A MONETARY SHOCK IN AN UNCERTAIN AND SEQUENTIAL TRADE MODEL

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I study the effects of a monetary shock in a simple version of the uncertain and sequential trade (UST) model. A negative money shock has: (a) no effect on quoted prices in the first period, (b) a negative effect on output, (c) a positive effect on the interest rate, (d) a negative effect on the real wages and (e) a negative effect on real profits. These effects are in accordance with the stylized facts in Christiano Eichenbaum and Evans (1997).

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INTRODUCTION

In a recent paper Christiano Eichenbaum and Evans (CEE, 1997) found that in response to a contractionary monetary shock:

(a) The aggregate price level initially responds very little;
(b) Aggregate output falls;
(c) Interest rates initially rise;
(d) Real wages decline by a modest amount;
(e) Profits fall.

These "stylized facts" seems rather robust. For a survey of the literature see CEE (1998).

In their article CEE (1997) compared the ability of two models to account for these stylized facts: A limited participation model and a sticky price model. They conclude that the two alternative models cannot account for all the above stylized facts. The limited participation model cannot explain the lack of an initial effect on prices (stylized fact [a]) while the sticky price model cannot explain the decline in profits (stylized fact [e]).

Here I show that a simple version of the uncertain and sequential trade (UST) model can account for all of the above stylized facts.

UST models are based on ideas in Prescott (1975) and Butters (1977). Prescott considers an environment in which sellers set prices before they know how many buyers will eventually appear. He assumes that less expensive goods will be sold before more expensive ones and obtains an equilibrium trade-off between the price and the probability of making a sale. A similar trade-off arises in Butters

(1977). In both models sellers commit to prices before the realization of demand. In the UST approach taken by Eden (1990), trade is sequential and an equilibrium distribution of prices is obtained even though sellers are allowed to change their prices during trade. The UST approach was recently used in monetary economics by Eden (1994), Lucas and Woodford (1994), Woodford (1996) and Williamson (1996). The approach here is closely related to Bental and Eden (1996).

Money is not neutral in the UST model because irreversible trading decisions are made before the resolution of uncertainty about the current period money supply. The interest effect of money in the current version is due to the cash-in-advance constraint in the bonds market. I make a straightforward use of the idea that "if cash is required for trading in securities, then the quantity of cash - of "liquidity" - available for this purpose will in general influence the prices of securities traded at that time" (Lucas [1990] page 237). See also Grossman and Weiss (1983), Rotemberg (1984) and Fuerst (1992).

THE MODEL

There are N infinitely lived households. Each household consists of two people: a seller and a buyer. At the beginning of each period they separate: The buyer takes the money and goes shopping. The seller produces and sells his output for money. After trade in the goods market is completed the seller takes his revenue in the form of cash. On the way home some sellers visit the bank (the bond market). At the end of the period the two members of the household reunite and consume the basket of goods that the buyer has bought.

Labor (L_t) is the only input and output equals labor input. It is assumed that the household is risk neutral and its single period utility is: $c_t - v(L_t)$, where the function v( ) has the standard properties of a cost function: $v' > 0$ and $v'' > 0$. The household's discount factor is $0 < \beta < 1$.

The amount of money available to household h at the beginning of period t is M_t^h dollars. The buyer takes the M_t^h dollars and goes shopping. Buyers arrive in the market sequentially. The order of arrival is determined by an i.i.d. lottery.

On the way to the market the buyer may receive a transfer of T_t dollars. The number of buyers that will receive a transfer is unknown. For simplicity it is assumed that either γN or $2\gamma N$ buyers will get a transfer with equal probability of occurrence. The identity of the buyers that will receive the transfer is determined by an i.i.d. lottery.[1]

The buyer spends all the available cash: M_t^h if he did not get a transfer and $M_t^h + T_t$ if he did get a transfer. Upon arrival, each buyer sees all the available selling offers and buys at the lowest price offer. Since cheaper goods are bought first, buyers that arrive late may face a higher price.

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[1] We may think in terms of the following story. The government is commited to a welfare program which pays T_t dollars to households which qualifies. The criteria for qualification is not well understood by the public and therefore ex-ante all households assign the same probability to winning the welfare lottery. This probability itself is a random variable that may take the realizations γ and 2γ.

The average holding of money at the beginning of period t (before the beginning of the transfer process) is: $M_t = (1/N)\Sigma_{h=1}^{N} M_t^h$. For simplicity I assume: $T_t = M_t$. The total amount available for spending per seller is: $M_t + \gamma T_t = (1 + \gamma)M_t$ if a fraction γ got a transfer and $(1 + 2\gamma)M_t$ if a fraction 2γ got a transfer.

I divide all nominal magnitudes by M_t. This normalization is equivalent to using the money supply as a unit of account. I therefore define a normalized dollar by the money supply per household. A price of one normalized dollar means that you must pay the whole money supply (average per household) to get a unit of whatever is being sold.

From the sellers' point of view purchasing power arrives in batches. The first batch of $1 + \gamma$ normalized dollars arrives with certainty and buys in the first market at the price p_1. The second batch of γ normalized dollars may arrive with probability $^1/_2$ and if it arrives, it buys in the second market at the price of p_2 normalized dollars per unit.

At the beginning of the period the representative seller chooses the amount of labor L and allocates the available supply across the two markets:

(1) $$k_1 + k_2 = L,$$

where k_s is the supply to market s. This allocation may be viewed as a contingent plan which specifies how much will be sold to each batch of demand that arrives.

The seller's total revenues if s markets open are:

(2) $$tr^s = \Sigma_{j=1}^{s} p_j k_j,$$

normalized dollars.

A fraction $0 < \alpha \leq 1$ of the sellers go to the bonds market after the closing of all the goods markets. The identity of the sellers who go to the bonds market is determined every period by an i.i.d. lottery.

There are only government bonds. A bond is a promise to pay one normalized dollar at the beginning of next period. The price of a bond if s markets open today is denoted by $1/R^s$ and the gross interest rate is R^s. Since nothing happens between the buying of a bond and the payment of the dollar it promises, agents who get an access to the bond market will invest all their idle money in bonds at the gross interest rate: $R^s \geq 1$. In what follows I assume that this condition is satisfied.

The supply of government bonds is constant over time and is given by b normalized dollars.[1] The interest on the government debt is paid out of lump sum taxes collected at the end of the period. The lump sum tax is: $g^s = b - b/R^s = b(1 - 1/R^s)$ normalized dollars if s markets were open this peirod.

The sequence of events within a typical period is illustrated by Figure 1.

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[1] The supply in terms of regular dollars does change and is given by $B_t = bM_t$.

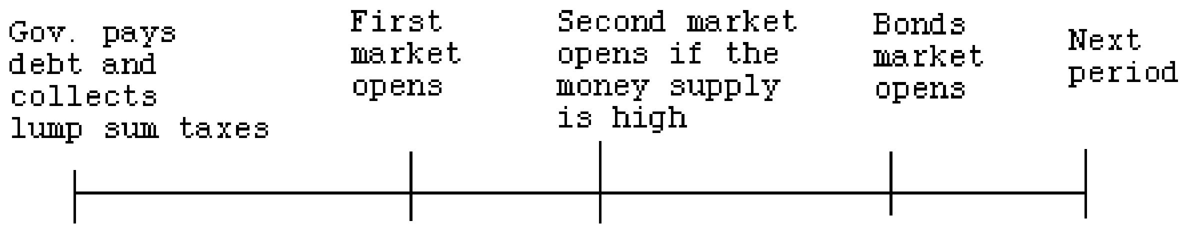

Figure 1

The amount of money available to the household at the beginning of next period is:

(3) $$m' = (R^s tr^s - g^s)/(1 + s\gamma)$$

if the seller buys bonds and $(tr^s - g^s)/(1 + s\gamma)$ if he does not buy bonds all in terms of next period's normalized dollars. Note that since the money supply increases at the rate of γs the number of regular dollars in the current period normalized dollar is a fraction $1/(1 + s\gamma)$ of the amount of regular dollars in next period's normalized dollars. Therefore, to convert current into next period normalized dollars we divide by $1 + s\gamma$.

A buyer who holds m normalized dollars at the beginning of the period will consume on average:

(4) $$E_{i,s}\{(m + i)/p_s\} =$$
$$= (1/2)(m + \gamma)/p_1 + (1/2)[\theta(m + 2\gamma)/p_1 + (1 - \theta)(m + 2\gamma)/p_2],$$

units of consumption, where i = 1 if the buyer gets a transfer and zero otherwise; Expectations are taken over both i = 0,1 and

$s = 1,2$ and $\theta = (1 + \gamma)/(1 + 2\gamma)$ is the fraction of dollars in batch 1 out of the post-transfer money supply when two markets open.[1]

The Bellman equation which describes the household's behavior is:

(5) $$V(m) = \max E_{i,s}(m + i)/p_s - v(k_1 + k_2)$$

$$+\beta\alpha E_s V\{[R^s(\Sigma_{j=1}^{s} p_j k_j) - g^s]/(1 + s\gamma)\}$$

$$+ \beta(1-\alpha)E_s V\{[(\Sigma_{j=1}^{s} p_j k_j) - g^s]/(1 + s\gamma)\},$$

where, as before, E_x denotes expectations with respect to the random variable x and the maximization is with respect to k_s. Here V(m) is the maximum expected utility that a household who starts the period with m normalized dollars can achieve. The first row is the current expected utility. The last two rows are the future expected utility: The second is for the case in which the seller buys bond (with probability α) and the third is for the case in which he does not buy bonds.

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[1] To compute (4), note that if only one market opens, the buyer will buy at the price p_1 with probability one. If he gets a transfer (with probability γ) he will buy $(m + 1)/p_1$ units of consumption. Otherwise, he will buy m/p_1 units. The expected consumption if only the first market opens is therefore:
$[\gamma(m + 1) + (1 - \gamma)m]/p_1 = (m + \gamma)/p_1$.
If two markets open in the current period, then the buyer will participate in the first market with probability
$\theta = (1 + \gamma)/(1 + 2\gamma)$ and receive a transfer with probability 2γ. The expected consumption given that two markets open is therefore:
$\theta(m + 2\gamma)/p_1 + (1 - \theta)(m + 2\gamma)/p_2$. And the unconditional expected consumption is (4).

To state the first order conditions for (5) it is useful to define the expected purchasing power of a normalized dollar held by the buyer at the beginning of the period. This is:

(6) $$z = (\tfrac{1}{2})(1/p_1) + (\tfrac{1}{2})[\theta/p_1 + (1 - \theta)/p_2].$$

The first term on the right hand side is the purchasing power when only one market opens times the probability of this event. The second expression applies for the case in which both markets open and the normalized dollar will buy in market 1 with probability θ.

It is also useful to introduce the coefficient:

(7) $$\omega^s = [(1-\alpha) + \alpha R^s]/(1 + s\gamma),$$

which converts current normalized dollars earned in the goods market to next period's normalized dollars. A current normalized dollar earned in the goods market will yield R^s if its owner buys bonds and 1 otherwise. It will therefore yield on average $(1-\alpha) + \alpha R^s$ current normalized dollars. To convert it to next period's normalized dollars we multiply by $M_t/M_{t+1} = 1/(1 + s\gamma)$.[1]

[1] Here we assume bonds that reach maturity immediately. The analysis can be extended to more conventional bonds that reach maturity after T periods by using an appropriate conversion coefficient. Specifically, let $d = \beta E_s[1/(1 + s\gamma)]$ denote the value of a normalized dollar that will be paid next period in terms of todays normalized dollar. Then if the government issues a T periods bonds the conversion coefficient (7) becomes:
$\omega^s = (1-\alpha)/(1 + s\gamma) + \alpha d^T R^s/(1 + s\gamma)$.

A normalized dollar earned in the second market will therefore buy on average $\omega^2 z$ units of consumption. A normalized dollar earned in the first market will buy ωz units, where $\omega = (1/2)\omega^1 + (1/2)\omega^2$.

The first order conditions which must be satisfied for an interior solution ($k_s > 0$) to (5) are:

$$(8) \qquad v'(k_1 + k_2) = \beta p_1 \omega z = (1/2)\beta p_2 \omega^2 z.$$

To interpret (8) note that a unit sold in market 2 will bring a revenue of p_2 normalized dollars which, on average, will become $p_2\omega^2$ next period's normalized dollars and buy $p_2\omega^2 z$ units of consumption. The expression $(1/2)\beta p_2\omega^2 z$ is therefore the expected discounted consumption that a unit of capacity supplied to market 2 will bring. Similarly, the expression $\beta p_1 \omega z$ is the expected discounted consumption that a unit supplied to market 1 will bring. Thus, (8) says that at the optimum the marginal cost (v') must equal the discounted expected consumption that an additional unit will bring.

A steady state equilibrium is a vector $(k_1, k_2, p_1, p_2, R^1, R^2, \omega^1, \omega^2, z)$ such that:

(a) $R^s \geq 1$; $z = (6)$; $\omega^s = (7)$;

(b) given $(p_1, p_2, R^1, R^2, \omega^1, \omega^2, z)$, the quantities (k_1, k_2) satisfy the first order condition (8);

(c) Goods markets which open are cleared:

$p_1 k_1 = 1 + \gamma$; $p_2 k_2 = \gamma$;

(d) The bond market is cleared: $\alpha(\Sigma_{j=1}^{s} p_j k_j) = b/R^s$ for $s = 1,2$.

<u>A numerical example:</u> To solve for the nine equilibrium magnitudes: $(k_1, k_2, p_1, p_2, R^1, R^2, \omega^1, \omega^2, z)$, we use the five equations in

(6) - (8) and the four market clearing conditions. Assuming for example (γ = 0.1, b = 1, α = $^3/_4$, v(L) = L^2, β = 1), yields the following solution: (k_1 = 0.458, k_2 = 0.019, p_1 = 2.397, p_2 = 5.195, R^1 = 1.21, R^2 = 1.11, ω^1 = 1.054, ω^2 = 0.902, z = 0.408).

Note that it is possible to solve the bond markets variables (R^1, R^2, ω^1, ω^2) before solving for the other variables.

The nominal interest rate:

The goods markets clearing conditions imply: $\Sigma_{j=1}^{s} p_j k_j = 1 + s\gamma$. Therefore, the clearing of the bonds market implies:

$$(9) \qquad R^s = b/\alpha(\Sigma_{j=1}^{s} p_j k_j) = b/\alpha(1 + s\gamma).$$

If the supply of bonds is strictly greater than the demand for it, $b > \alpha(1 + s\gamma)$, the gross nominal rate is greater than one and there will be an active bonds market.

The equilibrium relationship (9) implies a negative relationsip between R and the end of period money supply. What assumptions are required for this result? To answer this question let us consider an environment with no uncertainty and assume that the fraction of buyers who get the transfer payment is always γ. In this case there will be only one goods market. Let us denote the price in the single goods market by p (normalized dollars per unit) and let k denote the supply of the representative seller. Suppose further that α = 1 and all sellers buy bonds with the pk normalized dollars that they earn at the goods market. Under the assumption $b > 1 + \gamma$, there exists a market clearing interest rate: $R = b/pk = b/(1 + \gamma) > 1$. Clearly an increase in the end of period money supply (an increase in γ) leads to a reduction in the interest rate.

Thus limited participation ($\alpha < 1$) and trading uncertainty are not necessary for the negative relationship between the end of period money and the interest rate. This negative relationship arises as a special case of Lucas' argument cited in the introduction: The cash in advance in the bonds market implies that the amount of liquidity affects the interest rate.

The role of the limited participation assumption is to allow for the possibility that both cash and bonds are held while bonds have a higher rate of return. In this model all sellers invest their idle cash in bonds if they have a chance to do so. Idle cash is held only by sellers who for one reason or another did not make it to the bank.

<u>The "winner curse" effect:</u>

Substituting (9) in (7) yields:

(10) $$\omega^s = [(1-\alpha) + b/(1 + s\gamma)]/(1 + s\gamma),$$

which implies $\omega^1 > \omega^2$. Thus, a normalized dollar earned in the first market is worth more in terms of next period's normalized dollars than a normalized dollar earned in the second market: $\omega = (1/2)(\omega^1 + \omega^2) > \omega^2$. This result will play an important role in explaining stylized fact (d). It is in the spirit of the "winner curse" effect in the auctions literature: The success in buying dollars for goods implies a lower value of the dollar.

Accounting for the stylized facts (a) - (c):

In equilibrium, money affects quoted prices with a lag of one period. Thus, prices may appear to be "sticky" in spite of the fact that sellers do not have an incentive to change prices during trade. This accounts for the first stylized fact. When money is low and only one market opens capacity utilization is low and measured output is low. This accounts for the second stylized fact. The nominal interest rate (9) declines with s and the current money supply. The real interest rate, $R^S/(1 + s\gamma) = b/\alpha(1 + s\gamma)^2$, also declines with s (and the current money supply). This accounts for the third stylized fact.

I now add variable costs and the distinction between workers and firms to account for the stylized facts (d) and (e).

ADDING VARIABLE COSTS

It was implicitly assumed that capacity created at the beginning of the period can be costlessly converted into output at the time of sale. I now assume that it costs one unit of variable labor (l) to convert a unit of capacity into output at the time of sale. We may think of a restaurant as an example. It takes a unit of fixed labor to prepare a meal (create capacity) and a unit of variable labor to serve it and wash the dishes (convert capacity into output).

Unlike the previous section, here production is done by firms. Each household owns a firm which hires (fixed and variable) labor, but cannot supply labor to his own firm. The wage rate for the fixed factor is denoted by w_f. The wage rate for the variable factor supplied if market s opens is denoted by w_{vs}, all in normalized dollars.

The firm's profits if s markets open are:

(11) $$\psi^s = \Sigma_{j=1}^{s} (p_j - w_{vj})k_j - w_f(k_1 + k_2).$$

The firm maximizes the expected purchasing power of profits. It chooses $k_s \geq 0$ to solve:

(12) $$\max\ (1/2)\psi^1\omega^1 z + (1/2)\psi^2\omega^2 z.$$

An interior solution to (12) requires a strictly positive net price $(p_j - w_{vj})$ and

(13) $$(p_1 - w_{v1})\omega = (1/2)(p_2 - w_{v2})\omega^2 = w_f\omega.$$

This says that the expected net revenue (in terms of next period's normalized dollars) from supplying an additional unit to market 1 is the same as the expected net revenue from supplying an additional unit to market 2 and is equal to marginal cost.

For simplicity I consider the case in which the household's utility function is linear in variable labor:
$U(c, L, l) = c - v(L) - \eta l$, where L is the quantity of fixed labor, l is the quantity of variable labor and η is a strictly positive parameter. We may think of the variable input as being supplied by a new member of the household: a child.

I use l_s to denote the quantity of variable labor supplied to market s if it opens and describe the household's problem by the following Bellman's equation:

$$(14) \qquad V(m) = \max E_{i,s}(m + i)/p_s - v(L) - \eta l_1 - (1/2)\eta l_2$$

$$+ \beta\alpha E_s V\{[R^S(w_f L + \Sigma_{j=1}^{S} w_{vj} l_j + \psi^s) - g^s]/(1 + s\gamma)\}$$

$$+ \beta(1-\alpha)E_s V\{[(w_f L + \Sigma_{j=1}^{S} w_{vj} l_j + \psi^s) - g^s]/(1 + s\gamma)\},$$

where the maximization is with respect to L and l_s. The first order conditions for an interior solution to this problem are:

$$(15) \qquad v'(L) = \beta w_f \omega z,$$

and

$$(16) \qquad \eta = \beta w_{v1}\omega z = \beta w_{v2}\omega^2 z.$$

These first order conditions require that marginal utility cost equals expected discounted consumption. Note that since $\omega > \omega^2$,

$$(17) \qquad w_{v1} < w_{v2}.$$

The intuition is in the "winner curse" effect: A normalized dollar earned in market 1 is worth more in terms of next period's normalized dollars and therefore the wage rate for the variable factor in market 1 is lower, in spite of the fact that we assume constant marginal variable labor cost. The result will be only strengthen if instead of the linear cost $v(L) + \eta l$ we assume $v(L + l)$.

Substituting (15) in (13) leads to:

(18) $$v'(L) = \beta(p_1 - w_{v1})\omega z = (1/2)\beta(p_2 - w_{v2})\omega^2 z$$

which is the same as (8) when $w_{v1} = w_{v2} = 0$.

A steady state equilibrium is a vector (k_1, k_2, L, l_1, l_2, p_1, p_2, w_f, w_{v1}, w_{v2}, ψ^1, ψ^2, R^1, R^2, ω^1, ω^2, z) such that:

(a) $R^s \geq 1$; $p_j - w_{vj} \geq 0$; $L = k_1 + k_2$; $l_s = k_s$;

(b) z = (6); ω^s = (7); ψ^s = (11);

(c) given (p_1, p_2, w_f, w_{v1}, w_{v2}, ω^1, ω^2, z) the quantities (k_1, k_2) solve (12);

(d) given (p_1, p_2, R^1, R^2, w_f, w_{v1}, w_{v2}, ψ^1, ψ^2), the labor supplies (L, l_1, l_2) solve (14);

(g) markets are cleared:

$p_1 k_1 = 1 + \gamma$; $p_2 k_2 = \gamma$;

and $\alpha(\Sigma_{j=1}^{s} p_j k_j) = b/R^s$;

for s = 1,2.

<u>Back to the stylized facts:</u>

In this model sellers take into account the "winner curse" effect on the value of the dollar: A dollar earned in a higher index market is worth less in terms of next period's normalized dollars than a dollar earned in a lower index market. Therefore, the equilibrium wage of the variable factor in market 1 must be lower than the equilibrium wage of the variable factor in market 2 ($w_{v1} < w_{v2}$). The average real wage of the variable factor is lower when the money supply is low: $w_{v1} < (w_{v1}l_1 + w_{v2}l_2)/(l_1 + l_2)$. This is in accordance with stylized fact (d).

An interior solution to the firm's problem (12) requires a strictly positive net price ($p_j - w_{vj}$). Using the definition of profits in (11) this implies: $\psi^2 - \psi^1 = (p_2 - w_{v2})k_2 > 0$. Thus in accordance to the stylized fact (e) real profits are low when the money supply is low.

CONCLUSIONS

A simple UST model was presented. In this model a negative money supply shock causes all the effects documented in CEE (1997).

The lack of price level response (stylized fact [a]) occurs because quoted prices do not change in response to current changes in the money supply. At the beginning of the period the sellers put a price tag on each unit. This activity was described as allocating capacity over abstract markets. It reflects a contingent plan which specifies the amount that will be sold to each batch that may arrive. The contingent plan is time consistent and therefore the seller does not change quoted prices in response to information that arrives during the trading process.[1]

The fall in aggregate output (stylized fact [b]) occurs because when money is low, less markets are opened and capacity utilization is low. Measured output is therefore low for two reasons. First, unused capacity is often not measured as output: A meal which was prepared in a restaurant but was not sold is not measured as output even when fixed costs were used in the preparation of the meal.

[1] Average transactions prices do change but the data collected about prices is usually about quoted prices not transaction prices.

Second, low capacity utilization implies the use of less variable factors of production.

When the money supply is low, the sellers revenue is low and less idle money arrives at the bonds market. Since the supply of bonds does not depend on the end of period money supply, low money leads to a low price of bonds and high rate of return. This accounts for stylized fact (c).

The "winner curse" accounts for stylized fact (d): Money earned in the first market will buy more on average than money earned in the second market. Therefore, the wage paid for the variable factor is lower in the first market. When the money supply is low only market 1 opens and the average wage is low. Assuming fatigue (increasing marginal variable labor cost) will only strengthen this result.

When the money supply is low, capacity utilization is low and since the firm bears the full cost of low capacity utilization, profits are low. This accounts for stylized fact (e).

In addition to the above five stylized facts, Gertler and Gilchrist (1994) observed that inventories tend to rise after a monetary contraction. This is difficult to explain when using either the limited participation model or the fixed price model in CEE (1997). In these models price taking firms can sell as much as they want at the market clearing price and therefore storage occurs only if the expected rise in price covers the storage and interest costs of holding inventories. After a monetary contraction, prices are expected to fall and therefore inventories should not be held according to these models.

UST models allow for the possibility that sellers will fail to make a sale and hold "unwanted" inventories even if prices are expected to decline. Inventories are especially large after a monetary contraction because in this case many units are not sold. For a UST model with inventories, see Bental and Eden (1996) and Eden (forthcoming).

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