounted income to the worker when experienced, and γ the rate at which he becomes experienced (exogenous for now). W_n^t is given by (for later reference we write it as a function of w_e^t)

$$(r+s)W_e^t(w_e^t) = w_e^t + \max_e[ep^e(W^r - W_e^t(w_e^t)) - c(e)] \tag{6}$$

where W^r is the expected income for a worker in a raiding firm, e the search intensity of the worker and $c(e)$ the search cost. It follows that the worker's choice of e is given by $c'(e) = p(W^r - W_e^t)$. Let J_n^t denote the value of a training firm with a novice, and J_e^t the value of a training firm with an experienced worker. Then

$$(r+s)J_n^t = y_n^t - w_n^t + \gamma(J_e^t - J_n^t) \tag{7}$$

For a training firm with an experienced worker we have that

$$(r+s)J_e^t = y_e^t - w_e^t - ep^e J_e^t \tag{8}$$

We first consider the choice of w_e^t given that $W_n^t(w_n^t, w_e^t) \geq \bar{W}_n^t$ for some $\bar{W}_n^t$. It follows from (5) that $w_n^t = (r+s+\gamma)\bar{W}_n^t - \gamma W_e^t$. Inserted into (7) this gives

$$(r+s+\gamma)J_n^t = y_n^t - (r+s+\gamma)\bar{W}_n^t + \gamma(J_e^t + W_e^t) \tag{9}$$

The firm's only choice variable is w_e^t. Only the last term depends on w_e^t, and it follows that the firm chooses w_e^t so as to maximise $J_e^t + W_e^t$, the joint expected discounted income for the worker and the firm when the worker becomes experienced. The point is that w_e^t can be used to govern the search intensity of the worker. In the appendix we show the following lemma:

Lemma 2 For any $\bar{W}_n^t$, the optimal wage for experienced workers is given by $w_e^t = y_e^t$

The proof is given in the appendix. However, the intuition is simple: Workers choose e so as to maximise W_e^t. If $w_e^t \neq y_e^t$, the worker does not internalise the effect of quitting on the firm's profits. If $w_e^t < y_e^t$, on-the-job search gives rise to a negative externality for the current employer, and there is too much on-the-job search and turnover. On the other hand, if $w_e^t > y_e^t$, on-the-job search gives rise to a positive externality for the firm, and there will thus be too little on-the-job search and turnover. Or, put differently, given the overall compensation $\bar{W}_n^t$ to the worker, it is optimal for the firm to induce the worker to exert the on-the-job effort level that maximises the workers' and the firms' joint surplus, and where the gain from successful on-the-job search for the worker is a part of this surplus. The worker is induced to do this exactly when there is no externality from his on-the-job search on the firm, that is, when $y_e^t = w_e^t$.

Let Y^t denote the joint expected discounted income for a training firm and its novice employee. It follows that we can write it as

$$(r+s)Y^t = y_n^t + \gamma(W_e^t(y_e^t) - Y^t) \tag{10}$$

Here we use that firms set the wage to the experienced worker equal to his productivity y_e^t. Similarly, we can write the joint expected income for a raiding firm and an experienced worker employed in that firm as $Y^r = y^r/(r+s)$.

Let V^r denote the value of a raiding vacancy, and let V^t denote the value of a training vacancy. From the last section we know that in competitive search equilibrium, the income to the searching worker is maximised given that the firms break even. From (2) it thus follows that the equilibrium in the on-the-job search market solves the problem

$$\max_{p^e}(r+s)W_e^t(y_e^t) = \max_{p^e}\{y_e^t - c(e^*) + e^* p^e(Y^r - \frac{r+q(p^e)}{q(p^e)}K - W_e^t(y_e^t))\} \tag{11}$$

where p^e is the arrival rate of job offers (per unit of search intensity) in the on-the-job search market. Analogously, the equilibrium in the unemployed-search market is given by (with W_e^t given by (11))

$$\max_{p^u}(r+s)W_e^t(y_e^t) = \max_{p^u}\{y_e^t + p^u(Y^t - \frac{r+q(p^u)}{q(p^u)}K - W_e^t(y_e^t))\} \tag{12}$$

where p^u is the arrival rate of jobs in the unemployment search market. It is now easy to show the following

Proposition 2 Suppose $y_n^t + \gamma(Y^r - K) > (r + s + \gamma)K$. Then the equilibrium exists

It is now also easy to show that the equilibrium is efficient. First, consider the on-the-job search market. By introducing the choice of search intensity into the Bellman equation (4), it follows that the choice of search intensity in equilibrium is optimal. Since $y_e^t = w_e^t$ it follows that proposition 1 applies and the on-the-job search market is efficient. Furthermore, from the same proposition it follows that the social value of one more experienced worker is equal to the private value, $W_e^t(y_e^t)$. But proposition 1 then tells us that the unemployed-search market is efficient as well, and thus that the equilibrium as a whole is efficient.

Proposition 3 The equilibrium defined by (11) and (12) is socially efficient

Note that wage profile for a worker is steeper than his productivity profile. This seems to be consistent with empirical findings. Lazear argues that firms use steep wage profiles as an incentive device. In this paper, firms use a steep wage profile in order to economise on turnover.

Firm-specific human-capital investments

Suppose now that firms can invest in firm-specific human capital in their workers. We assume that the firms undertake the investments when workers are novices. Investing k dollar in a novice increases the productivity of an experienced worker, and we write $y_e^t = y_e^t(k)$. As the human capital is firm-specific, y^r is independent of k. It follows that we can write $W_e^t = W_e^t(k)$. For simplicity, we assume that the investments in firm-specific human capital are undertaken just before the worker becomes experienced. Since the entire return from the investments accrues to the worker, the socially optimal investment level is such that $W_e^{t\prime}(k) = 1$.

Proposition 4 When firms can advertise and commit to wages, the investments in firm-specific human capital are socially efficient

The first thing to note is that since the social and private value of getting one more experienced worker coincide, it is sufficient to show that the firm will set k so that $W_e^{t\prime}(k) = 1$. The second thing to note is that for a given wage w_e^t for experienced workers, the entire gain from firm-specific investments accrues to the firm, thus the firm has no commitment problem when it comes to firm-specific investments. For a given $\bar{W}_n^t$, a firm chooses wages and investment in firm-specific capital such that the profit is maximised given that $W_n^t = \bar{W}_n^t$. For any given y_e^t, we know that $w_e^t = y_e^t$ is optimal. The firm thus minimises $\frac{w_n^t + \gamma k}{r+s+\gamma}$ given that $W_n^t = \bar{W}_n^t$. Using (5) to eliminate w_n^t then gives that the firm will minimise $\bar{W}_n^t + \frac{\gamma(k - \bar{W}_e^t(k))}{r+s+\gamma}$, which obviously implies that $W_e^{t\prime}(k) = 1$.

It is worth noting that the worker in effect receives the entire gain from the firm-specific human capital through higher wages when experienced, and therefore also in effect pays the entire cost of the investment as a novice through a lower wage. This seems to contradict conventional wisdom that firm-specific human capital should be (at least partially) financed by the employer.

General human capital investments

Until now, all general human capital acquisition has been associated with learning by doing, in the sense that the rate at which the workers obtain human capital is exogenous. In this subsection we endogenise the rate at which the worker acquire general skills (i.e., the rate at which a novice becomes experienced).[1]

To this end, we assume that the rate at which a given worker becomes experienced, γ, can be written as a function of the flow h of investments in the worker. We also assume that the firm can advertise and commit to h. The social gain associated with having one more experienced worker is given by $W_e^t(y_e^t) - (W_n^t + J_n^t) = W_e^t(y_e^t) - Y_n^t$. The socially optimal value of h thus solves $\max_h \gamma(h)[W_e^t(y_e^t) - Y_n^t] - h$ with the first order condition

$$\gamma'(h)[W_e^t(y_e^t) - Y_n^t] = 1$$

The firm, on the other hand will chose an optimal mix of h and wages for experienced workers and inexperienced workers in order to minimise wage costs given that $W_n^t = \bar{W}_n^t$. Since the firm's profit is given by $Y_n^t - \bar{W}_n^t$, it follows that the firm

[1] We do not assume that the agents can manipulate the general human capital level for experienced workers, as this will take us into huge technical difficulties.

will choose h so as to maximise Y_n^t. Now $(r+s)Y_n^t = y_n^t + \gamma(h)(W_e^t - Y_n^t) - h$. Obviously, the first order conditions for maximum is given by $\gamma'(h)[W_e^t - Y_n^t] = 1$. We have thus shown the following proposition:

Proposition 5 Suppose firms can advertise and commit to wages and to the amount of general human capital investments. Then the equilibrium allocation is socially efficient

4 One wage rate

In the full commitment case, efficiency is obtained because firms can be compensated for a high wage for experienced workers by a low wage for inexperienced workers. In equilibrium, the costs K of opening a training firm is capitalised during the period where the worker is a novice.

However, for several reasons this compensation form may be impossible. This may be because there is a lower bound on wages because there exists a minimum wage law, or that having a low wage to novices is costly because the workers are credit constrained. A lower bound on wages is most likely to bind if the training period is relatively short. Firms may also be unable to commit to a wage for experienced workers that is higher (or sufficiently higher) than the wage for novices. We choose the latter interpretation, and thus assume that firms cannot commit to pay a higher wage for novices than for experienced workers. In this section we also assume that the parameters are such that firms, *ex post (*when the worker is experienced) do not want to increase the wage for experienced workers in order to economise on turnover (this assumption will be removed in the next section).

When a high wage rate for experienced workers cannot be compensated for by a low wage to novices, the same wage rate (the experienced wage rate) serves two purposes, to share income between workers and firms and to adjust the incentives to do on-the-job search. However, as a single wage rate cannot serve two purposes, the equilibrium wage turns out to be an inefficient compromise between these two ends.

Let w^t denoted the wage in the training firms, which now is independent of whether the worker is an experienced worker or a novice. It follows that we can write $e = e(w)$, with $e'(w) < 0$. Using (7) and (8) yields

$$(r+s+\gamma)\frac{\partial J_n^t}{\partial w^t} = -1 - \gamma\frac{1+pe'(w^t)J_e^t}{r+s+ep}$$

From equations (5), (6), and the envelope theorem it follows that

$$(r+s+\gamma)\frac{\partial W_n^t}{\partial w^t} = 1 + \frac{\gamma}{r+s+ep}$$

Now

$$\frac{\partial J_n^t}{\partial W_n^t} = \frac{\partial J_n^t/\partial w^t}{\partial W_n^t/\partial w^t}$$

It thus follows that

$$\frac{\partial J_n^t}{\partial W_n^t} = -1 - \gamma\frac{pe'(w^t)J_e^t}{r+s+ep+\gamma} \tag{13}$$

Since $e'(w^t) < 0$, the last term is positive, and it follows that $\frac{\partial J_n^t}{\partial W_n^t} > -1$. Thus, giving one unit more to the worker in terms of wages reduces the firm's profit with less than one unit. Training firms choose w so as to maximise V. Taking derivatives with respect to w and setting it equal to 0 thus gives

$$\eta\frac{J_n^t - K}{W_n^t - W^u} = 1 + \gamma\frac{pe'(w^t)J_e^t}{r+s+ep+\gamma} < 1 \tag{14}$$

It thus follows that the cost to the firm of increasing expected discounted wages with one unit is less than one unit. Thus, it is cheaper to increase wages in this case than in the full-commitment case, and for given values of J_n^t and W^u, the equilibrium wage is higher than in the full commitment case.

However, wages obviously will be set below y_e^t,otherwise firms will never capitalise on K. First, it will ($cet.par$) increase the workers' incentives to do on-the-job search. Second, more raiding firms will ($cet.par$) enter the market:

According to lemma 1, the equilibrium in the on-the-job search market maximises $W_e^t(w^t)$. The equilibrium thus solves the maximisation problem

$$\max_{e,p^e} w^t - c(e) + p^e e^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K) \tag{15}$$

It follows that p^e is decresing in w^t. To see this, note that from the envelope theorem, $W_e^{t\prime}(w^t) = 1/(r+s+pe)$. From (15) it follows that the equilibrium value of p maximises $p^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K)$ with respect to p^e, and when W_e^t falls, the optimal value of p^e increases as well.

It is also possible to show that the search intensity is decreasing in w^t. To see this, first note that we can write $W_e^t(w^t)$ as

$$(r+s)W_e^t(y_e^t) = \max_e \{w^t - c(e) + e^e[\max_{p^e} p^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K) - W_e^t(y_e^t)]\}$$

From the envelope theorem it follows that the derivative of $\max_{p^e} p^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K)$ with respect to w^t is equal to $-p^e/(r+s+pe) < 0$. Since the first order condition for e is given by $c'(e) = \max_{p^e} p^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K)$, it thus follows that $e'(w^t) < 0$.

We have thus showed the following proposition:

Proposition 6 *Suppose firms advertise one wage only, as described above. Then, relative to the socially efficient equilibrium, the following holds:*

1. *There is too much on-the-job search (e is too high)*
2. *Given the number of training firms in the market, there are too many raider firms entering the market (p is too high)*

It also follows that Y_n^t is lower in this case than with commitment, as the joint expected income for the worker and the firm falls for experienced workers. To see this, first recall that when $w = y_e^t$, J is zero, and maximising W_e^t is equvialent to maximising the joint income for the experienced worker and his employer. For $w^t < y_e^t$ it follows that

$$\begin{aligned}(r+s)[W_e^t + J_e^t] &= w^t + (y^t - w^t) + \max_{e,p^e}[-c(e) + e^e p^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K)] - e^e p^e J_e^t \\ &< y^t + \max_{e,p^e}[-c(e) + e^e p^e(Y^r - W_e^t - \frac{r+q(p^e)}{q(p^e)}K - J_e^t)] \\ &= W_e^t(y_e^t)\end{aligned}$$

It is straight-forward to show that in the competitive search equilibrium, a fall in productivity leads to increased unemployment. In our model, the unemployment rate therefore exceeds its first best level for two reasons: first, there is too much turnover, which reduces the incentives for training firms to create jobs. Second, given the level of turnover, unemployment increases because a larger proportion of the joint income is allocated to the worker, thus reducing the incentives to open up vacancies in training firms.

Proposition 7 In equilibrium, too few training firms enter the market

Still, although the market is inefficient compared to the first best solution, it is first best in the following sense: suppose the planner was setting the wage in training firms (not being allowed to discriminate between experienced and inexperienced workers), while all other decisions were carried out by the market. Then the planner would set the wage equal to the equilibrium wage in training firms. To see this, note that the social value of an experienced worker entering the on-the-job search market, leaving out the interests of training firms, is given by $W_e^t(w^t)$. When the interests of the training firms is included, it thus follows that the social value of an experienced worker is equal to $W_e^t(w^t) + J_e^t(w^t)$. It thus follows that the equilibrium wage w^t is still set so as to maximise W^u given that $V^t = K$. Thus, the training firms maximise welfare given the search behaviour of experienced workers and the entry decisions of raiding firms.

Lemma 3 Suppose the planner determines the wage level in training firms (not being able to discriminate between experienced workers and novices), while all other decisions were taken by the market participants. Then the planner would set the wage in training firms equal to the equilibrium wage in training firms.

By using exactly the same argument, it follows that the number of training firms entering the market, and the investment levels in general and in firm-specific human capital (given that they can be truthfully advertised) are second-best efficient:

Proposition 8 The unemployment search market is second-best efficient. Thus, the government cannot improve welfare by subsidising training firms or investments in general or firm-specific human capital.

It is interesting to note the difference between our result and the result in Salop (1979). Salop derives the wage that is optimal for the firms in order to economise on turnover. If this insider wage is higher than the outside wage (the wage necessary to attract applicants), job rationing occurs in equilibrium. However, if the outside wage is higher than the insider wage, the insider wage has no influence on the wage rate in the economy. In this paper, by contrast, the market-clearing wage has no natural definition. When setting wages, firms trade off search costs and wage costs. If a high wage, in addition to speeding up the hiring process, also has a positive impact on worker turnover, this tilts this trade-off in the direction of higher wages. This is true independently of the costs of turnover to the firm, although the effect in numerical terms of course is larger the more important are the costs to the firms associated with turnover.

Since Salop does not model the turnover explicitly, this prevents welfare analysis of his model. We model turnover explicitly, and show that the high wages paid in training firms, as well as the reduced number of training firms and the reduced human capital investments in each training firm, may be considered as an optimal respons to the imperfections caused by the assumption that firms cannot make wages contingent on experience.

Finally, note that welfare can be improved by policy measures that deal directly with the externalities caused by turnover. For instance, welfare can be improved if the government taxes wages in excess of wages earned in training firms (and thus only paid by workers employed in raiding firms). This will reduce the incentives to do on-the-job search for experienced workers and for raiding firms to enter the market instead of training their own workers.

5 *Ex post* determination of wages

In this section, we assume that the wages for experienced workers are determined *ex post*. This may for instance be because firms want to set a higher wage in period one than in period two in order to economise on turnover. When setting the wage for experienced workers, a firm has to take into account that lowering wages implies higher search intensity and thereby a higher quit probability rate.

Let us first derive the value of w_e^t that maximises J_e^t. Taking the derivative of (8) with respect to w_e^t gives

$$(r+s)\frac{dJ_e^t}{dw_e^t} = -1 - \frac{de}{dw_e^t}pJ_e^t - pe\frac{dJ_e^t}{dw_e^t}$$

The first order conditions for maximum is thus given by

$$-\frac{de}{dw_e^t}pJ_e^t = 1 \tag{16}$$

Due to the envelope theorem, we know from (6) that $dW_e^t/dw_e^t = -1/(r+s+pe)$. Taking the derivative of the first order condition for e with respect to w_e^t thus gives

$$c''(e)\frac{de}{dw_e^t} = -\frac{p}{r+s+pe}$$

which inserted into (16) gives

$$\frac{p^2}{r+s+pe}J_e^t = c''(e) \tag{17}$$

It is now possible to show the following lemma:

Lemma 4 For all values of p, firms set $w_e^t < y_e^t$. For sufficiently low values of p^e, the optimal *ex post* wage is below the equilibrium wage in the one-wage case, while for sufficiently high J_e^t (sufficiently large frictions in the unemployed-search market), the optimal *ex post* wage is above the equilibrium wage in the one-wage case

As wages for experienced workers are below y_e^t, proposition 6 still holds, and there is excess turnover in the market.

Now consider investments in firm-specific human capital. Higher firm-specific human capital implies that it is optimal for the firm to set a higher wage for experienced workers. From (17) it follows that w_e^t is increasing in y_e^t. Thus, investments in firm-specific human capital may be considered as a partial commitment device. Thus, although the firm sets the wage in the next period, the worker is willing to renege on wages in the training period, and thereby pay parts of the training costs, as the wage that the firm will set when the worker becomes experienced will increase.

More specifically, from (17) it follows that the wage the firm sets can be written as $w_e^t = w_e^t(k)$. From the envelope theorem, it follows that the return on H.C. investments to the firm is given by $J_e^{t\prime}(k) = \frac{y_e^{t\prime}(k)}{r+s+pe}$. Let k^* be defined by the equation $J_e^{t\prime}(k^*) = 1$, that is, the point at which the gain in terms of expected increased production value from the investments is equal to the marginal investment cost. It follows that firms set k above k^* in order to economise on the turnover.

However, given the inefficiencies caused by firms not being able to commit to a wage contract, the firm-specific human capital investments are optimal (since the proof is almost identical to the proofs in the last section, it is omitted):

Proposition 9 Suppose the planner can determine k, but no other variables in the economy. Then the planner will set k at the same level as generated by the market.

Similar effects are identified by other authors: however, these authors have missed that over-investments may be an optimal response to the problem that firms are unable to commit to a wage contract.

6 Appendix

Proof of lemma 2

Write J_e^t as a function of w_e^t and e; $J_e^t = J_e^t(w_e^t, e)$. Similarly, we write W_e^t as a function of w_e^t, $W_e^t = W_e^t(w_e^t)$. Since e is a choice variable for the worker, W_e^t does not depend on e. By the envelope theorem it follows that $\frac{\partial J_e^t}{\partial w_e^t} = -\frac{dW_e^t}{dw_e^t}$. Thus,

$$\begin{aligned}\frac{d}{dw_e^t}(J_e^t + W_e^t) &= \frac{\partial J_e^t}{\partial w_e^t} + \frac{\partial J_e^t}{\partial e}\frac{de}{dw_e^t} + \frac{dW_e^t}{dw_e^t} \\ &= \frac{\partial J_e^t}{\partial e}\frac{de}{dw_e^t}\end{aligned}$$

From (8) it follows that $(r+s)\frac{\partial J_e^t}{\partial e} = -\frac{p^e J_e^t}{r+s+ep^e}$, thus

$$\frac{d}{dw_e^t}(J_e^t + W_e^t) = -\frac{p^e J_e^t}{r+s+ep^e}e'(w_e^t) \tag{18}$$

The first order conditions for maximum is thus that $J_e^t = 0$, i.e., that $w_e^t = y_e^t$. Since $e'(w_e^t) < 0$ it follows that $\frac{d}{dw_e^t}(J_e^t + W_e^t) > 0$ for $w_e^t < y_e^t$, while $\frac{d}{dw_e^t}(J_e^t + W_e^t) < 0$ for $w_e^t > y_e^t$. This shows that $w_e^t = y_e^t$ is the optimal wage to set for experienced workers.

Since the worker's search effort is decreasing in w_e^t, it follows that the sign of the derivative is the opposite. We know that the worker sets e such that $c'(e) = p(W^r - W_e^t)$, which inserted into (9) gives

$$(r+s+\gamma)\frac{\partial J_n^t}{\partial w_e^t} = -\frac{\gamma p^e J_e^t}{r+s+ep^e} e'(w_e^t)$$

Since $e'(w_e^t) < 0$, it follows that J_n^t is increasing in w_e^t as long as $J_e^t > 0$ and decreasing in w_e^t when $J_e^t < 0$. Obviously, it is optimal to set $w_e^t = y_e^t$. Note that this result holds independently of W^r and $\bar{W}_e^t$. It follows that we can write J_n^t as

$$(r+s+\gamma)J_n^t = y - (r+s+\gamma)\bar{W}_e^t + \gamma W_e^t(y_e^t) \tag{19}$$

where $W_e^t(y_e^t)$ denotes the value of W_e^t that is obtained when $w_e^t = y_e^t$.

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