Politics and Efficiency of Separating Capital and Ordinary Government Budgets*

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Abstract

We analyze the democratic politics and competitive economics of a 'golden rule' that separates capital and ordinary account budgets and allows a government to issue debt to finance only capital items. Many national governments followed this rule in the 18th and 19th centuries and most U.S. states do today. We study an economy with a growing population of overlapping generations of long-lived but mortal agents. Each period, majorities choose durable and nondurable public goods. In a special limiting case with demographics that make Ricardian equivalence prevail, the golden rule does nothing to promote efficiency. But when the demographics imply even moderate departures from Ricardian equivalence, imposing the golden rule substantially improves the efficiency of democratically chosen allocations of public goods. We use some examples calibrated to U.S. demographic data and find greater benefits from adopting the golden rule at the state level or with 19th century demographics than under current national demographics.

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1 Introduction

A 'golden rule' prescribes that a government pay for nondurable goods and services out of current revenues and that it issue debt to finance durable goods. This paper studies how some alternative rules for financing public expenditures affect outcomes when durable and nondurable public goods are chosen each period by direct elections within competitive economies inhabited by overlapping generations of households who are not altruistic. When government financing restrictions do not distinguish among different public goods according to their durability, adverse incentives can confront voters. For example, because selfish voters ignore the services that future generations will receive, a comprehensive balanced budget rule makes voters prefer amounts of durable public goods that are smaller than Pareto efficient amounts. The golden rule offsets that adverse incentive by reducing the costs born by current voters and passing some costs on to future taxpayers.

This paper provides a reason for financing some government purchases by issuing debt that is distinct from, and complementary to, reasons studied by Barro [6] and Lucas and Stokey [19], who view government debt (either risk-free, as in Barro [6] and Aiyagari et. al. [2], or state-contingent, as in Lucas and Stokey [19]) as a way to reallocate distorting taxes across time or states of nature. To bring out the difference in our rationalizations of government debt from the ones in those studies, the present paper assumes that the government can levy lump sum taxes. This paper is about how, by affecting voters incentives, rules for administering deficits affect paths for government expenditures, something that the literature on tax smoothing takes as exogenous.

We use a 'guess and verify' strategy by positing a two-parameter class of fiscal constitutions that restrict government financing. One parameter tells the fraction of durable government goods the government can finance through borrowing, while the other measures the term to maturity of that debt. The golden rule, with 100% debt financing of public investment, represents a special case. We show that given any maturity of debt, a constitution of this form is flexible enough for there to exist values of debt financing that cause electoral outcomes to implement a Pareto efficient allocation. The parameters that work depend on the parameters of the economy and do not literally represent the golden rule. By asking how far from the golden rule are the parameters that support a Pareto efficient allocation, we are able to establish that for a quantitatively interesting range of economies, the golden rule gives a good approximation to a constitution that supports a Pareto efficient allocation.

The remainder of this paper is organized as follows. Section 2 shows the forces underlying our basic findings by using a two-period model with three cohorts of agents. The model in this section is rich enough to show how allowing government debt for financing durable public goods can align voters' preferences with those of a Pareto planner. However, the simple setting of this section omits subtle issues about the term structure of government debt, differences in incentives across generations alive, and formalizing beliefs about future political and economic outcomes that must be confronted in a model with a more realistic demographic structure and a longer time horizon. We analyze those issues in the context of a richer overlapping generations model with longer-lived agents in section 3. In this case, implementing an efficient level of spending through the political system does not necessarily require each household to 'pay for what it gets;' rather, efficient spending can be attained by properly aligning marginal costs and benefits of the pivotal voters. In section 4, we use the section 3 model as the basis for calculations that evaluate the quality of the golden rule as an approximation to a constitution that supports a Pareto efficient outcome. We find that the golden rule is a good approximation. By contrast, rules that fail to distinguish across durable and nondurable public goods generate significant distortions even with small departures from Ricardian equivalence. Interestingly, these distortions are much more prominent and robust for U.S. states than for the federal government. Mobility across states implies that households will discount future benefits from public goods much more than what would result from mortality and population growth alone; furthermore, the effect of mobility remains present even when generations are altruistically linked, as in Barro [5].[1]

[1] Bassetto with Sargent [7] tells about economists who have discussed the golden rule. Keynes long advocated

2 A two-period model

A simple two period model captures the main forces for intergenerational conflict that make the constitutional rules for government deficits important. The economy has three cohorts of households: the initial old who live only in period 1; the initial young who live in both periods; and the future young who live only in period 2. Each cohort is $1 + n$ times larger than the previous one. Households consume a nondurable private good and also enjoy the services of a public good, the durability of which matters in ways that we shall highlight.

Within each period of their lives, households have preferences ordered by

$$c_{a,t} + u(\Gamma_t),$$

where $c_{a,t}$ is consumption of the private good in period t by a household of age a (young or old), Γ_t is the amount of the public good per capita in period t, and u is strictly increasing, twice continuously differentiable, and strictly concave. The initial young discount future utility at the rate β. An individual in any cohort cares only about himself.[2]

Households are endowed with one unit of the private good in each period of their lives. The government can turn one unit per capita of the private good into one unit of the public good. The public good depreciates at a rate δ^Γ. To raise revenues, the government can collect lump sum taxes τ_t on all households alive.

The Pareto-efficient path of the public good solves:

$$\begin{aligned} u'(\Gamma_2) &= 1 \\ u'(\Gamma_1) &= 1 - \beta(1 - \delta^\Gamma) \end{aligned} \tag{1}$$

For the second period, Pareto efficiency simply requires the marginal benefit per capita of an extra unit of the public good to equal its marginal cost per capita in terms of forgone private consumption. For the first period, Pareto efficiency adjusts the marginal social cost downward to reflect the marginal present value of the public good left for period 2, namely, $\beta(1 - \delta^\Gamma)$.

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the rule, even after writing the *General Theory*. In a personal communication to us, Robert Skidelsky told us that he doubts that our model captures Keynes's reasoning.

[2]Because the lack of Ricardian equivalence is very important for our results, we will return to this assumption when discussing the quantitative implications of the full-fledged model in section 5.

We will contrast the Pareto-efficiency condition (1) with the policy choices that would be preferred by different cohorts.

2.1 Balanced budget and $\delta^\Gamma = 1$

First, consider the simplest scenario, in which the public good depreciates fully ($\delta^\Gamma = 1$) and a constitution requires the government to balance its budget in both periods. In this case, all households alive would support providing the public good in its efficient amount because all gain the same utility from the good and contribute equally to its cost through lump sum taxes.

2.2 Balanced budget and $\delta^\Gamma < 1$

Suppose next that the balanced-budget restriction is kept, but that $\delta^\Gamma < 1$, so that some of the first-period investment carries benefits into the second period. In this case, the initial old pay for the period-1 investment but receive no second-period benefits; the initial young pay for the investment in the first period and reap benefits for 2 periods;[3] and the cohort born in period 2 benefits without contributing to the cost.

Now, the balanced-budget restriction leads to intergenerational conflict over the provision of the public good. The initial old would prefer a level of the public good that satisfies $u'(\Gamma_1) = 1$ because they do not benefit from later services from the public good. Under a balanced budget, they would want less of the public good than occurs in the allocation that satisfies (1). The policy preferred by the young in period 1 depends on what they expect will occur in period 2. Suppose they have perfect foresight. Both generations alive in period 2 will agree on providing the public good in an amount that solves $u'(\Gamma_2) = 1$, which implies that any additional undepreciated public goods inherited from the previous period will be accompanied by decreased expenses for the public good in the second period. Therefore, the initial young face the following costs and benefits from an extra unit per capita of the public good in period 1:

- Additional tax in period 1 to provide public good equals 1.

[3] However, even for them, the costs and benefits need not be fully aligned.

- Additional benefit in period 1 equals $u'(\Gamma_1)$.
- Lower tax in period 2 resulting from the lower need to buy new units of the public good equals $(1 - \delta^\Gamma)/(1 + n)$.[4]

The period-1 policy preferred by the initial young equates the present value of their marginal costs and benefits and therefore satisfies

$$u'(\Gamma_1) = 1 - \beta \frac{1 - \delta^\Gamma}{1 + n}.$$

In selecting their preferred policy, the initial young neglect costs and benefits accruing both to the initial old and to the cohort that has not yet been born. When there is no population growth, these neglected effects cancel, inducing the young to support the efficient provision of the public good. But if there is population growth or decline, their preferences will be for underproviding or overproviding it.

The unborn generation in period 1 would support the highest possible investment in the public good because they pay nothing and get benefits in the second period. But their preferences would not affect government policy in period 1 because decisions will depend only on the preferences of living cohorts. Only under special circumstances would the outcome be Pareto efficient. In the case of positive population growth, both cohorts living in period 1 would support underprovision of the public good, so this is the most likely outcome.

2.3 Government debt allowed and $\delta^\Gamma < 1$

Continue to assume that $\delta^\Gamma < 1$, but now suppose that the government can finance a fraction x of its period-1 spending through debt issues that must be repaid in period 2. Because household preferences are linear in private consumption, the interest rate in a competitive equilibrium will be $1/\beta$.

[4]The assumed preferences make the public good subject to a congestion externality, that contributes the term $1 + n$. A similar analysis would apply if congestion were not present.

Under an institutional regime that allows debt level x, the initial old prefer a policy outcome that satisfies

$$u'(\Gamma_1) = 1 - x.$$

Debt shifts some of the cost of the initial investment onto future generations. Since this lowers the cost borne by the initial old, they will support greater provision of the public good. If $x = \beta(1 - \delta^\Gamma)$, their preferred policy coincides with the socially efficient level.

For the young, the presence of debt lowers the marginal cost of period-1 spending by x in the first period, but raises it by $\frac{x}{\beta(1+n)}$ in the second period when the debt becomes due. So the policy preferred by the young satisfies

$$u'(\Gamma_1) = 1 - \beta\frac{1 - \delta^\Gamma}{1 + n} - x\left(1 - \frac{1}{1 + n}\right). \tag{2}$$

As for the old, when $x = \beta(1 - \delta^\Gamma)$, the policy preferred by the young is Pareto efficient.

2.4 Discussion

The simple example above illustrates the benefits of a budgetary rule that imposes budget balance on government spending for nondurable goods, but allows issuing debt to pay for durable public investment. This rule more closely aligns the costs and benefits accruing to different generations;[5]

When $x = \beta(1 - \delta^\Gamma)$, the rule emulates a competitive rental market for public goods. If such a market existed, efficiency could be attained by imposing period-by-period budget balance and requiring the government to rent the flow of services of the durable good. That would make the distinction between durable and nondurable public goods disappear: all households alive would equally value the flow of services and would equally pay for it. There seem to be cases in which a rental market could operate. For example, our analysis provides a rationale for requiring the government to lease its vehicles or other standard durable equipment. But many major public investments are subject to monopoly and/or monopsony, and for these a rental market would not

[5]The benefits of the golden rule in aligning costs and benefits have been previously discussed; examples from very different sources are in Studensky [37], page 15, Secrist [35], and the Economist [1]. However, these benefits were mostly cast in terms of fairness, rather than efficiency.

work. Any major infrastructure project, such as a canal, would necessarily not face competition from a good substitute, generating monopoly power on the supply side. On the demand side, no other entity would be allowed to bid against the government for renting nuclear missiles.

This simple two-period model contains the main intuition behind many of our results, but there are good reasons to develop a model with longer lived agents and a longer-lived economy.

1. *Debt maturity.* In a multi-period environment, the maturity of debt plays an important role. Except for specific maturities, there will not be in general a fraction of debt financing x that will ensure *unanimous* support for an efficient outcome. However, we will be able to satisfy the weaker requirement that the pivotal voter supports efficient provision of the public good.

2. *Quantitative importance of Ricardian non-equivalence.* The simple model is enough to suggest that the old will favor underprovision of durable public goods under a balanced budget, but we need a multiperiod environment to evaluate the quantitative importance of this motive and other deviations from Ricardian equivalence.

3. *Adding important qualifications.* Some qualitative implications differ in a model with longer and more empirically plausible horizons. In the example above, there is a tight connection between population growth and the preferences of the initial young for over/underprovision under a balanced budget; this connection is looser in multiperiod environments. As we shall see, reasonable calibrations imply that all generations alive favor underinvestment under a balanced budget, even when population is constant.

3 The Benchmark Economy

Households can live at most $N + 1$ periods, where we allow $N = +\infty$. Conditional on having survived until then, each household faces a probability $1 - \theta_s$ of death in its s-th period of life. Population grows at a constant rate n and the demographic structure starts from a steady state and remains in it. Households consume a private good and enjoy the services of two public goods, one nondurable, the other durable. A household born in period t has preferences ordered by[6]

$$\sum_{s=t}^{N+t} \beta^{s-t} \left(\prod_{j=0}^{s-t-1} \theta_j \right) [c_{s-t,s} + v(G_s, \Gamma_s)] \tag{3}$$

where $c_{s-t,s}$ is consumption of the private good in period s by a household of age $s-t$ (born in period t), Γ_s is the per capita stock of the durable public good; G_s is the amount of nondurable public good per capita in period s. For brevity, we will often call Γ_s public capital and G_s public consumption. We assume that v is strictly concave, twice continuously differentiable, and that it satisfies Inada conditions. We also assume $\beta(1+n) < 1$. There is no uncertainty.[7]

From section 2, we retain the simplifying assumptions that preferences are linear in private consumption and that all living households value public goods equally. We will discuss how the results depend on both assumptions in section 5 and in the appendix.

Each household alive is endowed with 1 unit of labor that it supplies to a firm that can turn K units of private capital and L units of labor into $F(K, L)$ units of an intermediate good.[8] Firms can turn a unit of the intermediate good into one unit of either the private consumption good, private capital, nondurable public good, or public capital. Private capital depreciates at a rate δ^k, and public capital depreciates at a rate δ^Γ. The economy-wide resource constraints are

$$C_t + i_t + G_t + \gamma_t \leq F\left(\frac{K_{t-1}}{1+n}, 1\right) \tag{4}$$

$$K_t \leq (1-\delta^k)\frac{K_{t-1}}{1+n} + i_t \tag{5}$$

$$\Gamma_t \leq (1-\delta^\Gamma)\frac{\Gamma_{t-1}}{1+n} + \gamma_t \tag{6}$$

where i_t is investment in private capital per capita, γ_t is investment in public capital per capita, K_t is the capital stock per capita at the end of period t (to be used for production in period

[6]We adopt the convention $\left(\prod_{j=0}^{-1} \theta_j\right) \equiv 1$.

[7]Our working paper version (Bassetto with Sargent [7]) includes uncertainty, with minimal impact on the results stated in this paper.

[8]We assume F to be increasing, concave, continuously differentiable, linearly homogeneous and to satisfy Inada conditions.

$t+1$), and C_t is consumption per capita:

$$C_t \equiv \sum_{s=0}^{N} \lambda_s c_{s,t}. \tag{7}$$

Also,

$$\lambda_s \equiv \frac{(1+n)^{-s} \prod_{j=0}^{s-1} \theta_j}{\sum_{t=0}^{N} (1+n)^{-t} \prod_{j=0}^{t-1} \theta_j}$$

is the fraction of households of age s. The initial values of K_{-1} and Γ_{-1} are exogenous.

3.1 Pareto-optimal allocations

Either we assume that private consumption can be negative or, more realistically, we will restrict our analysis to a range of parameters and utility entitlements that make private consumption be positive.

We define a sequence $\{\{c_{s,t}\}_{s=0}^{N}, i_t, \gamma_t, G_t, K_t, \Gamma_t\}_{t=0}^{\infty}$ to be a *feasible (real) allocation* if it satisfies equations (4), (5), and (6) and is bounded.[9] We define a feasible allocation to be *efficient* or *ex ante Pareto optimal* if there is no other feasible allocation that delivers a weakly higher expected utility to each generation and a strictly higher utility to at least one generation. We assume that the planner can implement direct intergenerational transfers. When these are not possible, the golden rule has important distributional implications that we will discuss later.

The linearity of utility in consumption makes efficient allocations easy to characterize. In each period t, taking 1 unit of consumption per capita from each surviving household born in period $t-j$ allows a planner to award a consumption increase of λ_j/λ_s to each surviving household born in period $t-s$. This costs the former generation $\beta^j \prod_{m=0}^{j-1} \theta_m$ in expected utility and gives an expected utility gain to the latter of $\frac{\beta^s \prod_{m=0}^{j-1} \theta_m}{(1+n)^{j-s}}$.[10] The slope of the Pareto frontier is therefore

[9]The term "real" is introduced to distinguish this allocation from the asset allocation that we will introduce later. Wherever not specified, by allocation we mean real allocation. The assumption of boundedness could be relaxed, but bounded sequences are a particularly simple commodity space to work with. All infinite sums that appear below are well defined in this commodity space. Technological growth could be accommodated by suitably rescaling variables and introducing a shifter in preferences to ensure stationarity.

[10]If $t-j<0$, we measure utility from time 0, so the expected utility cost is $\beta^t \prod_{m=j-t}^{j-1} \theta_m$. Likewise, if $t-s<0$, the expected utility gain is $\frac{\beta^t \prod_{m=0}^{j-1} \theta_m}{(1+n)^{j-s} \prod_{m=0}^{s-t-1} \theta_m}$.

uniquely determined as the ratio $[\beta(1+n)]^{j-s}$. Movements along the frontier are achieved by redistributing consumption without affecting other variables. The difference between this situation and what prevails in a standard Pareto problem for consumers having strictly concave utility is illustrated in figure 1. In the standard problem, the slope of the utility frontier depends on the allocation, and the frontier is recovered by varying the Pareto weights assigned to each group in the Pareto problem; with our quasilinear preferences, a fixed Pareto weight corresponds to all utility pairs, so that the frontier cannot be traced out by varying the Pareto weight.

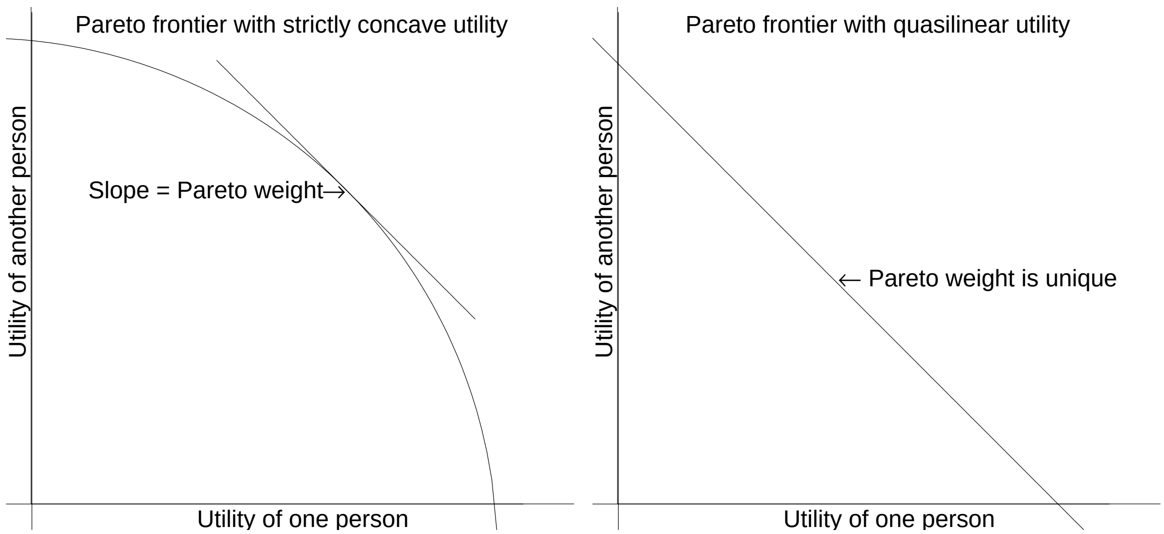

Figure 1: Pareto problem and the shape of the frontier

We consider only those allocations in which all households belonging to the same cohort are treated symmetrically. Among these, an efficient allocation solves the following

Pareto problem:

$$\max_{\{C_t,G_t,\Gamma_t,i_t,\gamma_t,k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} [\beta(1+n)]^t \{C_t + v(G_t,\Gamma_t)\} \tag{8}$$

subject to (4), (5) and (6).

The first-order conditions for this problem are

$$v_G(G_t,\Gamma_t) = 1, \quad t = 0, \ldots \tag{9}$$

$$v_\Gamma(G_t,\Gamma_t) = 1 - \beta(1-\delta^\Gamma), \quad t = 0, \ldots \tag{10}$$

$$F_k\left(\frac{K_t}{1+n},1\right)=\frac{1}{\beta}-(1-\delta^k)\quad t=0,\ldots \tag{11}$$

The assumptions about F guarantee that (11) has a unique solution K_t. Equations (9) and (10) imply that G_t and Γ_t are uniquely determined as well. All of these variables are independent of the initial levels of public and private capital, Γ_{-1} and K_{-1}. Investment can then be deduced from (5) and (6), and consumption per capita, but not its allocation across different cohorts, can be found from (4).

3.2 Implementing Pareto-efficiency through elections

A Pareto-efficient allocation can be attained under the following institutions. A government is empowered to levy taxes and purchase public goods. A constitution specifies that public goods be chosen by majority vote each period. The constitution names two parameters (α, x) that restrict debt repayment and government borrowing: a fraction α of government-issued consols is to be redeemed each period and a fraction x of public investment is paid for with debt. Decisions unfold as follows:

(i) In period t, the government purchases G_t and γ_t from competitive firms. The amounts are chosen by majority voting, subject to financing restrictions to be described in points (iii), (iv), and (v).

(ii) Taxes are lump sum and levied equally on all living households.

(iii) Each unit of nondurable spending G_t must be financed from taxes collected in period t.

(iv) For each unit of public investment γ_t, a fraction x is paid for by issuing new consols.[11]

(v) In each period, the government raises enough taxes to pay interest on outstanding consols and to repurchase a fraction $\alpha > 0$ of them.

[11] Different debt maturities and repayment schedules could be considered, provided that the amount of bonds is adjusted appropriately. In what follows, we will assume that the government issues exactly x. When median$(\theta_s) < 1+n$, as we will generally assume, the conditions that ensure that it is desirable to set x to a positive number also ensure that all the people alive will benefit from issuing debt, so that issuing the full amount x would be chosen even if the constitution prescribed a limit only on debt *issuance*, rather than a *level*.

(vi) Households and firms trade goods and factors of production in competitive markets.

Features (i), (iii), and (iv) capture the golden rule. The "constitution" does not specify amounts of public goods but restricts how to pay for them. By virtue of the ways that they align costs and benefits from public goods, the financing restrictions provide incentives for voters to choose efficient amounts.

An attractive feature of this institutional setup is its simplicity. Future generations require very little information about past behavior, making it easier to enforce such a rule, for example, by threatening to repudiate government debt. More complicated mechanisms might require future generations to know more to detect whether their predecessors had conformed to the rule. Therefore, it could take longer for a more complicated rule to become understood and accepted.

3.3 Equilibrium

We first describe the households' optimization problem, then define an equilibrium. Households can trade physical capital and government consols. A *tax-debt policy* is a bounded sequence $\{\tau_t, B_t\}_{t=0}^{\infty}$, where τ_t are lump-sum taxes per capita in period t and B_t is the amount of government-issued consols per capita at the end of period t. We set the coupon payment on each consol to 1. A *price system* is a sequence $\{w_t, r_t, p_t\}_{t=0}^{\infty}$, where w_t is the wage rate in period t, r_t is the rental rate of capital, and p_t is the price of a consol, exclusive of the time-t coupon payment. An *asset allocation* is a sequence $\{\{k_{s,t}, b_{s,t}\}_{s=0}^{N-1}\}_{t=0}^{\infty}$ consisting of the capital and the government consols held by a household of the cohort born in $t-s$ at the end of period t.

A household born in period $s \geq 0$ takes as given the price system and the tax policy and solves the following problem:

$$\max \sum_{t=s}^{s+N} \beta^{t-s} \left(\prod_{j=0}^{t-s-1} \theta_j \right) c_{t-s,t} \tag{12}$$

subject to the flow budget constraint

$$\begin{aligned} & c_{t-s,t} + k_{t-s,t} + p_t b_{t-s,t} + \leq w_t - \tau_t + \\ & \frac{[r_t + 1 - \delta^k]}{\theta_{t-s-1}} k_{t-s-1,t-1} + \frac{(p_t + 1)}{\theta_{t-s-1}} b_{t-s-1,t-1} \quad t = s, \ldots, s+N. \end{aligned} \tag{13}$$

This budget constraint assumes that households participate in a collective insurance agreement that redistributes the assets of people who die to the survivors in proportion to their holdings. This insurance scheme does not influence the resulting preferences over the public good. However, it is necessary to ensure that an interior solution exists at the equilibrium interest rate.[12] We assume that $k_{-1,s-1} = b_{-1,s-1} = 0$, so households start with no wealth.[13]

To prevent Ponzi schemes, it is necessary to impose a borrowing limit on households. If N is finite, we require $k_{N,s+N} = b_{N,s+N} = 0$. If N is infinite, we require a household's asset allocation to be essentially bounded from below.[14] We also require households to choose bounded real allocations.

Households that were born before period 0 solve a problem similar to problem (12)-(13) from period 0 onwards, except for their initial condition. A household born in period $j < 0$ starts from an (exogenous) initial condition $(k_{-j-1,-1}, b_{-j-1,-1})$ that is not necessarily 0.[15]

A *competitive equilibrium* is a real allocation $\{\{\hat{c}_{s,t}\}_{s=0}^{N}, \hat{i}_t, \hat{\gamma}_t, \hat{G}_t, \hat{K}_t, \hat{\Gamma}_t\}_{t=0}^{\infty}$, an asset allocation $\{\{\hat{k}_{s,t}, \hat{b}_{s,t}\}_{s=0}^{N-1}\}_{t=0}^{\infty}$, a tax-debt policy $\{\hat{\tau}_t, \hat{B}_t\}_{t=0}^{\infty}$, a price system $\{\hat{w}_t, \hat{r}_t, \hat{p}_t\}_{t=0}^{\infty}$, and initial conditions $\{\{k_{-s-1,-1}, b_{-s-1,-1}\}_{s=-N}^{-1}, K_{-1}, \Gamma_{-1}, B_{-1}\}$, such that:

(i) given the price system, the tax policy, the initial conditions, and the transfers, $(\{\hat{c}_{t-s,t}\}_{t=\max(0,s)}^{s+N}, \{\hat{k}_{t-s,t}, \hat{b}_{t-s,t}\}_{t=\max(0,s)}^{s+N-1})$ solves the maximization problem of the representative household born in period s;

[12] With quasilinear preferences, households would bunch all of their consumption at specific ages if their discount factor changed from period to period due to changes in the survival probability.

[13] This implies that the corresponding terms in (13) can be neglected, and the undefined term θ_{-1} does thus not appear.

[14] The household's asset allocation is *bounded from below* if $\{k_{t-s,t}, b_{t-s,t}\}_{t=s}^{\infty}$ form a sequence that is bounded from below; it is *essentially bounded from below* if there exists an alternative asset allocation $\{\tilde{k}_{t-s,t}, \tilde{b}_{t-s,t}\}_{t=s}^{\infty}$ that is bounded below, satisfies (13) for a suitable bounded choice of $\{c_{t-s,t}\}_{t=s}^{\infty}$, and is such that $[r_t + 1 - \delta^k]\tilde{k}_{t-s,t} + (p_t + 1)\tilde{b}_{t-s,t} + \leq [r_t + 1 - \delta^k]k_{t-s,t} + (p_t + 1)b_{t-s,t},\ t \geq s$.

[15] We also require $\sum_{s=0}^{N-1} \lambda_s k_{s,-1} = K_{-1}$ and $\sum_{s=0}^{N-1} \lambda_s b_{s,-1} = B_{-1}$.

(ii) at any time t, factor prices equal marginal products:

$$\begin{aligned}\hat{w}_t &= F_l\Big(\frac{\hat{K}_{t-1}}{1+n}, 1\Big)\\ \hat{r}_t &= F_k\Big(\frac{\hat{K}_{t-1}}{1+n}, 1\Big)\end{aligned} \tag{14}$$

(iii) at any time t, the allocation satisfies the feasibility conditions (4), (5) and (6);

(iv) at any time t, the asset markets clear:

$$\begin{aligned}\hat{K}_t &= \sum_{s=0}^{N} \lambda_s \hat{k}_{s,t}\\ \hat{B}_t &= \sum_{s=0}^{N} \lambda_s \hat{b}_{s,t}\end{aligned} \tag{15}$$

(v) the government budget constraint holds:

$$\hat{p}_t \hat{B}_t = \frac{\hat{B}_{t-1}}{1+n}(1+\hat{p}_t) + \hat{G}_t + \hat{\gamma}_t - \hat{\tau}_t. \tag{16}$$

The necessary conditions of the household maximization problem imply that in a competitive equilibrium[16]

$$\hat{p}_t = \frac{\beta}{1-\beta}, \quad t \geq 0 \tag{17}$$

$$1/\beta = \hat{r}_{t+1} + 1 - \delta^k, \quad t \geq 0 \tag{18}$$

and, if $N = \infty$,

$$\lim_{t\to\infty} \beta^t \Big\{ [\hat{r}_t + 1 - \delta^k]\hat{k}_{t-s,t-1} + (\hat{p}_t + 1)\hat{b}_{t-s,t-1} \Big\} = 0, \quad s \geq 0, \tag{19}$$

Because utility is linear in consumption, in any competitive equilibrium, individual consumption completely absorbs the impact of shocks at time t, making history irrelevant, just as it is in a Pareto-optimal allocation. Furthermore, substituting (14) into (18), shows that capital coincides

[16] These conditions include the assumption that there are no pricing bubbles. When $N = \infty$, introducing one-period debt would be sufficient to rule bubbles out. When N is finite, it is necessary to assume that households can set up an infinitely-lived intermediary. Huang and Werner [18] provide a complete treatment of this issue.

with its Pareto-efficient value. Working backwards through (14), we uniquely determine the equilibrium factor prices; like the labor supply and capital, they are independent of $\{G_t, \Gamma_t\}_{t=0}^{\infty}$.

Next we define a political-economic equilibrium in which households collectively choose public spending and investment each period. To evaluate those choices, households must form expectations about the evolution of the economy. A *history* of the economy is a sequence $h^t \equiv \{G_j, \Gamma_j\}_{j=0}^{t}$, for any $t \geq 0$.[17] We define a political-economic equilibrium in terms of a mapping Ψ that associates to each history h^t a time-t allocation (other than public spending and capital, which are already included in h^t) $(\{c_s(h^t)\}_{s=0}^{N}, i(h^t), \gamma(h^t), K(h^t))$, a time-$t$ tax-debt policy $(B(h^t), \tau(h^t))$, a time-t price vector $(w(h^t), r(h^t), p(h^t))$, a time-t asset allocation $(\{k_s(h^t), b_s(h^t)\}_{s=0}^{N-1})$, and a time-$t+1$ choice of government spending and capital $(G(h^t), \Gamma(h^t))$. To obtain the time-0 values of government spending and capital, we associate two values $G(\emptyset)$ and $\Gamma(\emptyset)$ with the null history. Given any history h^j, each mapping Ψ recursively generates a history from h^j: $h^t = (h^{t-1}, G(h^{t-1}), \Gamma(h^{t-1}))$. Each mapping Ψ and its associated history induce an allocation, a price system, and a tax/debt policy from initial conditions Γ_{j-1} and $\{k_s(h^{j-1}), b_s(h^{j-1})\}_{s=0}^{N-1}$.[18]

A mapping $\tilde{\Psi} \equiv (\{\tilde{c}_s\}_{s=0}^{N}, \{\tilde{k}_s, \tilde{b}_s\}_{s=0}^{N-1}, \tilde{i}, \tilde{\gamma}, \tilde{K}, \tilde{B}, \tilde{\tau}, \tilde{w}, \tilde{r}, \tilde{p})$ is a *political-economic equilibrium* when it has the following properties.

(i) (*Competitive equilibrium*) Given any history h^j, including the null history, the real and asset allocations, the price system, and tax/debt policy induced by $\tilde{\Psi}$ form a competitive equilibrium from the initial conditions $\Gamma_{j-1}, \tilde{K}(h^{j-1}), \tilde{B}(h^{j-1}), \{\tilde{k}_s(h^{j-1}), \tilde{b}_s(h^{j-1})\}_{s=0}^{N-1}$.

(ii) (*Self-interested voting*) Given any history h^{j-1}, including the null, $(\tilde{G}(h^{j-1}), \tilde{\Gamma}(h^{j-1})$ is a Condorcet winner over any alternative proposal (G, Γ), assuming that in the future the economy will follow the path implied by $\tilde{\Psi}$. That is, given any alternative (G, Γ), the

[17] Although other variables could be introduced as part of the history, it can be shown that their presence would be redundant with the definition of an equilibrium below. See e.g. Chari and Kehoe [12].

[18] h^{j-1} is the predecessor of the history h^j. When we consider an initial history h^0, $\tilde{k}_s(h^{-1})$ is $k_{s,-1}$, the initial level that is exogenously given; the same applies to other variables.

following inequality holds for more than 50% of the people alive at time j:

$$\begin{aligned}
&\tilde{c}_s(\tilde{h}^j_{G,\Gamma}) + v(G,\Gamma) + \\
&\sum_{t=1}^{N-s} \beta^t \left(\prod_{m=s}^{s+t-1} \theta_m \right) \left[\tilde{c}_{s+t}(\tilde{h}^{j+t}_{G,\Gamma}) + v\Big(\tilde{G}(\tilde{h}^{j+t-1}_{G,\Gamma}), \tilde{\Gamma}(\tilde{h}^{j+t-1}_{G,\Gamma}) \Big) \right] \leq \\
&\sum_{t=0}^{N-s} \beta^t \left(\prod_{m=s}^{s+t-1} \theta_m \right) \left[\tilde{c}_{s+t}(\tilde{h}^{j+t}) + v\Big(\tilde{G}(\tilde{h}^{j+t-1}), \tilde{\Gamma}(\tilde{h}^{j+t-1}) \Big) \right]
\end{aligned} \tag{20}$$

where $\tilde{h}^{j+t-1}_{G,\Gamma}$ and $\tilde{h}^{j+t-1}$ are defined recursively as follows:

$$\tilde{h}^j_{G,\Gamma} = (h^{j-1}, G, \Gamma)$$

$$\tilde{h}^{j+t}_{G,\Gamma} = \left(\tilde{h}^{j+t-1}_{G,\Gamma}, \tilde{G}\Big(\tilde{h}^{j+t-1}_{G,\Gamma}\Big), \tilde{\Gamma}\Big(\tilde{h}^{j+t-1}_{G,\Gamma}\Big) \right), \quad t \geq 1$$

$$\tilde{h}^{j+t} = \left(\tilde{h}^{j+t-1}, \tilde{G}\Big(\tilde{h}^{j+t-1}\Big), \tilde{\Gamma}\Big(\tilde{h}^{j+t-1}\Big) \right), \quad t \geq 0.$$

Here $\tilde{h}^{j+t}$ is the history induced by $\tilde{\Psi}$ from h^{j-1}; $\tilde{h}^{j+t}_{G,\Gamma}$ is the history induced by choosing (G, Γ) in period j and following $\tilde{\Psi}$ afterwards.

(iii) (*Budget balance*) Given any non-null history,

$$\tilde{\tau}(h^t) = \frac{(1 + \alpha \tilde{p}(h^t)) \tilde{B}(h^{t-1})}{1+n} + G_t + (1-x)\tilde{\gamma}(h^t) \tag{21}$$

where G_t is the appropriate element of the history h^t.

Requirement (i) states that no matter what happened in the past, $\tilde{\Psi}$ prescribes a path that is a competitive equilibrium. Requirement (ii) states that at each time t, the majority prefers not to deviate from an equilibrium $(\tilde{G}, \tilde{\Gamma})$, taking into account how a deviation effects contemporaneous utility and also how it affects the future by altering subsequent histories. Requirement (iii) embodies the budget rule restrictions that prevent borrowing to pay for G_t, limit the ability to borrow to pay for γ_t, and impose a repayment schedule on existing debt.

We focus on *Markov equilibria.*[19]

[19] Markov equilibria exclude "trigger-strategy" equilibria in which failure to deliver a certain amount of public goods in the current period leads households to unfavorable expectations in the future even though the "fundamentals" of the economy are unaffected.

Proposition 1 *There exist Markov equilibria in which G and Γ are independent of past variables. Following any history, all variables are the same in all these Markov equilibria, except for the distribution of consumption. Though individual consumption may vary across different equilibria, all generations achieve the same welfare ex ante in all these Markov equilibria.*

Proof. We prove existence by constructing an equilibrium. We focus here on the crucial step of determining government spending and relegate the determination of all other variables to the appendix. To get the values of $G(h^{t-1})$ and $\Gamma(h^{t-1})$, we propose a candidate equilibrium in which future levels of public consumption and capital are unaffected by the current ones. Within the equilibrium, an increase in the time-t provision of either public good benefits only the households that are alive in period t. On the cost side, equations (16) and (21) imply that an additional unit of G_t increases time-t taxes by 1 unit, with no effect on subsequent taxes. An additional unit of γ_t increases time-t taxes by $1-x$ units and leads to a reduction in γ_{t+1} of $(1-\delta^\Gamma)/(1+n)$ units, but has no further effect on public investment. This implies that time-$t+1$ taxes change by

$$\frac{x}{1+n}\left(\frac{1-\beta}{\beta}+\alpha\right)-\frac{(1-x)(1-\delta^\Gamma)}{1+n}$$

and taxes in period $t+j$, $j>1$ by

$$\frac{x}{(1+n)^j}\left[\frac{1-\beta}{\beta}+\alpha\right][\delta^\Gamma-\alpha](1-\alpha)^{j-2}.$$

To a person of age s, the expected present value of taxes per unit of public investment is therefore

$$\begin{aligned} Q_s \equiv & 1-x+\frac{\beta\theta_s}{1+n}\left[x\left(\frac{1-\beta}{\beta}+\alpha\right)-(1-x)(1-\delta^\Gamma)\right]+ \\ & \sum_{j=2}^{N-s}\beta^j\left(\prod_{m=s}^{s+j-1}\theta_m\right)\frac{x}{(1+n)^j}\left[\frac{1-\beta}{\beta}+\alpha\right]\left(\delta^\Gamma-\alpha\right)(1-\alpha)^{j-2}. \end{aligned} \tag{22}$$

The indirect utility function over government expenditure policy for this person is

$$v(G,\Gamma)-G-Q_s\Gamma. \tag{23}$$

If we order households by Q_s, the preferences associated with (23) satisfy the order restriction of Rothstein [32, 33]. This implies that a Condorcet winner exists, and that it corresponds to the policy preferred by the person with median Q_s. The candidate equilibrium therefore sets $G(h^{t-1})$ and $\Gamma(h^{t-1})$ to

$$\arg\max_{G,\Gamma} v(G,\Gamma) - G - \text{median}(Q_s)\Gamma. \tag{24}$$

Given our assumptions, a unique solution to problem (24) exists. This proves that, when households believe that future values of G and Γ will be unaffected by the history of play, the same will hold of the current values over which they vote. *QED.*

To save breath, we will refer to "the" Markov equilibrium and ignore the multiplicity that is irrelevant for the aggregate allocation and for welfare. We proceed to compare an equilibrium allocation with a Pareto-optimal one.

3.4 Analytical Results

Within the Markov equilibrium, public consumption is at its efficient level because the solution of (24) satisfies the Pareto-efficiency requirement (9). With quasilinear and identical preferences, the level of public consumption G will be efficient if households are forced to pay for it through contemporaneous taxes. The intuition underlying this result is that a balanced-budget restriction for nondurable public consumption converts the decision about its level into a static one. All living households agree about its benefits and share the costs. That leads to an efficient decision.

When a balanced-budget restriction is in effect, the marginal cost perceived by a household of age s for the durable public good is $Q_s = 1 - \frac{\theta_s\beta(1-\delta^\Gamma)}{1+n}$. Under the assumption that $\text{median}(\theta_s) < 1+n$, i.e., that population does not shrink too fast, the first-order condition of (24) implies a higher marginal value of public capital than does the efficiency condition (10). In expressing political preferences over public goods, households consider only their private costs and benefits; they neglect both costs and benefits of households of other ages, and the benefits of unborn cohorts who would gain from current investment but not be called to pay for it. When $\text{median}(\theta_s) < 1+n$, the ignored benefits of future generations dominate and underinvestment ensues, independently of the identity of the median voter.

It is natural to expect that allowing some debt financing of public investment would inspire the generations currently alive to support increased investment, since some of the cost could then be shifted to future generations. This is indeed the case for all the calibrated examples that we analyze below. For generic survival processes, however, debt financing not only shifts costs to the unborn cohorts; it also reshuffles costs within the generations who are alive when decisions are made. Since decisions are made by majority rule, allowing debt financing will be beneficial only if the cost perceived by the pivotal voter decreases. When population grows, a sufficient condition for increased debt financing – a higher x – to promote an increase in public investment is that $\alpha \leq \delta^{\Gamma}$: the repayment rate of debt should be lower than the depreciation rate of capital.[20] Low values of α imply that debt is repurchased slowly, indicating a long maturity of debt and shifting costs toward unborn generations. The condition above ensures that the costs passed on to future generations are big enough to reduce the marginal cost of public investment perceived by all living generations. Allowing *some* long-term debt is thus unequivocally beneficial, whereas short-term debt might reduce efficiency.

This analysis leaves open *how much* debt the government should be allowed to issue to improve efficiency or to achieve Pareto optimality. In the appendix, we prove that, given a maturity structure α of the debt, there exists a fraction of debt financing x^* that induces voters to implement the Pareto-efficient public investment. But in general little can be said about how x^* varies with the parameters of the model. To get tighter results, we move on now to consider two special cases in which there is unanimous consensus among all generations alive over public investment.

3.4.1 Mimicking Rental Markets.

When the repayment rate of debt and depreciation of public capital match ($\alpha = \delta^{\Gamma}$), all households support the efficient level of public investment when the fraction of debt financing is set at

$$x^* = \beta(1 - \delta^{\Gamma}). \tag{25}$$

In the two-period model of section 2, we noted that setting $x = \beta(1 - \delta^{\Gamma})$ served to give outcomes that mimic setting up a market in which the government rents the flows of services

[20] This can be proved through tedious algebra.

from public capital subject to a balanced budget restriction. This value of debt effectively ensured that the current generations would "sell" the undepreciated capital left after one period at its correct social value. The same intuition applies to our more general environment. However, since both government debt and public capital now last longer than two periods, mimicking a rental market requires matching costs and benefits in all periods that follow the initial investment. This can be achieved by matching the maturity of debt to the durability of public capital. Several authors have anticipated this result by advocating the principle that "people be made to pay what they enjoy."[21]

When $\alpha \neq \bar{\delta}^{\Gamma}$, the x that leads to efficiency does not make people pay for what they enjoy. To get efficiency, it is not important to match (marginal) costs and benefits of current investment for each cohort, born and unborn. What matters is matching marginal costs and benefits for the *pivotal* voters among the cohorts that actually participate in the investment decision.

While our main analysis shows that matching debt maturity to project durability is not necessary to achieve efficiency, the goal of unanimous agreement would offer a justification for trying to implement that match. Furthermore, determining the correct level of debt is much easier in this case, since it would be independent of the demographic structure of the economy.

Comparative statics when the government tries to mimic rental markets is also simple and intuitive: the less durable is public capital, the less debt should be issued, and the shorter its maturity should be. Also, the more households discount the future (including the benefits of future public capital), the less debt should be issued.

On the quantitative side, when $\alpha \approx \delta^{\Gamma}$, we find that 100% debt financing, as implied by the golden rule, closely approximates what is needed to support efficiency. While equation (25) suggests $x^* < 100\%$, if decisions are made every year, then it it is plausible to set β close to 1 and to set δ^{Γ} to be small. This provides intuition for the numerical results obtained below in which 100% debt financing performs very well.

A drawback to putting $\alpha = \delta^{\Gamma}$ is that it requires setting different maturities and different levels of debt financing for different types of public investment. Introducing a constitutional restriction

[21] See e.g. Secrist [35].

on debt financing that distinguishes multiple categories of investments may be cumbersome and could exacerbate politicians' temptation to misclassify public spending, a temptation that is already introduced by the broad distinction between current and capital public spending that is embedded in the golden rule.[22] In the calibrated examples, we will explore the quantitative consequences of setting the same maturity structure and the same level of debt financing (100%) for all types of investment.

3.4.2 Age-Independent Probability of Survival

We now consider the case in which $N = \infty$ and $\theta_s = \bar{\theta}$, $s = 0, \ldots$, which puts us into setting of the overlapping generation model of Blanchard [8], in which all households face a constant probability of death, independent of age. This case is interesting not only as a benchmark, but also because it leads to results that resemble those of the quantitative calibration of the next subsection. There, the calibrated probabilities of dying (or moving) from one year to the next are almost constant over much of the age spectrum, making the Blanchard model give a good approximation to the incentives faced by the pivotal voters.

Having ruled out wealth effects through the assumption of quasilinear preferences, age differences are the only potential sources of conflict among different living cohorts. When households face the same survival process, independently of age, they share the same preferences for public goods; hence, elections will be unanimous, independently of the value of x.

In this case, simple algebra shows that more debt financing always provides incentives for more public investment, independently of the maturity of debt. This is because extra debt creates no redistribution of costs among generations alive, but it rather only shifts costs to future cohorts.

The efficient level of investment is attained if

$$x^* = \frac{\beta(1-\delta^\Gamma)\,(1-\beta\theta(1-\alpha)/(1+n))}{1-\beta\theta(1-\delta^\Gamma)/(1+n)} > 0. \tag{26}$$

Alternatively, for a fixed value of x, (26) can be inverted to compute the maturity structure of

[22] Poterba [25] and Bohn and Inman [9] show that, if present, misclassification of expenses has not undone the bite of the golden rule in the case of U.S. states. More in general, evidence that constitutional clauses affect state government behavior is also presented in Poterba [24, 26] and Poterba and Rueben [27].

debt that would be needed to achieve an efficient investment:

$$\alpha^* = 1 - \frac{1}{\beta \frac{\theta}{1+n}} - \frac{x}{\beta} + \frac{x}{\beta^2(1-\delta^\Gamma)\frac{\theta}{1+n}} = \delta^\Gamma + \frac{1-\beta(1-\delta^\Gamma)\frac{\theta}{1+n}}{\beta^2(1-\delta^\Gamma)\frac{\theta}{1+n}}(x - \beta(1-\delta^\Gamma)) \tag{27}$$

Equations (26) and (27) imply a negative relationship between the maturity of debt and the amount of debt financing that achieves efficiency. Intuitively, more debt financing and a longer maturity are alternative ways of shifting costs away from the current generations; hence, they substitute for one another as ways of supporting an efficient level of investment. While a positive solution to (26) can be found for any maturity, some values of x are inconsistent with $\alpha \in [0, 1]$. If too much debt financing is allowed, even one-period debt ($\alpha = 1$) will generate an incentive for the current generations to overinvest. If instead x is too low, even consols that are never repurchased ($\alpha = 0$) are not sufficient to shift enough costs to future generations to restore efficiency.[23]

A higher depreciation rate for public capital implies that the efficient debt financing is smaller or that the efficient maturity is shorter, as one would expect. The effect of increasing the probability of survival θ and/or decreasing the population growth rate n is ambiguous.[24] The optimal level of x does not converge to 0 as $\theta/(1 + n)$ converges to 1; however, the inefficiency caused by a given x vanishes in the limit as $\theta/(1 + n) \to 1$, because then Ricardian equivalence holds and implies that any debt structure would deliver a Pareto-optimal outcome.

Finally, (27) implies that, in order for the golden rule to achieve exact efficiency, the maturity structure of debt should be shorter than the durability of capital: $\alpha > \delta^\Gamma$.

4 Calibrated Examples

The previous theoretical results suggest that financing *some* but not necessarily 100% of public investment with debt brings election outcomes close to Pareto efficiency. If we interpret the

[23]Debt stability requires $\alpha \in (-n, 2 + n)$, which is weaker than $\alpha \in [0, 1]$. However, values outside of [0,1] are harder to reinterpret in terms of a maturity structure of debt.

[24]The optimal amount of debt is decreasing in $\theta/(1+n)$ if and only if $\delta^\Gamma > \alpha$, and α^* is decreasing in $\theta/(1+n)$ if and only if $x > \beta(1 - \delta)$.

golden rule as prescribing that 100% of the cost of public investment be financed by debt issue, it is interesting to inquiry of our model: how close is the efficient debt allowance x^* to 100%? And what is the efficiency loss from not setting x exactly to x^*?

To answer these questions, we now consider three calibrated examples. Throughout the three examples, we assume:

- Decisions are taken every year, so 1 year is the appropriate period length;
- $\beta = 0.96$;
- In the baseline experiment, we take $\alpha = 0.04522$, corresponding to a half-life of debt of 15 years. We also report the value of α that would make the golden rule (100% debt financing) exactly optimal.
- For depreciation, we consider a low value of $\bar{\delta}^\Gamma = 0.03$ and a high value of $\bar{\delta}^\Gamma = 0.06$. The former is meant to capture depreciation for major infrastructure projects, the latter is a number commonly used for private capital.

The three examples differ in their demographic structures:[25]

1. For "U.S. now," we calibrate the survival process and the age distribution to that faced by the U.S. population in 2000.
2. For "U.S. 1880", we calibrate the survival process and the age distribution to that faced by the U.S. population in 1880. We will use this example to see to what extent demographic changes over the last two centuries affect the outcomes under the golden rule.
3. To consider how federal and state governments are affected differently by the budget rule, we calibrate the age structure to that of Illinois in 2000.[26] We will label this experiment as "Illinois now." The crucial difference between "U.S. now" and "Illinois now" is the degree

[25]The details of the demographic structure are contained in appendix B.

[26]The results presented here differ slightly from those in our working paper version (Bassetto with Sargent [7]). This discrepancy is due to correcting a mistake in computing the mobility by age.

of mobility. The model probability of "death" is calibrated by summing the probability of dying and that of moving out of the state.[27]

	U.S. Now	U.S. 1880s	Illinois Now
$\bar{\delta}^{\Gamma} = 0.03$	114%	108%	108%
$\bar{\delta}^{\Gamma} = 0.06$	77%	80%	80%

Table 1: Optimal Amount of Debt Financing x^*

Table 1 reports the optimal fraction of public investment that should be financed by debt.[28] The main conclusion that we draw from this table is that this fraction is not very sensitive to the demographic structure and evolution of the population: it is about the same in a calibration with low population growth and low mortality rate (U.S. now), high population growth and high mortality rate (U.S. in 1880) and low population growth with high mobility rate (Illinois now). By contrast, the table shows that the debt is sensitive to the depreciation rate.

A similar picture emerges if we set $x = 1$ and try to recover the value of α that would make it optimal. When $\delta = 0.03$, the implied value of α is about 0.035 for the United States now to 0.037 for Illinois; with $\delta = 0.06$, the range is from about 0.071 for the United States now to 0.074 for Illinois. As predicted by our analytical results, the golden rule is optimal when the maturity of debt is shorter than the durability of capital, but just slightly so.

The results support the case for distinguishing between very long-term investments, such as major infrastructures and investment in equipment, for which a lower degree of debt financing or a shorter maturity of debt is warranted.

Table 2 measures the efficiency wedge in the provision of public capital for a given level of debt financing. This is a measure of the discrepancy between the first-order condition of the

[27]It is straightforward to adjust the model to account for migration. Immigration requires assuming that some households are born at ages greater than 0; out-migration requires adjusting the annuitization so that households lose their private assets if they die, but not if they move out of state. Both adjustments do not affect the computations developed above to establish efficiency of the provision of public goods.

[28]Even when $\alpha < \bar{\delta}^{\Gamma}$, in all of the calibrated examples debt decreases the cost perceived by all generations alive, which implies that x^* is unique.

	U.S. Now	U.S. 1880s	Illinois Now
	Under Balanced Bugdet		
$\bar{\delta}^{\Gamma} = 0.03$	20%	46%	43%
$\bar{\delta}^{\Gamma} = 0.06$	14%	32%	30%
	Under Golden Rule		
$\bar{\delta}^{\Gamma} = 0.03$	3%	3%	3%
$\bar{\delta}^{\Gamma} = 0.06$	-4%	-8%	-7%

Table 2: Efficiency Wedge $= \frac{\text{Meq}-\text{Mopt}}{\text{Mopt}}$

pivotal voter and the first-order condition for Pareto optimality; formally, it is defined as

$$\text{Wedge} = \frac{\text{Meq} - \text{Mopt}}{\text{Mopt}}$$

where Meq = marginal value of public capital in the political-economic equilibrium outcome and Mopt = marginal value of public capital in an efficient allocation. As an example, a reading of 50% means that only those public infrastructures would be financed that generate a present value of benefits of more than \$1.50 per \$1 invested. We choose this as our main welfare measure because, as we discuss more in the appendix, it is likely to be robust to changes in preferences, and possibly to the inclusion of public capital in the production function.[29]

First, we consider the consequences of adopting a balanced budget provision that forbids the government ever to borrow. These results coincide to those of any rule in which the deficit limit is independent of current spending; a prominent example of such a rule is the European stability pact, which used to cap deficits at 3% of GDP independently of public spending.[30] The

[29] An alternative measure of the costs and benefits of switching policy is cast in (private) consumption equivalents. Using the calibration presented in appendix B.2, the cost estimates turn out to be quite small. As an example, for $\delta = 0.06$, the welfare gain, aggregated across generations, from moving from a balanced budget to the efficient policy is a permanent increase of about 0.01% of consumption for the U.S. government, and of about 0.09% of consumption for Illinois. These small magnitudes are not surprising, since public investment is only about 1% of GDP for the federal government and 2% for Illinois, and our model does not include any complementarities between private and public goods on either the consumption or the production side.

[30] For a discussion of fiscal policy rules in relation to the EMU, see Buiter [11].

table shows that the efficiency losses from this type of policy can be substantial. Unlike the optimal amount of debt, the efficiency wedge is quite different across calibrations. It is smallest for current U.S. demographics, which are much closer to Ricardian equivalence,[31] and larger for 1880s demographics or the case of Illinois. These results provide a rationale for adopting the 1800s rule in the 18th and 19th century rather than now, and for adopting the rule at the state or local level rather than at the national level.

By contrast, in the case of 100% debt financing, as prescribed by the golden rule, the efficiency losses from deviating from x^* are quite small; the worst loss comes from the 1880s calibration with $\delta = 0.06$, where projects are undertaken if they generate a present value of benefits of \$0.92 per \$1 invested.

In driving differences across calibrations, three factors are most important:

- The degree of mortality/mobility. As expected, higher mortality or mobility implies greater departures from Ricardian equivalence, since it makes it more likely that a household alive today will not enjoy the long-term benefits of public capital. High mobility is the main factor driving Illinois (and all other states) away from Ricardian equivalence.

- Population growth. Higher population growth implies that the benefit of current public investment will benefit larger cohorts in the future; this creates bigger departures from Ricardian equivalence. The high population growth in 1880 justifies the large wedges for that calibration. By contrast, because of its slow growth, Illinois is in fact closer to Ricardian equivalence than the median state.

- The age structure of population. The cohorts most likely to neglect future benefits of current investment are the old (because of mortality) and, in the case of states, the young (because of mobility). Hence, states with large young and/or old population are more likely to invest suboptimally if no provision for differential treatment of capital and ordinary expenses is present.

[31] A calibration to European data, with even lower population growth, would lead to even lower efficiency losses.

5 Extensions

Our analysis has relied on several assumptions. We discuss here the roles of the main ones and speculate on the consequences of changing them.

1. **Altruism.** We assumed that households alive do not care about future cohorts. To the extent that different generations are altruistically linked, as in Barro [5], the case of the federal government would be even closer to Ricardian equivalence than what we found in our calibration. A small departure from Ricardian equivalence would persist because of the nontrivial immigration into the United States. Intergenerational altruism would have a much smaller impact on state-level results, since it would not affect the consequences of mobility.

2. **Capitalization of Taxes and Amenities into Land Prices.** In our analysis, migration decisions are exogenous, and public goods have no impact on land prices. If land prices are fixed, endogenous mobility would strengthen the benefits of the golden rule: without borrowing, current generations would have even less incentive to invest in public capital, since additional investment would trigger more immigration, reducing the benefit through congestion.[32] However, this effect is likely to be swamped by the migration flows that occur for reasons that are independent of taxes and benefits.[33] If additional public investment raises land prices, households that move out of a jurisdiction (whether for exogenous or endogenous reasons) would be able to recoup their contribution when they sell their land to new immigrants. Hence, full land capitalization of public improvements could act as a substitute for the golden rule, ensuring efficiency even under a balanced budget. The degree of capitalization will be affected by two key parameters:

    - The price elasticity of the demand for land. At the extreme, if demand is infinitely elastic, the only difference between areas with more or fewer amenities would be the

[32] See e.g. Schultz and Sjöström [34].

[33] As an example, even for the very targeted AFDC transfer programs, Meyer [20] and Gelbach [16] find that endogenous mobility, while present, has a quantitatively small impact on the programs.

size of the houses (the better the amenities, the smaller the houses/lots). If instead the size of the houses and lots is fixed (e.g., by zoning laws), the price impact will be greatest.[34]

- The elasticity of immigration with respect to public goods. If location decisions are primarily driven by idiosyncratic shocks, then additional public goods will generate little net immigration, and correspondingly small price effects. By contrast, if location decisions are primarily driven by the amenities offered by the community, capitalization of public goods into land prices will be more prominent.

Both factors tend to generate an important role for land capitalization at the local jurisdiction level, a topic addressed by a vast literature that follows Tiebout [39]. As remarked by Tiebout, however, the same factors will lead to less land capitalization at the state level.[35]

3. **Alternative Preferences.** The results obtained above rely assuming quasilinear preferences and that households benefit equally from public goods at all ages. In appendix C, we compare the results for quasilinear preferences and for a more-standard log choice. Unfortunately, when preferences are not quasilinear, a Markov equilibrium will typically depend on the entire distribution of wealth across the living cohorts; computing equilibria with both endogenous policy and long-lived overlapping generations is infeasible. We therefore study economies in which overlapping generations live for (at most) two periods. The results turn out to be quite similar, which makes us optimistic also for the case of long-lived overlapping generations. Heterogeneity in the valuation of public goods is quite straightforward to introduce; very little can be said in general, except for the obvious point that more debt financing would be called for if the pivotal voters tend to like the public

[34] The importance of this channel is acknowledged by Fullerton and Gordon [15], Stiglitz [36] and Mieszkowski and Zodrow [21], among others.

[35] See footnote 9 in Tiebout [39]. However, if the public goods provided by the state are essentially local in nature, land capitalization would still occur, albeit not uniformly across the state. In this case, property taxes could be used as a form of "benefit taxation." Wallis [40] and Wallis and Weingast [41] argue that this was an important factor for U.S. states in the 1800s. For a case in which debt capitalization into land prices may occur and lead to improved efficiency at the national level, see Rangel [29].

good less than the average person in the economy, and vice versa.

4. **Intergenerational Transfers.** In computing efficient allocations, we assumed that the planner can transfer resources across different cohorts. However, the constitutional arrangement that we specified does not allow such transfers. This implies that constitutional changes in the level of debt financing would have redistributive consequences; specifically, a higher degree of debt financing would favor current generations at the expense of future ones that would be burdened by the extra debt. We thus have not completed a thorough study of the conditions that favor the introduction of the golden rule in the constitution. In the case of U.S. states, we speculate that the inability to borrow in the wake of the defaults of the 1840s made it less costly for the generations alive to agree to deficit restrictions; in fact, those restrictions may have eased access to credit markets, at least to finance public projects.[36]

5. **Distortionary Taxes.** In our analysis, we assumed all taxes to be lump sum. In the presence of distortionary taxes, a wide literature on optimal debt financing has emerged since Barro [6] and Lucas and Stokey [19]. Their main prescription is that distortionary taxes should be varied smoothly over time and/or across states; hence, periods of unusually high government spending (such as wars) should be financed by either deficits or state-contingent capital levies. Tax smoothing explains why many state constitutions have provisions allowing deficits under special circumstances, such as periods of civil unrest. We view our theory as complementary to theories of tax smoothing. Our theory would explain why durable and nondurable government purchases that turn out to be similar in terms of intertemporal volatility should be financed very differently, one by borrowing, the other out of current revenues.

6. **Distortions from the Political Process.** In our model, citizens directly choose both durable and nondurable purchases for that period by voting each period. Bigger departures

[36]Rangel [28] and Boldrin and Montes [10] consider explicitly intergenerational transfers as a tool to gain wider support for the provision of public goods. In a specific example, they view social security as a way to pay the elderly off in exchange for the provision of public education.

from Ricardian equivalence are generated by political-economy models with politicians who act as if they are more short-sighted than voters.[37] Some of these models may be easily extended to strengthen the case for adopting a rule,[38] while others do not clearly generate a distinction between government spending in durable and nondurable goods.[39] A more systematic analysis of the performance of the golden rule in such environments is a worthwhile extension of our research.

6 Concluding Remarks

The tax smoothing models of Barro [6], Aiyagari et.al. [2], and Lucas and Stokey [19] explain how issuing risk-free or state-contingent government debt allows a benevolent government to finance an exogenous process for government expenditures by levying flat rate taxes across time and states in the least distorting ways. By assuming that all taxes are lump sum and that government purchases of durable and nondurable public expenditures are chosen each period by majorities, we focus on a different reason for government debt: to realign the interests of majorities of voters in ways that will induce them to choose more efficient allocations of public and private goods. By studying the effects of our two-parameter class of constitutions that allow majorities to issue debt to finance only durable public goods, we thus offer a reason for government debt that is complementary to the one studied by Barro and Lucas and Stokey. Our model provides a framework for evaluating proposals, for example, to substitute a golden rule for the maximum percentage of GDP deficit rule stated in the Maastricht Treaty.

[37] Chari and Miller [13] rely on these models in their informal discussion to advocate the adoption of the golden rule at the federal level.

[38] We view extensions of Rogoff and Sibert [31] and Rogoff [30] as particularly good candidates because they imply a tendency to choose projects with immediate benefits over projects with delayed benefits.

[39] Among these, the most natural extension of models of partisan politics such as Alesina and Tabellini [3], Persson and Svensson [23], or Tabellini and Alesina [38] would generate overspending in both types of goods, unless government capital is perceived as a less partisan good than nondurable consumption, as assumed by Peletier, Dur and Swank [22] and by Azzimonti-Renzo [4].

Appendix

A Proofs

A.1 Proof of Proposition 1 (completion)

Our proof works by construction. We proved earlier that all competitive equilibria share the same values of p_t, K_t, w_t, and r_t, $t \geq 0$, and that these are all constant and independent of $\{G_t, \Gamma_t\}_{t=0}^{\infty}$. This implies that, for any history h^t, there is a unique way to set $p(h^t)$, $K(h^t)$, $w(h^t)$, and $r(h^t)$. We set investment to the only values that are consistent with (5) and (6): $i(h^t) = K(h^t) - (1-\delta^k)K(h^{t-1})$ and $\gamma(h^t) = \Gamma_t - (1-\delta^\Gamma)\Gamma_{t-1}$.

For any non-null history, we set B recursively:

$$B(h^t) = \frac{(1-\alpha)B(h^{t-1})}{1+n} + \frac{x\gamma(h^t)}{p(h^t)}. \tag{28}$$

We set $\tau(h^t)$ according to (21), and aggregate consumption such that

$$\sum_{s=0}^{\infty} \lambda_s c_s(h^t) = F\left(\frac{K(h^{t-1})}{1+n}, 1\right) - i(h^t) - G_t - \gamma(h^t). \tag{29}$$

Subject to (29), individual consumption is not uniquely pinned down.

To construct individual consumption, we partition the set of histories into sequences $\{h^v\}_{v=t}^{\infty}$, $t \geq -1$, with the property that, within elements of the same partition, $h^v = (h^{v-1}, G(h^{v-1}), \Gamma(h^{v-1}))$, while for each initial element of a sequence either $t = -1$ or $h^t \neq (h^{t-1}, G(h^{t-1}), \Gamma(h^{t-1}))$. Within each partition, a history represents a path that is induced by the functions $G(.)$ and $\Gamma(.)$ from the history h^t. Along each of these paths, we set the functions $c_s(.), k_s(.)$, and $b_s(.)$ arbitrarily, subject to the following restrictions:

(i) Given the taxes and prices specified above, the resulting sequences must satisfy the household budget constraints (13) for an economy that starts in period t with initial conditions specified by $(k_s(h^{t-1}), b_s(h^{t-1}))_{s=0}^{N-1}$;

(ii) if $N = \infty$, the implied asset allocation must be essentially bounded from below;

(iii) the implied real allocation must be bounded;

(iv) individual consumption schedules are such that (29) holds in the aggregate.

Examples of functions that satisfy these requirements are the following:[40] $k_s(h^v) = K(h^v)/(1-\lambda_N)$, $b_s(h^v) = B(h^v)/(1-\lambda_N)$, and

$$\begin{aligned} c_s(h^v) = &-\frac{K(h^v)+p(h^v)B(h^v)}{1-\lambda_N} + w(h^v) - \tau(h^v) + \frac{[r(h^v)+1-\delta^k]K(h^{v-1})}{\theta_{s-1}(1-\lambda_N)} + \\ &\frac{(p(h^v)+1)B(h^{v-1})}{\theta_{s-1}(1-\lambda_N)}, \quad v>0, s=1,\ldots,N-1 \end{aligned} \tag{30}$$

$$\begin{aligned} c_s(h^0) = &-\frac{K(h^0)+p(h^0)B(h^0)}{1-\lambda_N} + w(h^0) - \tau(h^0) + \frac{[r(h^0)+1-\delta^k]k_{s-1,-1}}{\theta_{s-1}} + \\ &\frac{(p(h^0)+1)b_{s-1,-1}}{\theta_{s-1}}, \quad s=1,\ldots,N-1 \end{aligned} \tag{31}$$

$$c_0(h^v) = -\frac{K(h^v)+p(h^v)B(h^v)}{1-\lambda_N} + w(h^v) - \tau(h^v), \quad v \geq 0 \tag{32}$$

$$c_N(h^v) = w(h^v) - \tau(h^v) + \frac{[r(h^v)+1-\delta^k]K(h^{v-1})}{\theta_{N-1}(1-\lambda_N)} + \frac{(p(h^v)+1)B(h^{v-1})}{\theta_{N-1}(1-\lambda_N)}, \quad v>0, \tag{33}$$

$$c_N(h^0) = w(h^0) - \tau(h^0) + \frac{[r+1-\delta^k]k_{N-1,-1}}{\theta_{N-1}} + \frac{(p(h^v)+1)b_{N-1,-1}}{\theta_{N-1}}. \tag{34}$$

By iterating equation (28) forward and using the facts that $p(h^v)$ and $\Gamma(h^v)$ are constant over time, we can show that $B(h^v)$ is bounded, which implies that the consumption plans of the households are bounded.[41]

It is now straightforward to see that the mapping that we have just constructed generates a competitive equilibrium starting from any history h^j, given the levels of public spending and capital implied by (24). Starting from any initial capital level, we previously showed that the values assigned to $p(h^t)$, $K(h^t)$, $w(h^t)$ and $r(h^t)$, $t \geq j$, are the unique choices that ensure that the household problem has a (bounded) solution, that factor prices equal marginal products, and that (17) holds. With these choices, households are indifferent among all possible consumption profiles. Given the previously determined variables and (21), equation (28) describes the unique

[40]If $N=\infty$, λ_N should be set to 0, and (33) and (34) do not apply.

[41]If $N=+\infty$, the proposed asset allocation also satisfies the no-Ponzi games restriction and the transversality condition (19).

sequence of government debt that satisfies the government budget constraint and (19), and (29) describes the unique sequence of aggregate consumption that is consistent with the resource constraint (4). The conditions on the consumption of individual households ensure that the individual budget constraints are met. Whenever the future levels of G and Γ depend only on the future shocks and not on any history, the mapping constructed above implies that the current choices of G and Γ do not affect private capital or factor prices in any period, nor do they affect subsequent values of public spending and capital. These are precisely the conditions under which households will vote for G and Γ according to (24), which implies that the mapping satisfies condition (ii) of a political-economic equilibrium. Finally, condition (iii) is met because it was imposed in the construction of the mapping.

To check that all Markov equilibria lead to the same ex ante welfare for all generations, notice that prices, taxes, and the provision of the public goods are the same in all Markov equilibria and in all histories. As a consequence, the individual household's optimization problem is identical in all of the equilibria, and the multiplicity of the Markov equilibria comes only from the fact that households are indifferent among many equivalent optimal consumption plans. *QED.*

A.2 Proposition 2

Proposition 2 *Generically in the death process, there exists a value x^* such that, if $x = x^*$, the allocation induced by any Markov equilibrium is Pareto optimal. A sufficient condition for x^* to be unique is that Q_s is decreasing in x for all ages.*

We already observed that in a competitive equilibrium private capital and investment are equal to their (unique) value implied by any Pareto-optimal allocation. We also proved that the balanced-budget restriction on G_t ensures that (9) is satisfied. We need only to prove that there exists a value x^* such that the solution to (24) satisfies (10). First, note that Q_s is linear in x. Matching the first-order condition of (24) with respect to Γ to (10) requires

$$\text{median}(Q_s^0 + Q_s^1 x) = 1 - \beta(1 - \delta^\Gamma) \tag{35}$$

where

$$Q_s^0 \equiv 1 - \frac{\beta\theta_s(1-\delta^\Gamma)}{1+n}$$

and

$$Q_s^1 \equiv -1 + \frac{\beta\theta_s}{1+n}\left[\frac{1}{\beta} + \alpha - \delta^\Gamma\right] +$$
$$\sum_{j=2}^{N-s} \beta^j \left(\prod_{m=s}^{s+j-1} \theta_m\right) \frac{1}{(1+n)^j}\left[\frac{1-\beta}{\beta} + \alpha\right](\delta^\Gamma - \alpha)(1-\alpha)^{j-2}$$

For a generic death process, we have median$(Q_s^1) \neq 0$. In this case, the left-hand side of equation (35) is a continuous function of x, and it diverges to infinity as x diverges, but with opposite signs as $x \to -\infty$ or $x \to +\infty$. This implies that a solution to equation (35) exists. If $Q_s^1 < 0$ for all ages, the median cost strictly decreases with x and the solution is unique. *QED.*

B Details of Calibration

B.1 Demographic Structure

1. U.S. now.

   We use the death rate by age in 2000, from the National Center for Health statistics. For the age structure we use data from the 2000 U.S. Census. We abstract from out-migration from the U.S., which is small.[42] However, the age structure reflects the effects of nontrivial immigration. We truncate the distribution at age 90, assuming that a 90-year old person dies for sure; we also do not consider people below 18 years of age, so a person is "born" when (s)he reaches age 18 or (s)he immigrates. Finally, population growth is taken from the 10-year growth from the 1990 to the 2000 Census, at an annualized rate.

2. U.S. 1880s.

   Mortality data come from Haines [17] (we use data from his "West Model"). The data are aggregated in 5-year intervals of age, so we used piecewise linear interpolation. For deaths

[42] We tried including it, with little change.

and the age structure (relevant for voters), we only consider males between 21 and 80. For the population growth (which we assume to be relevant for taxpayers), we use the growth of the total U.S. population from 1880 to 1890, annualized. Results change very little if different choices are made regarding the inclusion/exclusion of one sex from the calibration.

3. Illinois now.

   For the probability of death, we use the same as the U.S. 2000 example. The probability of death is swamped by the effect of out-migration, so any difference between Illinois and the rest of the U.S. would be quantitatively insignificant for the results. For out-migration, the U.S. Census reports the number of people that left Illinois between 1995 and 2000, by age. We use this to construct an annual probability of out-migration by dividing the number of migrants by 5. We are implicitly assuming that out-migration is permanent, i.e., a person that leaves Illinois will never return to be an Illinois resident. This is clearly an unrealistic assumption, but we expect the bias introduced by it to be quantitatively small.

B.2 Parameter Choices for Welfare Computations

We assume that $v(G, \Gamma) = \xi \frac{G^{1-\sigma}}{1-\sigma} + \eta \frac{\Gamma^{1-\sigma}}{1-\sigma}$.

To pin down σ, we need a measure of the price elasticity of the demand for public goods. DelRossi and Inman [14] survey related studies, quoting estimates between 0.17 and 5. We choose a unit elasticity and set $\sigma = 1$ (log preferences).

To calibrate ξ and η for the federal government, we match the fraction of government consumption expenditures to GDP and government gross investment to GDP from NIPA data, which are approximately 6% and 1% respectively. For Illinois, we match the ratio of the operating budget appropriations to GDP and of capital budget appropriations to GDP (source: BEA for GDP, State of Illinois for spending); these numbers are approximately 7% and 2%. We do not consider the U.S. in 1880, as the scope of the federal government was extremely limited back then.[43]

[43]For the 19th century, we chose to analyze the United States because very good mortality data by age were readily available to us. However, that experiment is mainly designed to inform us about the likely gains for other

We assume that the economy is in steady state, and normalize GDP to 1.[44] To infer the stock of public capital from the flow of gross investment, we take the depreciation rate to be 6% (our higher choice, which does not correspond to infrastructure only). We assume that Illinois is following the golden rule, and fix ξ and η to match the appropriate ratios under this assumption. Since the federal government is not mandated to follow any constitutional rule in linking taxes and spending, it is less obvious what to assume. Fortunately, the results for the federal government are less sensitive to the specific value chosen: as the wedges in table 2 imply, even drastic changes in the degree of debt financing have only a moderate impact on government spending. We thus assume that the federal government follows a practice that requires, at the margin, an extra dollar in current revenues for each extra dollar in spending of either type; this rule is equivalent to a balanced budget, or to the 3% deficit/GDP limit of the European stability pact, or to any deficit ceiling which is not conditional on spending.

The implied values for ξ are approximately .078 for the federal government and .093 for Illinois; the implied values for η are approximately .02 at the federal level and .036 for Illinois.

C Robustness Checks

In the computations below, we compare the outcomes of quasilinear preferences with the political outcome when households have preferences given by

$$\log c_{0,s} + \log(1 - l_{0,s}) + \xi \log(G_s) + \eta \log(\Gamma_s) +$$
$$\beta\theta_0 \Big[\log c_{1,s+1} + \log(1 - l_{1,s+1}) + \xi \log(G_{s+1}) + \eta \log(\Gamma_{s+1}), \Big]$$

where $l_{j,s}$ is the labor supply of a person of age j in period s. To simplify the analysis, we take factor prices as given. We will mainly comment results at the steady state, but similar numbers occur along transition paths.

The choice of overlapping generations of two period lived people is useful computationally, but it is also interesting because it stacks the odds against us. To see this, consider the first-order

countries in which national governments played a more prominent role.

[44]Note that, in a growing economy, ξ and η should grow at the GDP rate to have balanced growth.

condition that determines public investment in the quasilinear case:

$$v_\Gamma(G,\Gamma) = \text{median}(Q_s), \tag{36}$$

where Q_s in (22) is computed assuming that any additional investment undertaken today is reversed in the subsequent period. If the current median voter stayed in power for the subsequent period as well, the same first-order condition would hold, even without quasilinearity: the envelope condition would imply that, at the margin, the household would be indifferent whether the current additional investment is fully undone next period, or whether some of it is passed on. The presence of conflict generates two departures from this first-order condition: first, the envelope condition no longer holds, since the identity of the median voter changes from one period to the next; second, the median has to be taken over the Q_s computed according to the equilibrium process for taxes, so the identity of the median voter might change depending on the speed with which any additional investment is reversed in the subsequent periods. However, both of these factors play a limited role in the quantitative results of the previous section. Both mobility and mortality change slowly with age; as a consequence, the choice of next period's median voter is very close to the optimal choice of the current median voter, and even a change in the identity of the median voter is unlikely to cause drastic changes in policy. By contrast, with overlapping generations living for two periods, the conflict between the current median voter (which we will take to be the young, assuming population growth) and the future median voter is extreme, since their survival profiles are radically different.

We construct a sequence of numerical examples. Even though we occasionally chose some parameter values to match data, these are meant purely to illustrate the extent to which quasilinear preferences and log preferences yield similar results.

In example 1, we set one period to be 30 years; this gives households a reasonable life span, but it implies an unreasonably long lag between one policy session and the next. We thus choose the following parameter values:

- $\theta_0 = 1$: the young survive for sure 1 more period, leading to maximal conflict between the two generations alive;

- $n = 1$: population doubles from one period to the next;

- $r = 5$ and $\delta^k = .84$: this corresponds to a yearly depreciation of about 6% and a net rate of return of capital of 5.62% per year;

- $\beta = \frac{1}{1+r-\delta^k}$: this implies that the individual consumption profile is flat;

- $\alpha = 0.2$: this implies a half-life of debt of about 3 periods (93 years);

- $\delta^\Gamma = 0.6$, about 1.7% per year;

- $w = 10$;

- $\eta = 0.02$, $\xi = 0.133$; this implies that, in the steady state of the golden rule, public investment/GDP is 2% and public consumption/GDP is 7%.

- Throughout the example, the government is not allowed to issue debt to pay for public consumption; in steady state, this turns out to be efficient, since both generations alive have the same private consumption level and hence the same valuation of the public good.

	Optimal debt if starting at golden rule	Steady-state wedge under balanced budget	Steady-state wedge under Golden Rule
Log	6%	3.8%	-54%
Linear	7%	4.2%	-54.2%

Table 3: Public investment and efficiency in example 1

Table 3 illustrates the results for this case. The first column illustrates the following experiment. We assume that the economy starts from the steady state implied by the political-economic equilibrium that prevails under the golden rule. At time 0, the fraction x of investment financed through bonds is unexpectedly changed once and for all. We compute the compensating variation that makes each generation indifferent between the original value of x and the new one. Typically, early generations will be hurt by a sudden reduction in x, since they will be able to pass on less debt to future generations, while later generations benefit from the lower steady-state level of debt. We aggregate welfare using the interest rate as the discount factor, which would be appropriate if the government could use generation-specific lump-sum transfers for compensation. The optimal amount of debt financing turns out to be quite low in this case: this is not surprising, since most public capital depreciates from one period to the next. More interestingly, log and linear preferences yield almost identical answers.

In the second column, we look at the wedge in the marginal utility of public capital between the efficient provision level and the one prevailing in steady state when the government is not allowed to issue any debt ($x = 0$). In the third, we study the same wedge, but in the steady state when the government follows the golden rule ($x = 1$). Even for these measures of distortion, linear and log preferences yield essentially identical results.

The results of example 1 are driven to a significant extent by the high discount factor and the low durability of public capital. In example 2, we choose numbers closer to a yearly calibration; this implies of course that people in this artificial economy would live only for 2 years.

Parameter values are now: $\theta_0 = 1$, $n = 0.02$, $\beta = 0.96$, $\delta^k = \delta^\Gamma = 0.06$, $r = .102$ (such that $\beta = \frac{1}{1+r-\delta^k}$), $\alpha = 0.045$, and η, ξ and w at the same values of the first example.

	Optimal debt if starting at golden rule	Steady-state wedge under balanced budget	Steady-state wedge under Golden Rule
Log	66%	59%	-16%
Linear	53%	18%	-16%

Table 4: Public investment and efficiency in example 2

For this example, the difference between log and linear preferences is more significant. However, this difference almost vanishes if we reduce the probability of survival from young to old age to $\theta_0 = 0.4$, as shown in table 5. Since $\theta_0 = 0.4$ still implies a high degree of conflict across generations, we view this result as very encouraging for the accuracy of the predictions contained in section 4.

Finally, to assess the impact of wealth effects, we change the interest rate r to 22.7% for the

	Optimal debt if starting at golden rule	Steady-state wedge under balanced budget	Steady-state wedge under Golden Rule
Log	89%	564%	-66%
Linear	89%	562%	-66%

Table 5: Public investment and efficiency in example 2, with $\theta_0 = 0.4$

log economy only, so that, in the steady state, consumption of the old is 12% higher than that of the young.[45] The approximation remains acceptable, except for distortion levels that are very

	Optimal debt if starting at golden rule	Steady-state wedge under balanced budget	Steady-state wedge under Golden Rule
Log	81%	263%	-62%
Linear	89%	562%	-66%

Table 6: Public investment and efficiency in example 2, with $\theta_0 = 0.4$ and an increasing consumption profile

far from anything that we considered in section 4.

The presence of a wealth effect implies that a balanced-budget restriction will no longer lead to efficiency in public consumption; since the young are relatively poorer, they will choose a lower level of G than optimality would dictate. Table 7 explores a potential role for letting the government issue debt as a way of restoring efficiency. While allowing a tiny fraction of debt financing improves welfare, the results suggest again that the difference between linear and log preferences is small; the main implication that spending in nondurable public goods should not be financed through debt is upheld.

[45] For the linear economy, the consumption profile is indeterminate, and an increase in r with no change in β would lead to unbounded solutions.

	Optimal debt if starting at golden rule	Steady-state wedge under Golden Rule
Log	6%	3%
Linear	0%	0%

Table 7: Public consumption and efficiency in example 2, with $\theta_0 = 0.4$ and an increasing consumption profile

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