Forward Guidance: Communication, Commitment, or Both?*

Marco Bassetto†

Abstract

Faced with the constraint of the zero lower bound on interest rates, central banks around the world have engaged in forward guidance as one instrument to stimulate the economy. To properly ascertain the potential benefits of forward guidance as an independent tool of monetary policy, it is important to understand how it can work. I analyze the strategic interaction between households and the central bank as a game in which the central bank has access to cheap talk. In the absence of private information, the set of equilibrium payoffs is independent of the announcements of the central bank: forward guidance as a pure commitment mechanism (“Odyssean forward guidance”) is a redundant policy instrument. When private information is present, central bank communication can have social value, and a central bank’s communication strategy interacts with its credibility. Forward guidance emerges as a natural communication strategy when the private information in the hands of the central bank concerns its own preferences or beliefs.

*Preliminary and incomplete. I thank Gadi Barlevy, Jeffrey Campbell, Martin Cripps, Mariacristina De Nardi, Marco Del Negro, Charles L. Evans, Antonella Ianni, Luigi Iovino, Spencer Krane, Eric Mengus, Benoît Mojon, Emi Nakamura, Thomas J. Sargent, and Jón Steinsson for helpful conversations. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

†Federal Reserve Bank of Chicago and IFS
1 Introduction

With interest rates effectively at the zero lower bound in the United States, the Euro Area, Great Britain, and Japan, monetary authorities across the world have looked for alternative tools to stimulate the economy. The resulting unconventional monetary policy has revolved around two main strategies: forward guidance and quantitative easing. In this paper, we will consider the conditions which make forward guidance effective. We define forward guidance as a situation in which central banks provide direct statements about the future path of their policy tools. To some extent, central banks have provided such statements for years, as part of their broader discussion of their view on the underlying conditions of the economy. In fact, Campbell et al. [13] find evidence that forward rates often reflect ahead of time what would appear as a monetary policy shock in VARs which are purely based on spot-market interest rates and macroeconomic variables. What has been different recently is that announcements have become more explicit, and that they have been tied to a desire to precommit future policy.\footnote{Commitment is very often desirable when a policymaker faces forward-looking agents, as emphasized first by Kydland and Prescott [24]. Within the new Keynesian framework, commitment to keep interest rates at zero for longer than would be optimal ex post is particularly valuable; see e.g. Eggertsson and Woodford [18] and Werning [33].} Campbell et al. [13] emphasized this distinction by defining “Odyssean” forward guidance a situation in which monetary authorities make statements with the primary objective of committing their future policy, and “Delphic” forward guidance a situation in which statements about future policy are primarily meant to share with the public any superior information that the central bank may have about the future course of policy. In practice, forward guidance is of course never purely Odyssean nor ever purely Delphic. An important message of this paper is that the “Odyssean” and “Delphic” elements of forward guidance are intrinsically linked: when a central bank has no superior information compared to the public (the purely “Odyssean” case), forward guidance will be shown to be a redundant policy instrument: while expectations about future policy actions matter, an explicit message giving advance notice of those actions is not needed. Furthermore, while pure Delphic forward guidance could be valuable on its own, a central bank
will find it much easier to sustain truthful announcements about its future intentions when these announcements form part of a bigger equilibrium in which central bank credibility is at stake.

The approach that we pursue in this paper is based on the theoretical literature on cheap talk.\(^2\) Cheap talk refers to a situation in which a player in a game has the possibility of sending messages that have no direct consequences on the set of future actions available to the players nor on their payoffs. Because of the lack of direct consequences, equilibria in which these messages are ignored are always present, but cheap talk opens the possibility for Pareto superior equilibria to emerge, in which messages reveal some information. When the messenger and the receiver of the message have conflicting objectives, full disclosure of private information will often not be possible.

The current policies of forward guidance map well into the theoretical framework of cheap talk. While central banks around the world have stated their intention not to raise interest rates for extended periods of time, these statements have not directly affected their ability to do so. As an example, the Federal Open Market Committee has continued to meet eight times a year, and a simple vote on each of these occasions could have led to a rate increase, independently of previous statements. Naturally, such a course of action could be detrimental in that it would lead to a loss of credibility; this indirect, endogenous response of private-sector expectations will play an important role in our analysis. Similarly, the statements per se do not have a direct effect on macroeconomic fundamentals nor on welfare; to the extent that they have been successful, it is because they influenced the private sector’s expectations about the future course of policy. Furthermore, as in all interesting applications of cheap talk, the central bank’s incentives are likely to be misaligned with those of the public, at least in the short run: as an example, in the throes of a major recession, the monetary authorities would most likely prefer sending optimistic messages and try to prevent expectations from adding to a downward spiral. The temptation to manipulate messages for short-term gains must be tempered with the potential loss of credibility.

In this paper, we show that forward guidance will be a particularly valuable policy tool when two conditions are met: first, the central bank must have some private information; second,

\(^2\)See Crawford and Sobel [16].
this private information must concern the central bank’s preferences or beliefs. The first point is straightforward: cheap talk is redundant if all the players in a game have symmetric information. The second condition highlights situations in which central bank communication is naturally thought as forward guidance, rather than a more general notion of transparency. As an example, if the central bank has superior information about the current state of the economy, it would be most natural for the central bank to make statements about the state of the economy itself, rather than revealing it indirectly through its intended course of action, which may be a poor proxy for the information that the private sector needs. In such a context, we will show an extreme example in which messages about future policy are useless, whereas statements about the state of the economy help in coordinating the private sector.

When the central bank has private information about its own preferences and/or beliefs, the private agents do not care directly about them; rather, that information is valuable to the extent that it helps them predict how the central bank will behave. In this case, forward guidance is a natural message space, because it conveys precisely the information that the private sector needs.

Moving beyond the simple model developed here, the intuition developed in this paper is useful to think about the role of forward guidance for current monetary policy. Is forward guidance a way for the central bank to commit to keep interest rates effectively at the zero lower bound longer than it would otherwise be optimal ex post? If it were widely understood and accepted that optimal monetary policy calls for extended periods of zero interest rates, central banks could have simply staked their credibility on their actions, with no need to supplement them with promises. As an example, it is widely understood and accepted that keeping inflation low is desirable; many central banks around the world have not adopted an explicit inflation target, but they still have managed to build their credibility in sustaining low inflation. It is more plausible that forward guidance is needed because the models in which interest rates are optimally kept at zero for an extended period are not universally understood and accepted. In this case, forward guidance could play a role to signal that central banks believe in the prescriptions stemming from these models, and will set their policy based on these beliefs.
1.1 Related literature

Many recent papers have used forward guidance as a term to directly refer to a policy of keeping interest rates low for an extended period of time. As an example, this is the case in the recent analysis of the “forward guidance puzzle” (Del Negro, Giannoni, and Patterson [17], McKay, Nakamura, and Steinsson [26]). Implicitly, these papers have assumed that any announcement of a policy of extended low rates would be fully credible, and conversely also that the lack of such an announcement would imply a different path of the expectations of the private sector. Here, we draw a distinction between announcements about future policy and the actual implementation of the announcements. We then study the conditions under which there is an independent role for advance notice about the future path.

Several papers have modeled monetary policy statements as (imperfectly) binding commitments. In Bodenstein, Hebden, and Nunes [12], preannounced path can only be revised at (exogenous) random times; in King, Lu, and Pasten [22, 23], central bankers come in two types, one of which is fully committed to carry out its announced policy and one which is free to re-optimize after any announcement.\(^\text{3}\) In contrast, we study here a situation in which it is well understood that statements are not direct constraints on policy, and policymakers must find it in their interest to carry through what they previously promised.

Within the context of monetary policy, cheap talk was first analyzed by Stein [31] in an environment which shares many traits with section 5 in this paper. He focused purely on Delphic announcements, abstracting from credibility and the repeated game aspect of the interaction. He also only focused on asymmetric information about a central bank target, which would not allow to distinguish between statements about future policy and transparency more in general. In a similar context, Moscarini [29] analyzed the interaction between the precision of a central bank’s information and its ability to credibly communicate it to the public.

Theoretical models of credibility and cheap talk have focused on persistent private information and learning over time (Sobel [30], Benabou and Laroque [11]). Here, the ability to support

---

\(^3\)King, Lu, and Pasten extend previous work by Barro [8], who considered direct commitment to policy in the absence of any announcement.
greater disclosure is not supported by the presence of “honest” types, but by the infinitely-
repeated nature of the game, as in Abreu, Pearce, and Stacchetti [1, 2].

Finally, our model is designed so that full transparency is optimal under commitment. In
particular, the assumption of a quadratic loss function is instrumental to achieve this result.
Our point is that, even when full transparency is optimal, there may still be no role for forward
guidance. With more general objectives, policymakers may find it optimal even from an ex ante
perspective to withhold part of the information; this issue is analyzed in detail in Jehiel [21].

2 A simple model

We work within the context of the Barro-Gordon model [9]. The intuition developed here extends
to the standard new Keynesian model, where the zero lower bound offers a special role to forward
guidance. From an expositional perspective, the advantage of the Barro-Gordon model is that
inflation is directly determined by the central bank, whereas in the new Keynesian model current
inflation and output depend on expectations about future policy that need to ultimately include
fiscal as well as monetary policy, as emphasized by Woodford [34] and Cochrane [15]. In both
environments, what is essential to our analysis is that expectations about future policy affect
current household decisions.

Time is infinite and discrete. The economy is populated by a continuum of private agents
(“households”) and a government (or central bank). Fundamental uncertainty in the economy is
described by a state space $\Omega$, whose generic element is $\omega$. The state $\omega$ will contain a realization
of two sequences:

- A sequence of potential output $(y^*_t)_{t=0}^\infty$, which we assume to be contained in $[y^l, y^h]$; and

- A sequence of target inflation $(\pi^*_t)_{t=0}^\infty$, contained in $[\pi^l, \pi^h]$;

\[^{\text{4}}\text{In the case of the new Keynesian model, Fujiwara and Waki [19] show that it is not optimal for the central
bank to reveal information about future cost-push shocks, in that better information about these shocks generates
immediate noise without providing any social gain.}\]
In addition to a realization of these sequences, the state of nature may contain other variables, such as advance signals that the households or the government may receive about current and future realizations. In general, the state of nature will not be known by either the government or the agents as of time 0, but it will be gradually revealed over time. For now, we generically denote as \( \{F_t\}_{t=0}^{\infty} \) and \( \{G_t\}_{t=0}^{\infty} \) the filtration of what is known at the beginning of period \( t \) by the households and the government, respectively. Throughout the paper, we will retain the following assumption about \( F_t \) and \( G_t \):

**Assumption 1** \( F_{t+1} \) is a finer partition than \( G_t \), that is, any private information that the government may have at the beginning of period \( t \) becomes common knowledge at the beginning of period \( t + 1 \).

Assumption 1 implies that households eventually learn the same information that the government had; this will make it easier for them to detect government deviations from equilibrium play and will in turn greatly simplify our analysis. In what follows, it is not essential that this information is known by the households with a delay of at most one period; we could assume longer delays, as long as they are finite. While I conjecture that results extend to the case of persistent private information, in which households may never learn what the government observed, the analysis of this case is considerably more involved.

In addition to fundamental uncertainty, there are sunspots \( s_t \) which (without loss of generality) have a i.i.d. uniform distribution\(^5\) on \([0, 1]^n\), with \( n \) arbitrary. \( s_t \) is observed by both households and the government at the beginning of period \( t \). The role of sunspots is to provide a source of coordination between households and the government which is independent of any messages that may be sent by the government.

After information has been revealed to households and the government, the sequence of events within each period \( t \) unfolds as follows:

1. The government can send a message \( m_t \) to the households, out of some set \( M \). \(^7\)Forward

\(^5\)While the assumption of a uniform distribution is without loss of generality, it is important that the sunspot distribution is absolutely continuous, so that the sunspot is not restricted to put probability mass on specific values.
guidance” is a situation in which the message is directly about the inflation level that will be chosen in stage 4 (or the inflation level that will be chosen in future periods in stage 4). For generality, we allow the possibility that the government randomizes over messages, and denote by \( \mathcal{M} \) a \( \sigma \)-algebra with respect to which this randomization is measurable.

2. Households form expectations about inflation \( \pi_t \) and aggregate output \( y_t \), based on the information currently available to them; \( y_e^t \) and \( \pi_e^t \) represents the household average expectation. Without loss of generality, we do not consider the possibility that households randomize over their expectations, because their best response is always single valued.\(^6\)

3. The government sets inflation \( \pi_t \in [\bar{\pi}, \pi] \),\(^7\)

\[
y_t = \theta y_e^t + (1 - \theta) y_t^e + \lambda (\pi_t - \pi_e^t).
\]

Here too we allow the government to potentially randomize, in which case its choice is assumed to be measurable with respect to Borel’s \( \sigma \)-algebra \( \mathcal{B}([\bar{\pi}, \pi]) \).

Barro and Gordon’s original specification sets \( \theta = 1.\(^8\) In their setting of symmetric information, that we will study in section 3, this makes no difference. However, we allow for the possibility that the government has superior information about \( y_e^t \) and that a strategic complementarity among private households leads to higher output when private-sector output expectations are

\(^6\)For this reason, in equilibrium, all households will have the same expectation.

\(^7\)We impose exogenous bounds \( \bar{\pi} \) and \( \pi \) for convenience, so that it is not necessary to discuss the consequences of government strategies that lead to nonexistence or infinite losses. In order not to deal with corner solutions in some of the proofs below, we assume that \( \bar{\pi} \) is sufficiently large and \( \pi \) is sufficiently low, and in particular

\[
\pi > \pi^h + \frac{\lambda k}{\alpha} + \frac{\lambda(1 - \theta)}{\alpha + \lambda^2} (y^h - y^f)
\]

and

\[
\bar{\pi} < \pi^\ell - \frac{\lambda(1 - \theta)}{\alpha + \lambda^2} (y^h - y^f)
\].

\(^8\)Barro and Gordon’s model is also expressed in terms of unemployment, rather than output, but this makes no difference for the results.
more favorable.\footnote{There is a large literature that studies the role of dispersed information in coordinating individual actions, using the global games approach from Morris and Shin [27]. Particularly relevant are the macroeconomic applications appearing in Hellwig [20], Amador and Weill [3], Lorenzoni [25], and Angeletos and La’O [4]. In that literature, government disclosure of information need not be beneficial, in that it may lead the private sector to focus too much on public signals and not enough on private signals; these issues are discussed especially in Morris and Shin [28] and Angeletos and Pavan [5, 6]. Here, I abstract from this complication because information is symmetric within the private sector, so government disclosure will be unambiguously beneficial.}

We assume that the government’s\footnote{We will use “government” and “central bank” interchangeably. There is a single policymaker.} loss function is

\[
(1 - \beta) E \sum_{t=0}^{\infty} \beta^t [(y_t - y_t^* - k)^2 + \alpha (\pi_t - \pi_t^*)^2],
\]

where \( E \) represents the unconditional expectation before any information is revealed and \( k \) is a bias in the government’s output target. As in Barro and Gordon, the interesting case is when \( k \neq 0 \) (and usually we think \( k > 0 \)), so that the government has a temptation ex post to resort to unexpected inflation to stimulate the economy.

A household’s loss function is

\[
(1 - \beta) E \sum_{t=0}^{\infty} \beta^t [(y_t - y_t^e)^2 + (\pi_t - \pi_t^e)^2],
\]

which simply means that in an equilibrium households will set their expectations rationally.

While \( \omega \) describes the sequence of exogenous fundamental shocks, to define an equilibrium we also need to keep track of sunspots and (public) histories of play. As is standard (see e.g. Chari and Kehoe [14]), to describe the set of possible equilibrium payoffs it is sufficient to keep track of the history of play by the large agent in the economy (the government).\footnote{As discussed in Bassetto [10], this is no longer sufficient when we are interested in studying the set of allocations that can be uniquely implemented under commitment.} We will thus define a history at the message stage as \( h^t := (\{s_s, m_s, \pi_s\}_{s=0}^{t-1}, s_t) \), and a history at the expectations-setting stage as \( h^{et} := (\{s_s, m_s, \pi_s\}_{s=0}^{t-1}, (s_t, m_t)) \).\footnote{At time 0, \( h^0 = s_0 \) and \( h^{et}_0 = (s_0, m_0) \).} \( H^t \) and \( H^{et} \) are the corresponding sets of histories, and \( \mathcal{H}^t \) and \( \mathcal{H}^{et} \) are the corresponding filtrations.\footnote{Formally, \( \mathcal{H}^t := B([0,1])^{nt} \times M^t \times B([\bar{\pi}, \bar{\pi}])^t \), where the superscript on the right-hand side refers to Cartesian power.}
A government strategy $\sigma^g$ is a $\mathcal{G}_t \times \mathcal{H}_t^0$-measurable mapping from $(\Omega, H^t)$ into a distribution over $(M, \mathcal{M})$ and a $\mathcal{G}_t \times \mathcal{H}_t^0$-measurable mapping from $(\omega, H^t)$ into a probability distribution over $([\pi, \bar{\pi}], B[\pi, \bar{\pi}])$. A (symmetric) household strategy is a $\mathcal{F}_t \times \mathcal{H}_t^0$-measurable mapping $\sigma^p$ from $(\Omega, H^t)$ to $[\pi, \bar{\pi}] \times [y^t, y^h]$.

When the government and the households play a strategy profile $\sigma^g, \sigma^p$, their play induces a probability distribution over outcomes $((\omega, H^\infty), \bigcup_{t=0}^{\infty} \mathcal{G}_t \times \bigcup_{t=0}^{\infty} \mathcal{H}_t^t)$.

A strategy profile $(\sigma^g, \sigma^p)$ is a perfect Bayesian equilibrium if:

- Given any $(\omega, h^t)$ and given that future play will occur according to $(\sigma^g, \sigma^p)$, any message in the support of $\sigma^g(\omega, h^t)$ is optimal for the government.
- Given any $(\omega, h^t)$ and given that future play will occur according to $(\sigma^g, \sigma^p)$, any inflation rate in the support of $\sigma^g(\omega, h^t)$ is optimal for government.
- Given any $(\omega, h^t)$ and $\sigma^g$, household expectations are rational:

$$y^e_t = E[y_t | \mathcal{F}_t, \mathcal{H}^t; \sigma^g], \quad \pi^e_t = E[\pi_t | \mathcal{F}_t, \mathcal{H}^t; \sigma^g].$$ (3)

## 3 Odyssean forward guidance

We consider first the case in which the government has no private information. Then we obtain the following result:

**Proposition 1** Assume that $\mathcal{F}_i = \mathcal{G}_i$. If $M \subseteq \mathbb{R}^g$, then the set of equilibrium government payoffs is the same as that of an economy in which no messages are allowed ($M = \emptyset$).

*Proof. See appendix.*

---

14 We assume here that the government does not directly observe $y^e_t$ and $\pi^e_t$. When the government has all the information that households have, this makes no difference: within an equilibrium, $y^e_t$ and $\pi^e_t$ are deterministic functions of what the government knows. This assumption is more relevant when households have superior information, as in the example of section 6. This example is relevant when the government cannot infer the missing information directly from household expectations.
To better illustrate what this proposition does and does not imply, we consider a specific example. Suppose that $k = 0.01$, $\pi_t^* \equiv 0.02$, $\beta = 0.96$, $\alpha = 1$, and $\lambda = 40$. Furthermore, $y_t^*$ is known (along with the entire past history of play) to both the government and the private sector. In this case, the government inflation target is deterministic at 2%. Potential output may be random, but the government always wants to overstimulate the economy by 1%. In a repeated game context, this economy admits many equilibria, independently of the message space $M$. We focus on two of them:\(^{15}\)

1. Suppose that $M = [\pi, \bar{\pi}]$.\(^{16}\) The following is an equilibrium strategy profile. If $\pi_t \neq m_t$ never occurred in the past, the government announces $m_t = 0.02$; otherwise, the government can send an arbitrary message (it could be 0.02 again, or anything else, since the households will no longer condition their strategy on the government’s reports). Households set $y_t^e = y_t^*$ independently of the past history. If $\pi_t \neq m_t$ never occurred in the past and $m_t \leq 0.43$, households “believe” the government and set $\pi_t^e = m_t$. Otherwise, households set $\pi_t^e = \pi_t^* + (\lambda/\alpha)k = 0.43$. Finally, if $\pi_t \neq m_t$ never occurred in the past and $m_t \leq 0.43$, the government follows through on its announcement and sets $\pi_t = m_t$; otherwise, it sets $\pi_t = 0.43$.

2. Suppose that $M = \emptyset$: no messages can be sent by the government. The following is an equilibrium strategy profile. Households set $y_t^e = y_t^*$ independently of the past history. If $\pi_t \neq 0.02$ never occurred in the past, households set $\pi_t^e = \pi_t^* = 0.02$. Otherwise, households set $\pi_t^e = 0.43$. Finally, if $\pi_t \neq 0.02$ never occurred in the past, the government sets $\pi_t = 0.02$; otherwise, it sets $\pi_t = \pi_t^* + (\lambda/\alpha)k = 0.43$.

In both equilibria described above household expectations are set according to a trigger strategy and the outcome coincides with what would arise under government commitment ($\pi_t = \pi_t^*$ and $y_t = y_t^*$); in both cases, the threat that disciplines the government’s temptation to overstimulate the economy is future reversion to permanently repeating the equilibrium outcome of the static one-shot game.

\(^{15}\)The verification that the two strategy profiles are indeed equilibria is relegated to the appendix.

\(^{16}\)Assume $\bar{\pi} < 0.02$ and $\bar{\pi} > 0.43$. 

11
The first equilibrium resembles what Campbell et al. [13] call “Odyssean forward guidance:” the government announces future policy, and puts its credibility at stake. If the government fails to deliver on its announcements, it loses its ability to coordinate expectations favorably, and high inflation ensues. This equilibrium shows that Proposition 1 does not say that government messages are necessarily irrelevant.

The second equilibrium achieves the same outcome, but without resorting to forward guidance, and is based on the idea that “actions speak louder than words.” Notice that, in the first equilibrium, while the expectations that households form about future policy are essential, the private sector can perfectly forecast what message the government will send. It is thus possible to bypass the message and stake the government’s credibility directly on its actions. Pure Odyssean forward guidance is unnecessary. When the private sector and the government share the same information, the temptation that the government faces to renege on its promises is the same whether those promises have been made explicit or left implicit. The same is true in a new Keynesian framework: if it is common knowledge that holding interest rates at zero for an extended period of time is optimal, households could directly form expectations based on this appropriate policy, and the central bank’s credibility would rely on following through with the expected policy, without need of advance messages.

Finally, in the example above, forward guidance is only about the policy that the government will undertake subsequently within the period, but the proposition extends to announcements about policy further into the future. As an example, the following is also an equilibrium. Suppose again that \( M = [\bar{\pi}, \bar{\bar{\pi}}] \). Assume that there is some outstanding message \( m_{t-1} = 0.02 \). If \( \pi_t \neq m_{t-1} \) never occurred in the past, the government announces \( m_t = 0.02 \); otherwise, the government can send an arbitrary message (it could be 0.02 again, or anything else, since the households will no longer condition their strategy on the government’s reports). Households set \( y_t^e = y_t^* \) independently of the past history. If \( \pi_t \neq m_{t-1} \) never occurred in the past and \( m_{t-1} < 0.43 \), households “believe” the government and set \( \pi_t^e = m_{t-1} \). Otherwise, households set \( \pi_t^e = 0.43 \). Finally, if \( \pi_t \neq m_{t-1} \) never occurred in the past and \( m_{t-1} < 0.43 \), the government follows through on its announcement and sets \( \pi_t = m_{t-1} \); otherwise, it sets \( \pi_t = 0.43 \). In this equilibrium, the
government announces its inflation plan one period ahead of time. Nonetheless, the equilibrium outcome remains the same, and it coincides with what happens in the trigger-strategy equilibrium with no messages that we described above.

4 Private information about the state of the economy

In this section, we consider a case in which the government has superior information about the underlying state of the economy. We retain the assumption that the inflation target $\pi^*_t$ is known by both the private sector and the government at the beginning of period $t$. In contrast, potential output $y^*_t$ is not known at the beginning of period $t$; we denote by $F_{y^*_t}(\cdot | \mathcal{F}_t)$ its distribution, conditional on the private-sector information. At the beginning of the period, the government has access to the same information as the private sector, but it also receives a potentially noisy signal $\tilde{y}_t$. Conditional on the government’s superior information, the distribution of potential output is thus $F_{y^*_t}(\cdot | \mathcal{G}_t)$.

As a benchmark, suppose that the government could commit to a strategy for its future reports and inflation choices at the beginning of time, before any information is revealed. It is straightforward to prove that the best equilibrium outcome arises when the government commits to report truthfully its information to the private sector and to set $\pi_t \equiv \pi^*_t$. Formally:

**Proposition 2** Let $M = [y^f, y^h]$ be the message space. Suppose that the government commits to a strategy $\sigma^g$ that reports $m_t = E[y^*_t | \mathcal{G}_t]$ and sets $\pi_t = \pi^*_t$ with probability 1. Let $(y_t, \pi_t, y^e_t, \pi^e_t)_{t=0}^\infty$ be the resulting equilibrium outcome. Then there is no other message space and/or government strategy that generates an equilibrium outcome that strictly dominates $(y_t, \pi_t, y^e_t, \pi^e_t)_{t=0}^\infty$.

**Proof.** See appendix.

Under commitment, the government realizes that it cannot fool the private sector; any deterministic misreporting of its signals would be undone by the private agents, and any garbling of the signals would simply increase the variance of output around potential, which is undesirable.

---

17 Observing $\tilde{y}_t$ implies that the $\sigma$-algebra $\mathcal{G}_t$ that represents the government information at time $t$ is finer than the one of the households, $\mathcal{F}_t$. 

13
When the government cannot commit, the typical cheap talk conflict emerges: since incentives are not aligned, the government has a temptation to misreport its signal to induce the households to increase their output. Nonetheless, the ability to send messages will in general be valuable, and superior equilibria in which some information is revealed will emerge.\footnote{We evaluate welfare from the perspective of the government, but the same is true for the households, whose expectations become more precise.} This will happen in either of two situations. First, it will happen when households face sufficiently high uncertainty about potential output compared to the government, so that the benefits from coordination trump the incentive to misreport. Alternatively, it will happen when the government is sufficiently patient, so that information revelation may be supported by trigger strategies in which the future credibility of the government is at stake.

We formalize these points in the following proposition.

**Proposition 3** Fix $\alpha, \lambda, k, \pi, \bar{\pi}$, and the stochastic process for $(y_t, \tilde{y}_t, \pi^*_t)_{t=0}^\infty$.

(i) As long as there exists a period $t$ such that $\text{Prob}(E[y^*_t|F_t] \neq E[y^*_t|G_t]) > 0$, there exists a value $\bar{\beta} \in [0, 1)$ and a message space $M \neq \emptyset$, such that for all $\beta > \bar{\beta}$ the set of equilibrium payoffs attainable in the game in which the government can send messages from $M$ is strictly larger than the corresponding set if no messages are allowed.

(ii) Given any $\beta \in [0, 1)$, there exists a value $\tilde{y}$ and a message space $M \neq \emptyset$ such that, if $E[y^*_t|F_t] > E[y^*_t|G_t] + \tilde{y}$ with positive probability, the set of equilibrium payoffs attainable in the game in which the government can send messages from $M$ is strictly larger than the corresponding set if no messages are allowed.

(iii) In both cases above, the expansion of the set includes higher equilibrium payoffs than what can be supported without messages. If in addition $\bar{\pi}$ is sufficiently high, $\bar{\beta}$ in part (i) can be chosen so that the set of payoffs for the game with messages does not include any payoffs that are worse than those achievable in equilibria of the game without messages.

**Proof.** See appendix.
Proposition 3 is not unambiguously optimistic. First, as is always the case in games with cheap talk, there will be equilibria in which the government “babbles,” sending the same message independently of its information, and households in turn disregard the government message, defeating any attempt to convey extra information. Perhaps even more disappointingly, there are instances in which allowing for a nontrivial message space will create the possibility of equilibria whose welfare is worse than the worst possible equilibrium under no messages. We know from the work of Abreu, Pierce, and Stacchetti [1, 2] that there often is a link between the payoff of the best and the worst equilibrium: the worst equilibrium represents a threat that can be used to support the best equilibrium, and the best equilibrium can be used as a reward for the government to be willing to endure the worst punishment. The ability to send messages offers a way for the government to better coordinate the private sector, reducing the volatility of output; paradoxically, by increasing the payoff in the best equilibrium, this ability opens the door for the worst equilibrium to become worse. The last part of the theorem proves that this will not happen if the maximal level of attainable inflation is sufficiently high and the government is sufficiently patient. In this case, the “punishment stage” of the worst equilibrium will not last a single period and the continuation of the worst equilibrium will not be the best equilibrium. Then, the government cannot do worse than what happens when households expect it to babble and ignore its messages. This case is reassuring: government communication may lead to better equilibrium payoffs, but not to worse ones.\footnote{Of course, we are silent on the process by which households and the government coordinate to one among many equilibria. If the introduction of government communication leads to coordination toward equilibria with a worse payoff, the ability to send messages might still be harmful.}

From here on, we will take the optimistic view that the economy coordinates on superior outcomes, in which case the government messages are unambiguously helpful. However, the next question is whether these messages take the form of “forward guidance” versus a generic need for “transparency.” A transparent government discloses (truthfully) a variety of information that is not publicly available. This information takes the form of forward guidance if it concerns the future path of policy. The example at hand is designed to be particularly stark. For $\beta$ sufficiently high, the Folk theorem implies that the best equilibrium outcome will coincide with the outcome
under commitment described in proposition 2. In this equilibrium, inflation is always at its

target value $\pi_t^*$ and it does not depend on the private information available to the central bank.

As a consequence, a message reporting future policy would not allow the households to infer

the information that they need to form the appropriate expectations: while communication is

potentially valuable, it is not about future policy.

The example above is clearly extreme. In richer environments, optimal government policy

will depend on the realization of the shocks about which the government has superior knowledge.

However, even in this case, reporting future policy, as opposed to the underlying information

that rationalizes the policy choice, is at best an indirect way to convey the information that the

households need to form their expectations. For the sake of concreteness, consider an example

in which, for some reason, the government objective function is

$$20 \left(1 - \beta \right) E \sum_{t=0}^{\infty} \beta^t \left\{ \left( y_t - y_t^* - k \right)^2 + \alpha \left[ \pi_t - \pi_t^* - f \left( y_t^* \right) \right] \right\} ,$$

so that optimal inflation under commitment and full transparency will be $\pi_t = \pi_t^* + E[f(y_t^*)|G_t]$. In this case, sending a message about future policy $\pi_t$ will reveal some information to the private sector about what the government observed through $\tilde{y}_t$. However, even in this case, unless $f$ is affine, knowing $E[f(y_t^*)|G_t]$ need not be the same as knowing $E[y_t^*|G_t]$, which is what the households really need. Announcing future policy is an imperfect and roundabout way of announcing the underlying information that the private sector requires to properly coordinate.

In the next two sections, we consider two cases in which the government’s private information

is not about the underlying state of the economy, but rather about its objective or its beliefs.

We will show that in this case sending messages about future policy is a natural way to convey

the information that households need to make their decisions.

---

20 We do not consider microfoundations for this example. It is simply meant as an illustration of a situation in which optimal government policy depends on the underlying information about the exogenous state of the economy.
5 Delphic forward guidance: private information about the government’s objective

We now assume that the government and the private sector have symmetric information about \( y_t^* \), but the government has private information about \( \pi_t^* \). Without loss of generality, we assume that the government knows \( \pi_t^* \) perfectly (since the only role of \( \pi_t^* \) is to act as the government’s preferred inflation rate). At the beginning of period \( t \), conditional on the information available to the private sector, \( \pi_t^* \) has a distribution \( F_{\pi_t^*}(\cdot | F_t) \). As in the previous section, we rule out persistent private information by assuming that \( \pi_t^* \) becomes common knowledge at the beginning of period \( t + 1 \).\(^{21}\) We can repeat the same steps as Propositions 2 and 3 and establish the following:\(^{22}\)

- If the government can commit to its strategy ahead of time, the best outcome arises when the government commits to report \( \pi_t^* \) truthfully, and sets \( \pi_t = \pi_t^* \).

- Whenever household uncertainty about \( \pi_t^* \) is sufficiently dispersed or the government is sufficiently patient (\( \beta \) sufficiently close to 1), the ability to send messages expands the set of possible equilibrium payoffs. This expansion will always include payoffs that are strictly higher than the best payoff attainable with no messages.

In this case, households have all the information about the underlying state of the economy that they need to make decisions, given government policy. While the government could report its underlying information that leads it to prefer \( \pi_t^* \), this is more information than necessary: all they need is to know the policy choice \( \pi_t \) that the government will take.\(^{23}\) In other words, what

\(^{21}\) That \( \pi_t^* \) becomes common knowledge is once again not essential, although it greatly simplifies the arguments. However, for reaping the benefits of repeated interaction, it is important that at least some additional information about \( \pi_t^* \) will become available to the private sector after the government policy choice. When this is not the case, Athey, Atkeson, and Kehoe [7] show that the optimal mechanism would involve a static provision of government incentives. Even in that environment, cheap talk, which is ruled out in their paper, could still be valuable. Within a new Keynesian model, this issue is revisited in Waki, Dennis, and Fujiwara [32].

\(^{22}\) Precise statements of the propositions and the proof are in the appendix.

\(^{23}\) Under commitment, \( \pi_t = \pi_t^* \), but this need not be the case when the government acts under discretion.
the private sector needs is precisely forward guidance about monetary policy.

6 Delphic forward guidance: private information about the government’s beliefs

In section 4, we assumed that the government had superior information about the underlying state of the economy. Here, we assume instead that the households receive a perfect signal of $y^*_t$ at the beginning of period $t$, while the government only observes a noisy signal $\tilde{y}_t$, so that it faces nontrivial uncertainty described by the conditional distribution $F_{y^*_t}(.|G_t)$. Even though households know $y^*_t$ perfectly, the government still has private information, about its own imperfect beliefs.

When government preferences are given by (2), optimal government policy under commitment requires the government to set $\pi_t = \pi^*_t$ unconditionally, and households do not need any report from the government to achieve perfect foresight, setting $y^e_t = y^*_t$ and $\pi^e_t = \pi^*_t$.

However, suppose instead that the loss function is once again distorted as in (4). In this case, the best equilibrium outcome when the government can commit to its strategy involves setting $\pi_t = \pi^*_t + E(f(y^*_t)|G_t)$ and a truthful report by the government. The government’s report could be about the signal $\tilde{y}_t$, but it can also be directly about $\pi_t = \pi^*_t + E(f(y^*_t)|G_t)$: the only reason households need to know the imperfect signal observed by the government is to form expectations about government policy. Once again, in this setting forward guidance is a natural message space for the government to communicate the information that can coordinate households towards desirable equilibria.

Formally, it would be incorrect to say that households have superior information compared to the government. If households unambiguously had superior information, $\mathcal{F}_t$ would be a finer $\sigma$-algebra than $G_t$; but in this case, households would know what the government knows. In our example, instead, households do not know what the government observed.
7 Conclusion

In this paper, we have shown that forward guidance is unnecessary if its purpose is purely as a commitment tool. When the government and the private sector have symmetric information, actions speak louder than words, and the same outcomes can be achieved when the government stakes its credibility on its actions directly, with no need to have interim messages. When the government has private information, optimal disclosure may call for transparency, but not necessarily forward guidance: in many instances, households learn more if the government discloses the actual information, rather than its future policy plans. We identified two circumstances in which a policy of forward guidance is potentially beneficial: these arise when the primary source of asymmetric information concerns directly the preferences or the beliefs of the policymaker.

References


Appendix

A.1 Proof of proposition 1.

Let $V$ be the set of equilibrium government payoffs for an economy with a message space $M$. First, we show that $V$ is nonempty, since there is always at least an equilibrium associated with playing in period $s$ the static equilibrium that would arise in a one-period game.

In general, the static equilibrium is given by the following:\footnote{Equation (5) is quite involved when it is not possible to establish that either the government or the households have superior information, since in this case higher-order beliefs matter explicitly. In our examples, either $G_t \subseteq F_t$}

$$
\pi_t = \frac{\lambda}{\alpha} k + \frac{\alpha}{\alpha + \lambda^2} \sum_{i=0}^{\infty} \left( \frac{\lambda^2}{\alpha + \lambda^2} \right)^i \hat{E}_t^i (\pi_t^*) + \frac{\lambda(1 - \theta)}{\alpha + \lambda^2} \sum_{i=0}^{\infty} \left( \frac{\lambda^2}{\alpha + \lambda^2} \right)^i \hat{E}_t^i \left[ E(y_t^*|G_t) - \hat{E}(y_t^*) \right]
$$

(5)
and

\[ y_t = \theta y_t^* + (1 - \theta)E(y_t^*|F_t) + \lambda (\pi_t - E[\pi_t|F_t]), \]

(6)

where, given any random variable \(X\), \(\hat{E}_t(X) := E[E(X|F_t)|G_t]\), and the \(i\) power in the sum indicates repeated application of the \(\hat{E}_t\) operator.

Let \(\hat{V}\) be the set of equilibrium payoffs for an economy where the message space is empty.

We first prove that \(\hat{V} \subseteq V\). This is straightforward, because in cheap talk games babbling is always an equilibrium. Specifically, let \(\hat{\sigma}_g, \hat{\sigma}_p\) be an equilibrium of the game with no messages. Let \(m^0\) be an arbitrary message from the set \(M\). We construct an equilibrium of the game with messages as follows:

- \(\sigma^g(\omega, h^t) = m^0 \ \forall h^t\);
- \(\sigma^p(\omega, h^{et}) = \hat{\sigma}^p(\omega, \hat{h}^{et}) \ \forall h^t\), where \(\hat{h}^{et}\) is the history that shares the same values of \((\pi_s, \pi_e^s)_{s=0}^{t-1}\) as \(h^{et}\);
- \(\sigma^g(\omega, h^{et}) = \hat{\sigma}^g(\omega, \hat{h}^{et}) \ \forall h^t\), where \(\hat{h}^{et}\) and \(h^{et}\) are related as above.

In this equilibrium, household expectations and government inflation choices are independent of the messages being sent, so the government is indifferent over which message to send, and \(m^0\) is (weakly) optimal. Households do not learn any information from government messages, both because the government and the households share the same information sets and because the government message is independent of its information. By construction, the probability distribution over future outcomes for inflation and inflation expectations is the same in the two games, given any choice of \(\pi_t\) for the government. Hence, the choices that were optimal for \(\hat{\sigma}^g\) remain optimal under \(\sigma^g\). Similarly, since the probability distribution over future outcomes remains the same, household expectations remain rational.

or \(F_t \subset G_t\). In the former case, equation (5) simplifies to \(\pi_t = (\lambda/\alpha)k + \pi_t^*\) and in the latter to

\[ \pi_t = \frac{\lambda}{\alpha}k + \frac{\alpha}{\alpha + \lambda^2} \pi_t^* + \frac{\lambda^2}{\alpha + \lambda^2} E(\pi_t^*|F_t) + \frac{\lambda(1 - \theta)}{\alpha + \lambda^2} \left[ E(y_t^*|G_t) - E(y_t^*|F_t) \right]. \]
Consider now the converse: \( V \subseteq \hat{V} \). Let \((\sigma^g, \sigma^p)\) be an equilibrium that attains the value \( v \in V \) for a game with messages. Let the sunspot in the original economy with messages be \( s_t \in [0, 1]^n \). In the economy without messages, households and the government will coordinate based on a sunspot \( \hat{s}_t \in [0, 1]^{n+q} \); the additional \( q \) dimensions of the sunspot will replace government messages as a source of coordination.\(^{26}\)

In each period \( t \), and for (almost) each information set in \( G_t \), we create a mapping \( \eta_t \) from \( \hat{s}_t \in [0, 1]^{n+q} \) to \((s_t, m_t) \in [0, 1]^n \times \mathbb{R}^q\):

\[
\begin{align*}
\eta_{it}(\hat{s}_t) &= s_{it} & i &= 1, \ldots, n \\
\eta_{n+1t}(\hat{s}_t) &= \inf_v \{v : \text{Prob}(m_{it} \leq v) \geq \hat{s}_{n+1t}\} \\
\eta_{n+it}(\hat{s}_t) &= \inf_v \{v : \text{Prob}[m_{it} \leq v| (m_{i-1t}, \ldots, m_{i-1t}) = (\eta_{n+1t}(\hat{s}_t), \ldots, \eta_{n+i-1t}(\hat{s}_t))] \geq \hat{s}_{n+it}\} & i &= 2, \ldots, q.
\end{align*}
\]

Next, define recursively a mapping \( \eta \) from histories of the game without messages at the expectation-setting stage to the game with messages at the same stage as follows:

\[
\eta(\hat{s}_0) := \eta_0(s_0, m_0),
\]

\[
\eta(\hat{h}^{e_t-1}, (\pi_{t-1}, \hat{s}_t)) := (\hat{h}^{e_t-1}, \pi_{t-1}, \eta_t(\hat{s}_t)).
\]

Define the following strategy profiles for the government and the households in the game without messages:

\[
\hat{\sigma}^g(\hat{h}^{e_t}) := \sigma^g(\eta(\hat{h}^{e_t})),
\]

\[
\hat{\sigma}^p(\hat{h}^{e_t}) := \sigma^p(\eta(\hat{h}^{e_t})).
\]

By the construction of the mapping \( \eta \), the probability distribution over \((\omega, \{\pi_t, y_t\}_{i=0}^\infty)\) remains the same, so the government attains the same payoff when \((\hat{\sigma}^g, \hat{\sigma}^p)\) is played in the game without messages as it does according to \((\sigma^g, \sigma^p)\) in the game with messages. Similarly, the distribution

\(^{26}\)The power of the theorem is that, while government messages may be correlated with the fundamentals of the economy and/or the past history of play, sunspots are not by construction: since messages play at most a pure coordination role and convey no extra information, there is no need for a strategic player to be the sender. The theorem could easily be extended to more general message spaces than \(\mathbb{R}^q\), but this requires identifying the equivalent of the mapping \( \eta \) below, which is difficult to characterize at the current level of generality.
over future outcomes in the two games remain the same starting from arbitrary histories \( \hat{h}^t \) and \( \eta(\hat{h}^t) \), whether these histories do or do not occur on the path of play of the corresponding strategy profiles. Hence, if \( \sigma^g(\eta(\hat{h}^t)) \) is an optimal distribution of inflation for the government at \( h^t \) in the game with messages, then the same distribution \( \hat{\sigma}^g(\hat{h}^t) \) is optimal in the game without messages. Similarly, if \( \sigma^p(\eta(\hat{h}^t)) \) represents rational expectations for the households about \((\pi_t, y_t)\), then the same expectations \( \hat{\sigma}^p(\hat{h}^t) \) are also rational in the game without messages. QED.

A.2 Proof of proposition 2

Let \( \mathcal{F}_t^m \) be the \( \sigma \)-algebra representing the households’ information after the government has sent its message for period \( t \). We have \( \mathcal{F}_t \subseteq \mathcal{F}_t^m \subseteq \mathcal{G}_t \). From the law of iterated expectations,

\[
E[(y_t - y_t^* - k)^2 + \alpha(\pi_t - \pi_t^*)^2] = E\left\{ E[(y_t - y_t^* - k)^2 + \alpha(\pi_t - \pi_t^*)^2 | \mathcal{F}_t] \right\}.
\]  

(8)

Taking the expectation of equation (1) conditional on \( \mathcal{F}_t^m \), we obtain

\[
y_t = \theta y_t^* + (1 - \theta) E(y_t^* | \mathcal{F}_t^m) + \lambda(\pi_t - E(\pi_t | \mathcal{F}_t^m))
\]  

(9)

Using equation (9) and the law of iterated expectations, we obtain

\[
(1 - \theta)^2 E\left\{ [y_t^* - E(y_t^* | \mathcal{G}_t)]^2 | \mathcal{F}_t \right\} + \frac{k^2}{E\left\{ [y_t^* - E(y_t^* | \mathcal{F}_t^m)]^2 | \mathcal{F}_t \right\}} + \lambda^2 E\left\{ (\pi_t - \pi_t^*)^2 | \mathcal{F}_t \right\} \geq
\]

(10)

The lower bound in equation (10) is exogenous to government policy, and it is attained when the government strategy sets \( \pi_t = \pi_t^* \) and \( m_t = E(y_t^* | \mathcal{G}_t) \), which ensures \( E(y_t^* | \mathcal{F}_t^m) = E(y_t^* | \mathcal{G}_t) \). QED.
B Proof of proposition 3.

(i) Proposition 2 proved that the commitment outcome when messages are allowed requires the government to disclose truthfully its information. Since there exists a period $t$ such that $\text{Prob}(E[y^*_t|F_t] \neq E[y^*_t|G_t]) > 0$, there is at least one period $t$ and a set of states of nature of positive probability in which the government has superior information and this disclosure reveals information to the households: this disclosure cannot be replicated without messages, so the commitment outcome when messages are allowed strictly dominates the commitment outcome when no messages are allowed.

We now use the standard logic of the Folk theorem to prove that there exists a value $\bar{\beta}$ such that, if $\beta > \bar{\beta}$, then the commitment outcome with messages is an equilibrium outcome of the game (with messages) under discretion.

Repeating the steps of the algebra in proposition 2, the payoff of following the commitment outcome from any arbitrary period $s$ is

$$k^2 + (1 - \theta)^2 E \left[ \sum_{t=s}^{\infty} \beta^{t-s} (E(y^*_t|G_t) - y^*_t)^2 | G_s \right].$$

(11)

We look for an equilibrium in which the commitment outcome is supported by the threat of permanent reversion to the equilibrium of the static one-shot game in which the government babbles, which we now compute. In this equilibrium, household expectations are independent of past history and of the message sent by the government. Since government messages carry no consequences, we can assume without loss of generality that the government always reports some default message $m_0$. In setting inflation, the government takes into account that its action has no consequences on the future, and it thus solves

$$\min_{\pi_t} E \left[ (y_t - y^*_t - k)^2 + \alpha(\pi_t - \pi^*_t)^2 | G_t \right]$$

subject to (1). The first-order condition yields

$$\lambda[(1 - \theta) (y^*_t - E(y^*_t|G_t)) + \lambda (\pi_t - \pi^*_t) - k] + \alpha(\pi_t - \pi^*_t) = 0.$$

(12)
Taking expected value of (12) and imposing the equilibrium conditions $y_t^e = E(y_t^e | F_t)$ and $\pi_t^e = E(\pi_t | F_t)$ we obtain

$$E(\pi_t | F_t) = \pi_t^* + \frac{\lambda k}{\alpha}. $$

Substituting this into (12) we solve for equilibrium inflation:

$$\pi_t = \pi_t^* + \frac{\lambda k}{\alpha} - \frac{\lambda (1 - \theta)}{\alpha + \lambda^2} \left[ E(y_t^e | F_t) - E(y_t^* | G_t) \right].$$

Substituting equilibrium inflation and equilibrium expectations in turn into (1), equilibrium output is given by

$$y_t = y_t^* + (1 - \theta) \left[ \frac{\lambda^2}{\alpha + \lambda^2} E(y_t^* | G_t) + \frac{\alpha}{\alpha + \lambda^2} E(y_t^* | F_t) - y_t^* \right].$$

The payoff of following the static babbling equilibrium from period $s$ into the indefinite future is thus

$$(1 - \beta) \sum_{t=s}^{\infty} \beta^{t-s} E \left[ (y_t - y_t^* - k)^2 + \alpha (\pi_t - \pi_t^*)^2 | G_s \right] =

k^2 (1 + \lambda^2 / \alpha^2) + (1 - \beta) (1 - \theta)^2 \sum_{t=s}^{\infty} \beta^{t-s} \left\{ \left( \frac{\lambda^2}{\alpha + \lambda^2} \right)^2 E \left[ (y_t^* - E(y_t^* | G_t))^2 | G_s \right] + \left( \frac{\alpha}{\alpha + \lambda^2} \right)^2 E \left[ (E(y_t^* | F_t) - E(y_t^* | G_t))^2 | G_s \right] \right\} \geq

k^2 (1 + \lambda^2 / \alpha^2) + (1 - \theta)^2 E \left[ \sum_{t=s}^{\infty} \beta^{t-s} (E(y_t^* | G_t) - y_t^*)^2 | G_s \right],

(13)

where the last inequality follows from

$$E \left[ (y_t^* - E(y_t^* | F_t))^2 | G_s \right] \geq E \left[ (y_t^* - E(y_t^* | G_t))^2 | G_s \right],$$

since the government information is superior.

\footnote{Note that our assumptions on $\pi$ and $\bar{\pi}$ ensure that the solution for inflation above is interior. Furthermore, there are no other solutions in which (12) may hold as an inequality due to inflation being sometimes at a corner: this is because $\partial \pi / \partial \pi_t^e \in [0, 1)$ according to (12) when a corner may be binding, so that, starting from $\pi_t^e = \pi_t^* + \frac{\lambda k}{\alpha}$, moving in either direction would be incompatible with a fixed point.}
Comparing equations (11) and (13), a government which chooses to deviate from the commitment play in period $s$ will lose at least $k^2\lambda^2/\alpha^2$ from period $s+1$ onwards. The best that the government can achieve through such a deviation is to set the loss in period $s$ to zero, which yields a one-period gain of

$$k^2 + (1 - \theta)^2 \text{Var}(y^*_t|G_t) \leq k^2 + (1 - \theta^2)(y^h)^2.$$  

We can thus choose

$$\bar{\beta} = \frac{k^2 + (1 - \theta^2)(y^h)^2}{k^2(1 + \lambda^2/\alpha^2) + (1 - \theta^2)(y^h)^2},$$

for any value of $\beta \geq \bar{\beta}$, the one-shot gain of deviating from the commitment outcome is smaller than the loss from subsequent reversion to the static babbling equilibrium. Hence, for such values of $\beta$, the commitment outcome is an equilibrium outcome even without commitment; this outcome is better than the best outcome that can be attained without messages, completing the proof.

(ii) (to be completed)

(iii) We first establish a couple of properties which will be used in the remainder of the proof.

- Given any period $t$ and any government information set $G_t \in \mathcal{G}_t$, the set of expected payoffs of continuation equilibria from period $t+1$ onwards is convex. To prove this, consider two continuation equilibria with payoff $L^1_{t+1}$ and $L^2_{t+1}$. Let these equilibria sunspot processes $\{s_s\} \in \mathbb{R}^n$ (assuming the same $n$ is without loss of generality, since the equilibrium play can simply ignore some of the components of $s_t$). To attain a continuation payoff $qL^1_{t+1} + (1-q)L^2_{t+1}$, we construct a continuation equilibrium based on a sunspot process $\hat{s}_s \in \mathbb{R}^{n+1}$. Specifically, whenever $s_{n+1,t+1} \leq q$, the continuation equilibrium from period $t+1$ coincides with the equilibrium attaining $L^1_{t+1}$, whereas otherwise it coincides with the equilibrium $L^2_{t+1}$.

- In part (i) of the proof, we established a threshold $\bar{\beta}$ that ensured that, in the game with messages, the commitment outcome can be sustained as an equilibrium outcome of the game with discretion using the threat of reversion to the static babbling
equilibrium. We now show that the same threshold is also sufficient to sustain the commitment outcome under the same threat even in the game without messages. Simple algebra proves that the commitment outcome of the game without messages is given by the following inflation policy:

$$\pi_t = \pi^*_t - \frac{\lambda(1 - \theta)}{\alpha + \lambda^2} \left[ E(y^*_t | F_t) - E(y^*_t | G_t) \right].$$

It follows that, in the game without messages, the difference in payoffs between the commitment outcome and repeating the static equilibrium outcome is exactly \(k^2 \lambda^2 / \alpha^2\), the value that we used as a lower bound for the game in which messages are allowed. The one-time gain from deviating is bounded above by the same value \(k^2 + (1 - \theta^2)(y^h)^2\) as before, and so \(\bar{\beta}\) from (14) will once again ensure that the one-time benefit of deviation from the commitment outcome is smaller than the subsequent loss.

We now reason by contradiction. Let \((\sigma^g_w, \sigma^p_w)\) be an equilibrium of the game with messages that attains a loss \(L_w\) higher than the supremum of the losses attainable in the game without messages, which we denote \(\hat{L}\). Since there is no link in the game across different information sets of the private sector, there necessarily is a period-0 information set \(F_0 \in \mathcal{F}_0\) for which the loss according to the equilibrium \((\sigma^g_w, \sigma^p_w)\) is worse than the conditional supremum of the losses attainable in the game without messages; we establish a contradiction for this information set. Let \(L_1(F_0), F_0 \in \mathcal{F}_0\) be the expected loss attained by the equilibrium \((\sigma^g_w, \sigma^p_w)\) from period 1 onwards, conditional on \(F_0\). We distinguish two cases, and rule out each of them in turn:

1. Consider first the case in which \(L_1(F_0)\) is smaller than the lowest attainable conditional expected loss from period 1 in the game without messages. This is impossible, provided \(\beta\) is sufficiently high. Specifically, the period-0 loss is bounded above by\(^{28}\)

$$L_{\text{worst}} := (1 - \beta)([(y^\ell - y^h - k)^2 + (\bar{\pi} - \pi^\ell)^2]).$$

\(^{28}\)The worst-case could in principle be either \(\pi\) or \(\bar{\pi}\), but we assume that \(\bar{\pi}\) is sufficiently large, so that high inflation is worst. The proof goes through with appropriate replacements if \(\pi\) is the worst-case scenario. In a model with deeper microfoundations, the zero lower bound imposes a limit on expected deflation, which implies that high inflation would be the worst-case scenario.
For $\beta > \bar{\beta}$, the commitment outcome of the game without messages is sustainable from period 1, so the loss from then on must be smaller than such outcome. However, when

$$\beta > \max\left\{ \frac{L_{\text{worst}}}{L_{\text{worst}} + \frac{k\lambda}{\alpha} \bar{\beta}}, \bar{\beta} \right\},$$

the loss from receiving $L_{\text{worst}}$ in period 0 and the commitment outcome of the game without messages from period 1 is smaller than the loss from playing the repeated static babbling equilibrium. Since the repeated static babbling equilibrium is attainable in the game without messages, this contradicts the premise that $L_w$ is higher than the supremum of the losses in the game without messages.

2. Suppose instead that $L_1(F_0)$ is not smaller than the lowest attainable conditional expected loss from period 1 in the game without messages.

To find the supremum of the losses in the game with messages as of period 0 and conditional on $F_0$, we follow the logic of Abreu, Pierce, and Stacchetti [1, 2]. Define $\sigma_0^g$ a government strategy for period 0 (a mapping from $G_0$ to a probability distribution over messages $m_0$ and inflation rates $\pi_0$) and let $E^{\sigma_0^g}$ denote expectations conditional on the fact that the government will follow the strategy $\sigma_0^g$ in period 0. The supremum of the loss will solve the following problem:

$$\sup_{\sigma_0^g, L_1} (1 - \beta) E^{\sigma_0^g} \left\{ E^{\sigma_0^g} \left[ \left( (1 - \theta)(E^{\sigma_0^g}(y_0^* | F_0, m_0) - y_0^*) - k + \lambda(\pi_0 - E^{\sigma_0^g}(\pi_0 | F_0, m_0)) \right)^2 \right] | G_0 \right\} +$$

$$\alpha(\pi_0 - \pi_0^*)^2 | F_0 \right\} + \beta E(L_1(G_0) | F_0)$$

(16)
subject to the incentive-compatibility constraint

\[ (1 - \beta)E^{\sigma_0^g}\left[(1 - \theta)(E^{\sigma_0^g}(y_0^*|F_0, m_0) - y_0^*) - k + \lambda(\pi_0 - E^{\sigma_0^g}(\pi_0|F_0, m_0))\right]^2 |G_0\] + 
\[ \alpha(\pi_0 - \pi_0^*)^2 + \beta L_1(G_0) \leq 
(1 - \beta) \inf_{\tilde{m}_0, \tilde{\pi}_0} \left\{ E \left[ (1 - \theta)(E^{\sigma_0^g}(y_0^*|F_0, \tilde{m}_0) - y_0^*) - k + \lambda(\tilde{\pi}_0 - E^{\sigma_0^g}(\pi_0|F_0, \tilde{m}_0))\right]^2 |G_0\] + 
\[ \alpha(\tilde{\pi}_0 - \pi_0^*)^2 \right\} + \beta \sup \mathcal{L}(G_0) \] (17)

and subject to \( \pi_t \in [\bar{\pi}, \bar{\pi}] \) and \( L_1(G_0) \in \mathcal{L}(G_0) \), where \( \mathcal{L}(G_0) \) is a convex set that represents all possible values from continuation equilibria from period 1 onwards conditional on the government’s information as of period 0.

In the problem above, we first observe that equation (17) must be binding almost surely: if this were not the case on a set of values of \( G_0 \) of positive measure and \( L_1(G_0) < \sup \mathcal{L}(G_0) \) on those values, an equilibrium with a higher loss could be attained by selecting a continuation equilibrium from period 1 that corresponds to a higher value of \( L_1(G_0) \); if instead \( L_1(G_0) = \sup \mathcal{L}(G_0) \) almost surely, then by the definition of its terms (17) could not hold as a strict inequality.

Second, we prove that \( L_1(G_0) \geq \inf \mathcal{L}(G_0) \) is not binding when

\[ \beta > \max\{ \frac{L_{\text{worst}}}{L_{\text{worst}} + \frac{k\lambda}{\alpha^2}}, \tilde{\beta} \}. \]

This follows the proof of step 1:

\[ E^{\sigma_0^g}\left[(1 - \theta)(E^{\sigma_0^g}(y_0^*|F_0, m_0) - y_0^*) - k + \lambda(\pi_0 - E^{\sigma_0^g}(\pi_0|F_0, m_0))\right]^2 |G_0\] + \( \alpha(\pi_t - \pi_t^*)^2 \leq L_{\text{worst}} \)

and

\[ \inf_{\tilde{m}_0, \tilde{\pi}_0} \left\{ E \left[ (1 - \theta)(E^{\sigma_0^g}(y_0^*|F_0, \tilde{m}_0) - y_0^*) - k + \lambda(\tilde{\pi}_0 - E^{\sigma_0^g}(\pi_0|F_0, \tilde{m}_0))\right]^2 |G_0\] + \( \alpha(\tilde{\pi}_0 - \pi_0^*)^2 \right\} \geq 0; \]

\footnote{If the supremum of \( \mathcal{L}(G_0) \) cannot be attained, then the incentive-compatibility constraint should hold as an inequality. However, it is straightforward to show that the theorem of the maximum implies that the overall problem is continuous in \( \sup \mathcal{L}(G_0) \) (endowed with the \( L^1 \) norm), so that the solution of the supremum in (16) will be the same if equality is included in (17).}
furthermore, for the given range of \( \beta \), sup \( \mathcal{L}(G_0) - \inf \mathcal{L}(G_0) \geq k^2 \lambda^2 / \alpha \), since the commitment outcome is sustainable and the worst equilibrium outcome is at least as bad as the repeated static babbling equilibrium. It then follows that (17) would not hold as an equality if \( L_1(G_0) = \inf \mathcal{L}(G_0) \), contradicting what we already proved.

Having proved that \( \inf \mathcal{L}(G_0) \leq L_1(G_0) \leq \sup \mathcal{L}(G_0) \) are not binding constraints, while (17) is, the plan that maximizes (16) subject to (17) is equivalent to the plan that solves

\[
\sup_{\sigma_0^g} \mathcal{E} \left\{ \inf_{m_0, \pi_0} \mathcal{E} \left[ \left[ (1 - \theta) (E^{\sigma_0^g}(y_0^*|F_0, \tilde{m}_0) - y_0^*) - k + \lambda (\tilde{\pi}_0 - E^{\sigma_0^g}(\pi_0|F_0, \tilde{m}_0)) \right]^2 | G_0 \right] + \alpha (\tilde{\pi}_0 - \pi_0^*)^2 | F_0 \right\},
\]

with \( L_1(G_0) \) determined as a residual so as to ensure that (17) holds with equality.

To solve the problem in (18), we first compute the infimum with respect to \( \tilde{\pi}_0 \), for given \( \tilde{m}_0 \) and \( \sigma_0^g \). The first-order condition yields

\[
\lambda \left[ (1 - \theta) \left( E^{\sigma_0^g}(y_0^*|F_0, \tilde{m}_0) - E(y_0^*|G_0) \right) - k + \lambda \left( \tilde{\pi}_0 - E^{\sigma_0^g}(\pi_0|F_0, \tilde{m}_0) \right) \right] + \alpha (\tilde{\pi}_0 - \pi_0^*) = 0,
\]

which (given the assumptions on \( \pi \) and \( \tilde{\pi} \) in footnote 7) yields an interior solution

\[
\tilde{\pi}_0 = \frac{1}{\alpha + \lambda^2} \left[ \alpha \pi_0^* + \lambda^2 E^{\sigma_0^g}(\pi_0|F_0, \tilde{m}_0) - \lambda \left[ (1 - \theta) \left( E^{\sigma_0^g}(y_0^*|F_0, \tilde{m}_0) - E(y_0^*|G_0) \right) - k \right] \right].
\]

Substituting (19) into (18) and simplifying the algebra, the problem becomes

\[
\sup_{\sigma_0^g} \mathcal{E} \left\{ \inf_{m_0} \frac{\alpha}{\alpha + \lambda^2} \left[ (1 - \theta) \left( E^{\sigma_0^g}(y_0^*|F_0, \tilde{m}_0) - E(y_0^*|G_0) \right) - k + \lambda (\pi_0^* - E^{\sigma_0^g}(\pi_0|F_0, \tilde{m}_0)) \right]^2 + (1 - \theta)^2 \text{Var}(y_0^*|G_0)|F_0 \right\}.
\]
We have
\[ E^{\sigma^0} \left\{ \inf_{\tilde{m}_0} \left[ (1 - \theta) \left( E^{\sigma^0}(y_0^*|F_0, \tilde{m}_0) - E(y_0^*|G_0) \right) - k + \lambda (\pi_0^* - E^{\sigma^0}(\pi_0|F_0, \tilde{m}_0)) \right]^2 |F_0 \right\} \leq \]
\[ E \left\{ \left[ (1 - \theta) \left( E^{\sigma^0}(y_0^*|F_0, m_0) - E(y_0^*|G_0) \right) - k + \lambda (\pi_0^* - E^{\sigma^0}(\pi_0|F_0, m_0)) \right]^2 |F_0 \right\} = \]
\[ (1 - \theta)^2 E \left\{ \left( E^{\sigma^0}(y_0^*|F_0, m_0) - E(y_0^*|G_0) \right)^2 |F_0 \right\} + \left[ -k + \lambda (\pi_0^* - E^{\sigma^0}(\pi_0|F_0, m_0)) \right]^2 \leq \]
\[ (1 - \theta)^2 E \left\{ (E(y_0^*|F_0) - E(y_0^*|G_0))^2 |F_0 \right\} + \left[ -k + \lambda (\pi_0^* - \bar{\pi}) \right]^2 . \]

(21)

The last line holds provided \( \bar{\pi} \) is sufficiently high, given \( \alpha, \lambda, k, \bar{\pi} \), and the stochastic process for \( (y_t, \tilde{y}_t, \pi_t^*)_{t=0}^\infty \) (but, importantly, independently of \( \beta \), which can thus be set sufficiently high in turn as to ensure that the bounds of the continuation payoff are not binding).

If the CB plays a strategy that sets \( \pi_0 = \bar{\pi} \) with probability 1 and babbles (i.e., it sends a message that is independent of its information), the resulting payoff attains exactly the last line of equation (21): hence, this is the strategy that achieves the supremum in problem (20). We thus conclude that a strategy that achieves the supremum of the losses in period 0 involves babbling,\(^{30}\) so that the ability or inability of the CB to send messages in period 0 has no consequences for the upper bound of the set of equilibrium payoffs. Hence, if the supremum of the equilibrium payoffs in the game with messages is higher than in the game without messages, this must be because the continuation payoff \( E[L_1(G_0)] \) is worse than the worst possible equilibrium payoff in the game without messages. But we can repeat the same steps as of period 1 and reach the same conclusion: that the worst equilibrium payoff as of period 1 is the same whether messages are or are not allowed in period 1, so that, if \( E[L_1(G_0)] \) is worse than the worst possible equilibrium in the game without messages, it must be because there is some information set \( F_1 \) such that the continuation payoff \( E[L_2(G_1)|F_1] \) is worse than the worst possible equilibrium payoff in the game without messages. We can proceed

\(^{30}\)If \( L(G_0) \) is a closed set, the supremum is attained and the strategy just described is the unique strategy that attains the worst payoff
iteratively and prove that the ability of sending messages up to any arbitrary period $t$ does not affect the worst possible equilibrium payoff. This establishes a contradiction: if the upper bound of the equilibrium payoffs in the game with messages is strictly worse than in the game without messages, then there must be a period $t$ and an information set $F_t$ in which the government sends informative messages.