# Online Appendix for: "Trade in Intermediate Inputs and Business Cycle Comovement" 

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This appendix includes supplemental results that are referenced in the main text. The supplemental results include: (1) stylized facts concerning gross output and value added correlations in the data (referenced in Section 4.1), and (2) robustness checks for the main trade-comovement regressions, in which I vary both the data and model simulations used to estimate trade-comovement regression coefficients, and (3) figures underlying the discussion of model mechanics in Section 4.3.3.

## 1 Value Added and Gross Output Correlations

Figure $\mathrm{S1}$ plots the bilateral correlation of gross output against the bilateral correlation of value added, for the aggregate economy and individual sector pairs. These figures illustrate the point that cross-country correlations in value-added and gross output are similar. The correlation is 0.82 in the aggregate, and larger than 0.75 for all sectors.

Figure 52 plots the bilateral correlation of value added or gross output in the services sector against the bilateral correlation of value added or gross output in the goods sector. These figures illustrate the point that country pairs with high bilateral comovement in goods production also tend to have high comovement in services production. The correlation is 0.56 for value added and 0.49 for gross output.

## 2 Trade-Comovement Robustness Checks

Table S1 presents variations on the aggregate and sector-level trade comovement regressions in the data and baseline version of the model. Panel A presents results in the data and model using the subsample of country pairs with gross output data in the EU KLEMS database. Panel B presents results in the data and model using the level of bilateral trade, rather than logs. Panel C presents results in the data and model using sector-level trade to define
bilateral trade intensity in the sector-level regressions. I define bilateral trade intensity as: $\log \left(\frac{E X_{i j}(s)+E X_{j i}\left(s^{\prime}\right)}{G D P_{i}(s)+G D P_{j}\left(s^{\prime}\right)}\right)$, for sector $s$ in country $i$ and sector $s^{\prime}$ in country $j$. This definition corresponds to Measure II in Di Giovanni and Levchenko (2010).

Table S 2 presents trade-comovement regressions for simulated data from alternative versions of the model, with the same specification as in Table 1.

In Panels A and B of Table S2, I report results for a version of the model with incomplete financial markets. In these simulations, I hold the nominal trade balance constant. To do this, I assume that the consumer budget constraint takes the form: $p_{i t}^{f} F_{i t}=w_{i t} L_{i t}+r_{i t} K_{i t}+T_{i}$, where $T_{i}$ is a constant nominal transfer across countries (equal to the trade balance in the steady state). Linearizing the budget constraint yields:

$$
\begin{equation*}
0=\hat{p}_{t}^{f}+\hat{F}_{t}-\operatorname{diag}\left(\frac{\bar{w}_{i} \bar{L}_{i}}{\bar{p}_{i}^{f} \bar{F}_{i}}\right)\left(\hat{w}_{t}+\hat{L}_{t}\right)-\operatorname{diag}\left(\frac{\bar{r}_{i} \bar{K}_{i}}{\bar{p}_{i}^{f} \bar{F}_{i}}\right)\left(\hat{r}_{t}+\hat{K}_{t}\right) . \tag{1}
\end{equation*}
$$

This equation replaces Equation (A22) in simulation of the model.
In Panel B and C, I return to the baseline model with complete markets and simulate the model with an alternative productivity process. Specifically, I use the productivity process estimated using HP filtered productivity data with smoothing parameter set to 100, as discussed in Appendix B.

## 3 Changes in Bilateral Correlations in Simulations with Alternative Elasticities

In Section 4, I simulated the model with alternative elasticities. In Section 4.3.3, I described the mechanics underlying changes in output correlations for changes in the elasticity of substitution between primary factors and intermediate inputs, and changes in the elasticity of substitution for inputs relative to final goods. Figures S3.S6 summarize those mechanics graphically.

The approach I take is to compute differences between bilateral correlations in alternative versions of the model and correlations in the baseline model. For example, if $\rho_{i j}^{v}(\sigma, \eta, \rho)$ is the correlation for variable $v$ in a simulation with parameters $(\sigma, \eta, \rho)$, then I compute differences $\rho_{i j}^{v}(\sigma, \eta, \rho)-\rho_{i j}^{v}(\bar{\sigma}, \bar{\eta}, \bar{\rho})$, where the bars denote the baseline value of the parameters. To summarize these differences as a function of trade intensity, I fit a locally weighted regression to the differences, and plot the predicted values (with standard errors) in the figures.

Table S1: Robustness Checks for Trade-Comovement Regressions with the Baseline Model

$\underline{\text { Panel C: Bilateral Trade Intensity Measured using Sector-Level Trade Data }}$

| Log Sectoral Bilateral Trade | 0.112*** | 0.104*** | 0.041*** | 0.060*** | 0.093*** | 0.096*** | 0.054*** | 066*** |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $(0.013)$ | (0.010) | (0.010) | (0.008) | (0.011) | (0.011) | (0.012) | (0.008) |
| Observations | 231 | 231 | 231 | 462 | 171 | 171 | 171 | 342 |
| R -squared | 0.26 | 0.33 | 0.08 | 0.13 | 0.27 | 0.30 | 0.10 | 0.16 |
| Panel C2: Model with Correlated Shocks |  |  |  |  |  |  |  |  |
| Log Sectoral Bilateral Trade | $\begin{gathered} 0.015 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.073^{* * *} \\ (0.010) \end{gathered}$ | $\begin{aligned} & -0.010 \\ & (0.010) \end{aligned}$ | $\begin{gathered} 0.022^{* * *} \\ (0.008) \end{gathered}$ | $\begin{aligned} & 0.022^{*} \\ & (0.013) \end{aligned}$ | $\begin{gathered} 0.079 * * * \\ (0.011) \end{gathered}$ | $\begin{aligned} & -0.007 \\ & (0.010) \end{aligned}$ | $\begin{gathered} 0.025^{* * *} \\ (0.008) \end{gathered}$ |
| Observations | 231 | 231 | 231 | 462 | 231 | 231 | 231 | 462 |
| R-squared | 0.01 | 0.18 | 0.00 | 0.02 | 0.01 | 0.20 | 0.00 | 0.02 |
| Panel C3: Model with Uncorrelated Shocks |  |  |  |  |  |  |  |  |
| Log Sectoral Bilateral Trade | $\begin{gathered} 0.003^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.006 * * * \\ (0.001) \end{gathered}$ | $\begin{gathered} -0.001^{* *} \\ (0.000) \end{gathered}$ | $\begin{gathered} 0.006 * * * \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.008^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.020^{* * *} \\ (0.002) \end{gathered}$ | $\begin{aligned} & -0.000 \\ & (0.000) \end{aligned}$ | $\begin{gathered} 0.010^{* * *} \\ (0.001) \end{gathered}$ |
| Observations | 231 | 231 | 231 | 462 | 231 | 231 | 231 | 462 |
| R -squared | 0.13 | 0.30 | 0.02 | 0.20 | 0.35 | 0.56 | 0.00 | 0.24 |

In Panel A: $\log$ bilateral trade is $\log \left(\frac{E X_{i j}+E X_{j i}}{G D P_{i}+G D P_{j}}\right)$. In Panel B: Bilateral trade is $\frac{E X_{i j}+E X_{j i}}{G D P_{i}+G D P_{j}}$. In Panel C: Log Sectoral Bilateral Trade is: $\log \left(\frac{E X_{i j}(s)+E X_{j i}\left(s^{\prime}\right)}{G D P_{i}(s)+G D P_{j}\left(s^{\prime}\right)}\right)$. All regressions include a constant. Robust standard errors in parentheses. Significance levels: * $p<.1,{ }^{* *} p<.05,{ }^{* * *} p<.01$.

Table S2: Trade-Comovement Regressions in Alternative Versions of the Model

|  | Real Value Added |  |  |  | Gross Output |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Aggregate <br> (1) | Goods <br> (2) | Services <br> (3) | Cross <br> (4) | Aggregate <br> (5) | Goods <br> (6) | Services <br> (7) | Cross <br> (8) |
| Panel A: Model with Constant Trade Balances |  |  |  |  |  |  |  |  |
| Panel A1: Correlated Shocks |  |  |  |  |  |  |  |  |
| Log Bilateral Trade | $\begin{gathered} 0.018 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.070^{* * *} \\ (0.013) \end{gathered}$ | $\begin{aligned} & -0.005 \\ & (0.012) \end{aligned}$ | $\begin{gathered} 0.020^{* *} \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.030^{* *} \\ (0.012) \end{gathered}$ | $\begin{gathered} 0.072^{* * *} \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.005 \\ (0.012) \end{gathered}$ | $\begin{gathered} 0.030^{* * *} \\ (0.009) \end{gathered}$ |
| Observations R-squared | $\begin{gathered} 231 \\ 0.01 \end{gathered}$ | $\begin{gathered} 231 \\ 0.14 \end{gathered}$ | $\begin{aligned} & 231 \\ & 0.00 \end{aligned}$ | $\begin{aligned} & 462 \\ & 0.01 \end{aligned}$ | $\begin{gathered} 231 \\ 0.02 \end{gathered}$ | $\begin{aligned} & 231 \\ & 0.13 \end{aligned}$ | $\begin{gathered} 231 \\ 0.00 \end{gathered}$ | $\begin{gathered} 462 \\ 0.03 \end{gathered}$ |
| Panel A2: Uncorrelated Shocks |  |  |  |  |  |  |  |  |
| Log Bilateral Trade | $\begin{gathered} 0.005^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.003^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.005^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.004^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.013^{* * *} \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.015^{* * *} \\ (0.002) \end{gathered}$ | $\begin{gathered} 0.010^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.012^{* * *} \\ (0.001) \end{gathered}$ |
| Observations <br> R-squared | $\begin{gathered} 231 \\ 0.24 \end{gathered}$ | $\begin{gathered} 231 \\ 0.09 \end{gathered}$ | $\begin{gathered} 231 \\ 0.16 \end{gathered}$ | $\begin{gathered} 462 \\ 0.13 \end{gathered}$ | $\begin{aligned} & 231 \\ & 0.43 \end{aligned}$ | $\begin{gathered} 231 \\ 0.46 \end{gathered}$ | $\begin{gathered} 231 \\ 0.30 \end{gathered}$ | $\begin{aligned} & 462 \\ & 0.40 \end{aligned}$ |
| Panel B: Model with AR(1) productivity process and Complete Markets |  |  |  |  |  |  |  |  |
| Panel B1: Correlated Shocks |  |  |  |  |  |  |  |  |
| Log Bilateral Trade | $\begin{gathered} 0.011 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.057^{* * *} \\ (0.012) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.017^{* *} \\ (0.008) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.013) \end{gathered}$ | $\begin{gathered} 0.066^{* * *} \\ (0.013) \end{gathered}$ | $\begin{aligned} & -0.005 \\ & (0.012) \end{aligned}$ | $\begin{gathered} 0.023^{* * *} \\ (0.009) \end{gathered}$ |
| Observations | 231 | 231 | 231 | 462 | 231 | 231 | 231 | 462 |
| R -squared | 0.00 | 0.10 | 0.00 | 0.01 | 0.01 | 0.12 | 0.00 | 0.02 |
| Panel B2: Uncorrelated Shocks |  |  |  |  |  |  |  |  |
| Log Bilateral Trade | $\begin{gathered} 0.003^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.004^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} -0.002^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.006^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.009^{* * *} \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.019 * * * \\ (0.002) \end{gathered}$ | $\begin{gathered} -0.000 \\ (0.001) \end{gathered}$ | $\begin{gathered} 0.011^{* * *} \\ (0.001) \end{gathered}$ |
| Observations | 231 | 231 | 231 | 462 | 231 | 231 | 231 | 462 |
| R-squared | 0.15 | 0.19 | 0.06 | 0.23 | 0.37 | 0.51 | 0.00 | 0.31 |

Log bilateral trade in all regressions is $\log \left(\frac{E X_{i j}+E X_{j i}}{G D P_{i}+G D P_{j}}\right)$, and all regressions include a constant. Robust standard errors in parentheses. Significance levels: ${ }^{*} p<.1,{ }^{* *} p<.05,{ }^{* * *} p<.01$.

Figure S1: Value Added and Gross Output Correlations for Aggregate Economy and Individual Sectors in Data


Figures labeled 'Goods' present correlations for the goods sector in $i$ with the goods sector in $j$. Figures labeled 'Services' present correlations for the services sector in $i$ with the services sector in $j$. Figures labeled 'Cross' present correlations for the goods sector in $i$ and the services sector in $j$, and vice versa.

Figure S2: Value Added and Gross Output Correlations in Data


Figure S3: Aggregate Bilateral Correlations with Alternative Elasticities Between Primary Factors and Intermediate Inputs (Differences from Baseline Model with Uncorrelated Shocks)


Midline indicates locally weighted regression of differences in bilateral correlations between simulations in columns (2) and (3) of Table 3 and the baseline model. Upper/lower lines indicate two standard deviation confidence intervals. Log Bilateral Trade is defined as: $\log \left(\frac{E X_{i j}+E X_{i i}}{G D P_{i}+G D P_{j}}\right)$.

Figure S4: Goods Bilateral Correlations with Alternative Elasticities Between Primary Factors and Intermediate Inputs (Differences from Baseline Model with Uncorrelated Shocks)


Midline indicates locally weighted regression of differences in bilateral correlations between simulations in columns (2) and (3) of Table 3 and the baseline model. Upper/lower lines indicate two standard deviation confidence intervals. Log Bilateral Trade is defined as: $\log \left(\frac{E X_{i j}+E X_{j i}}{G D P_{i}+G D P_{j}}\right)$.

Figure S5: Aggregate Bilateral Correlations with Alternative Elasticities for Final Goods and Intermediate Inputs (Differences from Baseline Model with Uncorrelated Shocks)


Midline indicates locally weighted regression of differences in bilateral correlations between simulations in columns (2) and (3) of Table 2 and the baseline model. Upper/lower lines indicate two standard deviation confidence intervals. Log Bilateral Trade is defined as: $\log \left(\frac{E X_{i j}+E X_{j i}}{G D P_{i}+G D P_{j}}\right)$.

Figure S6: Goods Bilateral Correlations with Alternative Elasticities for Final Goods and Intermediate Inputs (Differences from Baseline Model with Uncorrelated Shocks)





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\text { - Comp. Inputs and Subs. Final }-- \text {. Subs. Inputs and Comp. Final }
$$

Midline indicates locally weighted regression of differences in bilateral correlations between simulations in columns (2) and (3) of Table 2 and the baseline model. Upper/lower lines indicate two standard deviation confidence intervals. Log Bilateral Trade is defined as: $\log \left(\frac{E X_{i j}+E X_{j i}}{G D P_{i}+G D P_{j}}\right)$.

