

# Global Liquidity Trap\*

Ippei Fujiwara<sup>†</sup>, Tomoyuki Nakajima<sup>‡</sup>, Nao Sudo<sup>§</sup>, and Yuki Teranishi<sup>¶</sup>

First draft February 2009; This draft July 2010

## Abstract

In this paper we consider a two-country New Open Economy Macroeconomics model, and analyze the optimal monetary policy when countries cooperate in the face of a “global liquidity trap” – *i.e.*, a situation where the two countries are simultaneously caught in liquidity traps. Compared to the closed economy case, a notable feature of the optimal policy in the face of a global liquidity trap is its *international dependence*. Whether or not a country’s nominal interest rate is hitting the zero bound affects the target inflation rate of the other country. The direction of the effect depends on whether goods produced in the two countries are Edgeworth complements or substitutes. We also compare several classes of simple interest-rate rules. Our finding is that targeting the price level yields higher welfare than targeting the inflation rate, and that it is desirable to let the policy rate of each country respond not only to its own price level and output gap, but also to those in the other country.

*JEL Class:* E52; E58; F41

*Keywords:* Zero interest rate policy; two-country model; international spillover; monetary policy coordination

---

\*We thank Klaus Adam, Joshua Aizenman, Michele Cavallo, Larry Christiano, Michael Devereux, Marty Eichenbaum, Yiping Huang, Takatoshi Ito, Jinill Kim, Michael Klaue, Giovanni Lombardo, Bartosz Mackowiak, Shinichi Nishiyama, Paolo Pesenti, Andrew Rose, Lars Svensson, Kazuo Ueda, Tack Yun, Tsutomu Watanabe, Mike Woodford and seminar participants at University of Tokyo, Osaka University, Far East and South Asia Meeting of the Econometric Society 2009, Bank of Canada, Federal Reserve Board, Summer Workshop on Economic Theory 2009 in Otaru University of Commerce, Bank of Italy, ECB-IFS-Bundesbank seminar, Bank of Japan, and NBER-EASE for helpful comments and discussions. Nakajima thanks financial support from the Murata Science Foundation and the Japanese Ministry of Education, Culture, Sports, Science and Technology. Views expressed in this paper are those of the authors and do not necessarily reflect the official views of the Bank of Japan.

<sup>†</sup>Financial Markets Department, Bank of Japan. E-mail: [ippei.fujiwara@boj.or.jp](mailto:ippei.fujiwara@boj.or.jp)

<sup>‡</sup>Institute of Economic Research, Kyoto University. E-mail: [nakajima@kier.kyoto-u.ac.jp](mailto:nakajima@kier.kyoto-u.ac.jp)

<sup>§</sup>Institute for Monetary and Economic Studies, Bank of Japan. E-mail: [nao.sudou@boj.or.jp](mailto:nao.sudou@boj.or.jp)

<sup>¶</sup>Institute for Monetary and Economic Studies, Bank of Japan. E-mail: [yuuki.teranishi@boj.or.jp](mailto:yuuki.teranishi@boj.or.jp)

# 1 Introduction

The world economy now faces the largest economic downturn since World War II. To prevent further economic deterioration, most central banks in developed economies, including the United States, the United Kingdom, and Japan, have reduced policy interest rates to unprecedentedly low levels, acting promptly and in unison. The liquidity trap is no longer an extreme event only for one country, but has become an international concern that needs to be solved through international monetary cooperation. Figure 1 shows nominal interest rates from year 2008 to 2009 in several advanced countries. All nominal interest rates exhibit drastic decreases from their levels in 2008.

In this paper we investigate how monetary authorities of different countries should coordinate with each other when they find themselves simultaneously caught into a liquidity traps; that is, how they should coordinate policy measures in the case of a *global liquidity trap*. For this purpose, we consider a two-country version of the model of Eggertsson and Woodford (2003), and study the optimal monetary policy coordination in an environment where the zero lower bound for the nominal interest rate binds in both countries.

We start by asking under what conditions countries in an open economy separately might fall into a liquidity trap. We assume producer currency pricing and complete international asset markets. Then, if the nominal interest rate never hits the zero bound, the optimal policy is given by setting the producer price index (PPI) inflation rate to be zero for each country at all times, as in previous studies such as Clarida, Gali, and Gertler (2001) and Benigno and Benigno (2003). Under such a policy, the nominal interest rate in each country is set equal to the real interest rate associated with its own PPI. Thus we can define the “natural rate of interest” for each country as the PPI-based real interest rate in the equilibrium with zero PPI inflation. The first best can be attained as long as the natural rate of interest defined in this way is positive for each country. However, once a country’s natural rate becomes negative, its nominal interest rate will hit the zero bound and its economy will fall into a liquidity trap. Thanks to complete international asset markets, every household in every country has access to the same set of assets at

the same prices. Nevertheless monetary authorities in different countries will face different real interest rates, because PPIs will vary if countries produce different goods. This explains why it is possible for countries to fall separately into liquidity traps even though international asset markets are complete.

We then show that the optimal monetary policy in the case of a global liquidity trap exhibits two notable features: *history dependence* and *international dependence*. The importance of the history dependence in the conduct of monetary policy during a liquidity trap has been noted in previous studies on the closed economy.<sup>1</sup> The adverse effect of the liquidity trap can be mitigated if the monetary authority commits to generate some inflation and stimulate production in the future. Such a mechanism is also at work in a global liquidity trap.

The international dependence of the optimal monetary policy discussed in this paper is a new feature of interdependence that arises in a global liquidity trap. In open economies, countries are naturally interrelated through trades in goods and services. In the context of optimal monetary policy, several aspects of such interdependence across countries have been analyzed in previous studies such as Clarida, Gali, and Gertler (2001) and Benigno and Benigno (2003). One of our main findings is that a global liquidity trap brings about a new form of interdependence in the optimal monetary policy in open economies. In our model, if the nominal interest rate never hits the zero bound, the optimal policy could be implemented by a purely inward-looking inflation-targeting policy in which the target inflation rate for each country depends only on its own output gap.<sup>2</sup> This is no longer the case once the possibility that the zero bound binds is taken into account. The target inflation rate for each country necessarily depends on conditions in the other country.

---

<sup>1</sup>Examples on previous work on the closed economy include, among others, Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), and Adam and Billi (2006, 2007). As discussed in Eggertsson and Woodford (2003), there could be fiscal policies to ease the adverse effect of liquidity trap. In the current paper, however, we focus on the outcomes the monetary policy can bring about taking the fiscal policies as given.

<sup>2</sup>This has been shown previously, for instance, by Clarida, Gali, and Gertler (2001). In other word, even if the two countries trade goods each other, international dependence is obtained only in the liquidity trap.

In this sense, the global liquidity trap introduces a new form of international dependence in the optimal monetary policy, which is what we shed light on in this paper. The direction to which the international dependence works depends on whether goods produced in the two countries are Edgeworth complements or substitutes. For instance, suppose that they are substitutes. Then if one country is in a liquidity trap, it exerts downward pressure on the target inflation rate in the other country. In addition, a country pursuing an inflationary policy in order to extricate itself from a liquidity trap will exert upward pressure on the other country's inflation target. If goods produced in different countries are complements, the effects work in opposite directions.

We then examine if the optimal monetary policy can be approximated by a simple interest-rate rule. For a closed economy where there is no possibility of a liquidity trap, Schmitt-Grohé and Uribe (2007) show that the optimal policy is replicated fairly well by the class of interest-rate rules that respond only to the inflation rate. Taking the liquidity trap into consideration, Eggertsson and Woodford (2003) argue that a simple price-level targeting policy performs well for the closed economy. Our question here is whether such a similarly simple monetary policy rule can be identified in our model of a global liquidity trap. What we find, in line with Eggertsson and Woodford (2003), is that an interest-rate rule with a price-level target performs much better than the corresponding rule with an inflation target. Furthermore, improved performance is obtained when we allow the interest-rate rule for each country to depend on the other country's price level and output gap. This is because such a rule helps to capture, at least to some extent, the international dependence that a desirable policy should possess when faced with a global liquidity trap.

The structure of the paper is as follows. Section 2 discusses the existing literature on liquidity traps. Section 3 describes our two-country NOEM model and derives the world-loss function. In section 4, we analyze the optimal policy coordination problem, and show that history and international dependences are crucial features of the optimal policy when the zero bound may bind. We also present a numerical example, which allows further investigation into the properties of the optimal policy. Section 5 considers simple interest-rate rules, and examines how well they can approximate the optimal policy in the face of

a global liquidity trap. Section 6 concludes.

## 2 A Brief Survey of the Related Literature

The BOJ's adoption of what was effectively a zero interest rate policy in the late 1990s renewed interest in the liquidity trap. For the case of the closed economy, the properties of the optimal (or at least desirable) monetary policy under such circumstances have been investigated, for instance, by Reifschneider and Williams (2000), Eggertsson and Woodford (2003), Jung, Teranishi, and Watanabe (2005), Kato and Nishiyama (2005), Adam and Billi (2006, 2007), and Nakov (2008).

Reifschneider and Williams (2000) examine how to conduct monetary policy when the non-negativity condition for the nominal interest rate may bind using the FRBUS model. In order to mitigate the deflationary impact of a liquidity trap, they show that it is desirable for the monetary authority to commit to maintain a zero interest rate for some periods even after the adverse shock that triggered the liquidity trap has disappeared. They did not derive the optimal monetary policy in their model, but later studies show that such history dependence is indeed one of its important characteristics.

Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005) derive the optimal monetary policy in a standard New Keynesian model. As suggested by Reifschneider and Williams (2000), they show that the optimal policy possesses the history-dependence property that the nominal interest rate remains zero for a while even after the adverse shock itself disappears. The commitment to such a policy ameliorates the deflationary pressure in earlier periods where the adverse shock exists, because it raises the expected inflation rate and lowers the real interest rate. Their models have been extended to different stochastic environments by Kato and Nishiyama (2005), Adam and Billi (2006) and Nakov (2008). The basic message that history dependence is the key feature of the optimal monetary policy in the face of a liquidity trap is unchanged in these extensions.

Coenen and Wieland (2003), Svensson (2001), and Nakajima (2008) study a liquidity trap in open economies. Coenen and Wieland (2003) and Svensson (2001) emphasize the importance of raising the expected rate of inflation in an open economy context, and explore

its implications for the nominal exchange rate. Nakajima (2008) analyzes the optimal monetary policy in a two-country version of the model of Eggertsson and Woodford (2003). These studies, however, restrict attention to a “local liquidity trap,” where there is only one country in the liquidity trap. The contribution of our paper is to extend the enquiry to encompass a global liquidity trap, where two countries are simultaneously caught in liquidity traps.

The optimal policies in the face of, respectively, global and local liquidity traps turn out to be different not only during periods when both countries are stuck in liquidity traps, but also in other periods when one country has successfully escaped. This is because we are considering the optimal policy commitment. Even in an environment where only one country remains in a liquidity trap, the optimal policy committed to is different depending on whether the commitment was made in a period when both countries were caught in a liquidity trap, or when one country had already escaped the liquidity trap. In this sense, the optimal policy problem in the face of a global liquidity trap is a distinct problem, for which we provide new insights from both analytical and computational viewpoints.

### 3 The Model

#### 3.1 Households

The model economy is an open-economy version of the sticky-price model developed by Woodford (2003), and closely related to the ones considered by Clarida, Gali, and Gertler (2001), and Benigno and Benigno (2003), among others.

The world economy consists of two countries; the home country ( $H$ ), and the foreign country ( $F$ ). The size of population in country  $j \in \{H, F\}$  is  $n_j$ , where  $n_H + n_F = 1$ . A set of differentiated products are produced in each country and they are traded between the two countries. Let  $N_j$  denote the set of those products. We assume that  $N_H = [0, n_H]$ , and  $N_F = (n_H, 1]$ .

In each country identical households reside, who consume differentiated commodities, supply differentiated labor, and own firms in their country. Monetary policy is set by the

monetary authority in each country. Details of monetary policy are discussed later.

### 3.1.1 Preferences

A representative household in the home country  $H$  has preferences given by

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tilde{u}(C_t) - \frac{1}{n_H} \int_{N_H} \tilde{v}[\ell_t(i)] di \right\}, \quad (1)$$

where  $0 < \beta < 1$ ,  $\sigma > 0$ , and  $\ell_t(i)$ ,  $i \in N_H$ , is the supply of type- $i$  labor, which is used to produce differentiated product  $i$ . We assume that  $\tilde{u}$  and  $\tilde{v}$  have constant elasticity:

$$\tilde{u}(C) \equiv \frac{C^{1-\sigma}}{1-\sigma}, \quad \tilde{v}(\ell) \equiv \frac{1}{1+\omega} \ell^{1+\omega}.$$

The consumption index for the home household,  $C_t$ , is given by

$$C_t = \left( \frac{C_{H,t}}{n_H} \right)^{n_H} \left( \frac{C_{F,t}}{n_F} \right)^{n_F}, \quad (2)$$

where  $C_{H,t}$  and  $C_{F,t}$  are the consumption indexes for, respectively, home and foreign goods consumed by the home household; they are defined by

$$C_{j,t} = \left[ n_j^{-\frac{1}{\theta}} \int_{N_j} c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad j = H, F. \quad (3)$$

Here,  $\theta > 1$  and  $c_t(i) \in N_j$  is the home household's consumption of good  $i$  produced in country  $j \in \{H, F\}$ . It is convenient to define the function  $u(C_H, C_F)$  by

$$u(C_H, C_F) \equiv \tilde{u} \left( \left[ \frac{C_H}{n_H} \right]^{n_H} \left[ \frac{C_F}{n_F} \right]^{n_F} \right).$$

The lifetime utility of a representative household in the foreign country  $F$  takes the same form as that of the home household:

$$U_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \tilde{u}(C_t^*) - \frac{1}{n_F} \int_{N_F} \tilde{v}[\ell_t^*(i)] di \right\}. \quad (4)$$

The consumption indexes for the foreign household,  $\{C_t^*, C_{H,t}^*, C_{F,t}^*\}$ , are defined as in equations (2) and (3):

$$C_t^* = \left( \frac{C_{H,t}^*}{n_H} \right)^{n_H} \left( \frac{C_{F,t}^*}{n_F} \right)^{n_F},$$

$$C_{j,t}^* = \left[ n_j^{-\frac{1}{\theta}} \int_{N_j} c_t^*(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \quad j = H, F.$$

Corresponding to the consumption indexes in the home country,  $C_t$ ,  $C_{j,t}$ ,  $j = H, F$ , the prices indexes,  $P_t$ ,  $P_{j,t}$ ,  $j = H, F$ , are defined as

$$P_t = P_{H,t}^{n_H} P_{F,t}^{n_F},$$

$$P_{j,t} = \left[ \frac{1}{n_j} \int_{N_j} p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \quad j = H, F,$$

where  $p_t(i)$ ,  $i \in N_j$ ,  $j \in \{H, F\}$ , is the price of good  $i$  produced in country  $j$  quoted in the home currency. The price indexes in the foreign country,  $P_t^*$ ,  $P_{j,t}^*$ ,  $j = H, F$ , are defined similarly by individual good prices,  $p_t^*(i)$ ,  $i \in N_j$ ,  $j \in \{H, F\}$ , quoted in the foreign currency.

We assume that the law of one price holds:

$$p_t(i) = \mathcal{E}_t p_t^*(i),$$

for all  $i \in N_j$ ,  $j \in \{H, F\}$ , where  $\mathcal{E}_t$  is the nominal exchange rate, defined as the price of foreign currency in terms of home currency. It follows that  $P_{j,t} = \mathcal{E}_t P_{j,t}^*$ ,  $j = H, F$ , and  $P_t = \mathcal{E}_t P_t^*$ .

### 3.1.2 Utility maximization

We assume worldwide complete financial markets. The flow budget constraint for the home household is

$$P_t C_t + E_t[Q_{t,t+1} W_{t+1}] = W_t + \int_{N_H} [w_t(i) \ell_t(i) + \Pi_t(i)] di + T_t, \quad (5)$$

where  $E_t$  is the conditional expectation operator,  $Q_{t,t+1}$  is the stochastic discount factor between dates  $t$  and  $t+1$  for nominal payoffs in the home country,  $W_{t+1}$  is the portfolio of one-period state-contingent bonds,  $w_t(i)$  is the date- $t$  nominal wage rate for type  $i \in N_H$  labor,  $\Pi_t(i)$  is the date- $t$  nominal profits from sales of good  $i \in N_H$ , and  $T_t$  is the nominal lump-sum transfer from the home government. Given the initial asset holding,  $W_0$ , the home household maximizes its lifetime utility as expressed in equation (1) subject to equation (5).

The flow budget constraint for the foreign household is expressed analogously as:

$$\mathcal{E}_t P_t^* C_t + \mathbb{E}_t[Q_{t,t+1} \mathcal{E}_{t+1} W_{t+1}^*] = \mathcal{E}_t W_t^* + \int_{N_F} \mathcal{E}_t [w_t^*(i) \ell_t^*(i) + \Pi_t^*(i)] di + T_t^*,$$

where  $W_{t+1}^*$  is the portfolio of state-contingent bonds in the foreign currency,  $w_t^*(i)$  is the nominal wage rate for type  $i \in N_F$  labor,  $\Pi_t^*(i)$  is the nominal profit from sales of good  $i \in N_F$ , and  $T_t^*$  is the nominal lump-sum transfer from the foreign government. Given the initial asset holding  $W_0^*$ , the utility maximization problem for the foreign household is defined as for the home household.

The first-order conditions that  $\{C_t, C_t^*, \ell_t(i), \ell_t^*(i)\}$  must satisfy are given by

$$\frac{\beta \tilde{u}_c(C_{t+1})}{\tilde{u}_c(C_t)} = \frac{\beta \tilde{u}_c(C_{t+1}^*)}{\tilde{u}_c(C_t^*)} = Q_{t,t+1} \frac{P_{t+1}}{P_t},$$

and

$$\frac{1}{n_H} \frac{\tilde{v}_\ell[\ell_t(i)]}{\tilde{u}_c(C_t)} = \frac{w_t(i)}{P_t}, \quad i \in N_H, \quad \frac{1}{n_F} \frac{\tilde{v}_\ell[\ell_t^*(i)]}{\tilde{u}_c(C_t^*)} = \frac{w_t^*(i)}{P_t^*}, \quad i \in N_F.$$

Here,  $\tilde{u}_c(C_{t+1})$  denotes the partial derivative of  $\tilde{u}(C_{t+1})$  with respect to  $C_{t+1}$ . We use corresponding notation for other derivatives. As we shall see, policy makers' stabilization efforts turn out to be best targeted not at  $P_t$  and  $P_t^*$ , but at  $P_{H,t}$  and  $P_{F,t}^*$ . Thus it is more convenient to rewrite the first-order conditions in terms of  $C_{j,t}$  and  $C_{j,t}^*$ ,  $j = N, H$ :

$$\frac{\beta u_H(C_{H,t+1}, C_{F,t+1})}{u_H(C_{H,t}, C_{F,t})} = \frac{\beta u_H(C_{H,t+1}^*, C_{F,t+1}^*)}{u_H(C_{H,t}^*, C_{F,t}^*)} = Q_{t,t+1} \frac{P_{H,t+1}}{P_{H,t}}, \quad (6)$$

$$\frac{\beta u_F(C_{H,t+1}, C_{F,t+1})}{u_F(C_{H,t}, C_{F,t})} = \frac{\beta u_F(C_{H,t+1}^*, C_{F,t+1}^*)}{u_F(C_{H,t}^*, C_{F,t}^*)} = Q_{t,t+1} \frac{P_{F,t+1}}{P_{F,t}}, \quad (7)$$

$$\frac{1}{n_H} \frac{\tilde{v}_\ell[\ell_t(i)]}{u_H(C_{H,t}, C_{F,t})} = \frac{w_t(i)}{P_{H,t}}, \quad (8)$$

$$\frac{1}{n_F} \frac{\tilde{v}_\ell[\ell_t^*(i)]}{u_F(C_{H,t}^*, C_{F,t}^*)} = \frac{w_t^*(i)}{P_{F,t}^*}, \quad (9)$$

$$\frac{u_F(C_{H,t}, C_{F,t})}{u_H(C_{H,t}, C_{F,t})} = \frac{u_F(C_{H,t}^*, C_{F,t}^*)}{u_H(C_{H,t}^*, C_{F,t}^*)} = \frac{\mathcal{E}_t P_{F,t}^*}{P_{H,t}}. \quad (10)$$

### 3.1.3 Equilibrium shares of consumption

Under the standard assumption that the representative households of the two countries are equally wealthy in the initial period, their equilibrium consumption levels are identical for all  $t$ :

$$C_{H,t} = C_{H,t}^*, \quad C_{F,t} = C_{F,t}^*, \quad C_t = C_t^*.$$

Let  $y_t(i)$ ,  $i \in N_H$ , and  $y_t^*(i)$ ,  $i \in N_F$ , denote the aggregate supply of home and foreign goods, respectively:

$$y_t(i) = n_H c_t(i) + n_F c_t^*(i), \quad i \in N_H, \quad y_t^*(i) = n_H c_t(i) + n_F c_t^*(i), \quad i \in N_F.$$

The corresponding production indexes for home and foreign goods are

$$\begin{aligned} Y_{H,t} &\equiv \left[ n_H^{-\frac{1}{\theta}} \int_{N_H} y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = n_H C_{H,t} + n_F C_{H,t}^*, \\ Y_{F,t} &\equiv \left[ n_F^{-\frac{1}{\theta}} \int_{N_F} y_t^*(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} = n_H C_{F,t} + n_F C_{F,t}^*, \\ Y_t &\equiv \left( \frac{Y_{H,t}}{n_H} \right)^{n_H} \left( \frac{Y_{F,t}}{n_F} \right)^{n_F} = n_H C_t + n_F C_t^*. \end{aligned}$$

It follows that

$$C_{H,t} = C_{H,t}^* = Y_{H,t}, \quad C_{F,t} = C_{F,t}^* = Y_{F,t}, \quad C_t = C_t^* = Y_t. \quad (11)$$

## 3.2 Aggregate supply

### 3.2.1 Technology

For simplicity, we assume that the technology to produce each good is linear in labor:

$$\begin{aligned} y_t(i) &= A_t n_H \ell_t(i), \quad i \in N_H, \\ y_t^*(i) &= A_t^* n_F \ell_t^*(i), \quad i \in N_F, \end{aligned}$$

where  $A_t$  and  $A_t^*$  represent country-specific technology shocks.

For later use, it is convenient to define random variables  $\xi_t$  and  $\xi_t^*$  by

$$\xi_t \equiv -(1 + \omega) \ln A_t, \quad \xi_t^* \equiv -(1 + \omega) \ln A_t^*,$$

and also functions  $v(y; \xi)$  and  $v^*(y^*; \xi^*)$  by

$$\begin{aligned} v(y; \xi) &\equiv \frac{e^\xi}{1 + \omega} \left( \frac{y}{n_H} \right)^{1+\omega} = \tilde{v} \left( \frac{y}{n_H A} \right), \\ v^*(y^*; \xi^*) &\equiv \frac{e^{\xi^*}}{1 + \omega} \left( \frac{y^*}{n_F} \right)^{1+\omega} = \tilde{v} \left( \frac{y^*}{n_F A^*} \right). \end{aligned}$$

Thus,  $v(y; \xi)$  and  $v^*(y^*; \xi^*)$  measure the disutility of producing  $y$  and  $y^*$  in the home and foreign countries, respectively, when their technology shocks are  $\xi$  and  $\xi^*$ . Note that

$$v_y(y; \xi) = \frac{\tilde{v}_\ell(\ell)}{n_H A}, \quad v_y^*(y^*; \xi^*) = \frac{\tilde{v}_\ell(\ell^*)}{n_F A^*}.$$

The first-order conditions (8) and (9) can then be rewritten as

$$\frac{v_y[y_t(i); \xi_t]}{u_H(Y_{H,t}, Y_{F,t})} = \frac{1}{A_t} \frac{w_t(i)}{P_{H,t}}, \quad \frac{v_y^*[y_t^*(i); \xi_t^*]}{u_F(Y_{H,t}, Y_{F,t})} = \frac{1}{A_t^*} \frac{w_t^*(i)}{P_{F,t}^*}, \quad (12)$$

where we have used equilibrium conditions given by equations (11).

### 3.2.2 Natural rates of output

Each producer takes the wage rate as given.<sup>3</sup> Using the equations (11) and the household's demand function for  $c_t(i)$ , which cost minimization determines as

$$c_t(i) = \frac{1}{n_j} C_{j,t} \left( \frac{p_t(i)}{P_{j,t}} \right)^{-\theta}, \quad j = H, F,$$

the nominal profits of a home supplier of good  $i \in N_H$  at date  $t$  are given by

$$\left[ (1 - \Omega) p_t(i) - \frac{w_t(i)}{A_t} \right] y_t(i) = \left[ (1 - \Omega) p_t(i) - \frac{w_t(i)}{A_t} \right] \frac{Y_{H,t}}{n_H} \left[ \frac{p_t(i)}{P_{H,t}} \right]^{-\theta} = n_H \Pi_t(i),$$

where  $\Omega$  is the constant tax rate on firms' revenue. The monopoly profits of a foreign firm are defined similarly with  $\Omega^*$  as the tax rate on its revenue.

Let us define the “natural rates of output” (Woodford, 2003) at date  $t$ ,  $Y_{H,t}^n$  and  $Y_{F,t}^n$ , as the levels of home and foreign output which would prevail in the flexible-price equilibrium. Suppose, momentarily, that all prices are fully flexible. Profit maximization leads to

$$\begin{aligned} (1 - \Phi) \frac{p_t(i)}{P_{H,t}} &= \frac{w_t(i)}{P_{H,t} A_t} = \frac{v_y[y_t(i); \xi_t]}{u_H(Y_{H,t}, Y_{F,t})}, & i \in N_H, \\ (1 - \Phi^*) \frac{p_t^*(i)}{P_{F,t}^*} &= \frac{w_t^*(i)}{P_{F,t}^* A_t^*} = \frac{v_y^*[y_t^*(i); \xi_t^*]}{u_F(Y_{H,t}, Y_{F,t})}, & i \in N_F, \end{aligned}$$

where we have used equations (12) and  $\Phi$  and  $\Phi^*$  are the measures of distortion due to market power defined by

$$1 - \Phi = \frac{\theta - 1}{\theta} (1 - \Omega), \quad 1 - \Phi^* = \frac{\theta - 1}{\theta} (1 - \Omega^*).$$

---

<sup>3</sup>See Woodford (2003, Section 3.1) for how to make this assumption consistent with the supposition that each producer uses a different type of labor.

When  $\Omega$  and  $\Omega^*$  are set so that  $\Phi = \Phi^* = 0$ , the flexible-price equilibrium is efficient.

In the flexible-price equilibrium,  $p_t(i) = P_{H,t}$  and  $y_t(i) = Y_{H,t}/n_H$  for all  $i \in N_H$ , and  $p_t^*(i) = P_{F,t}^*$  and  $y_t^*(i) = Y_{F,t}/n_F$  for all  $i \in N_F$ . Thus, the natural rates of output,  $Y_{H,t}^n$  and  $Y_{F,t}^n$ , are determined by

$$\frac{v_y[Y_{H,t}^n/n_H; \xi_t]}{u_H(Y_{H,t}^n, Y_{F,t}^n)} = 1 - \Phi, \quad \frac{v_y^*[Y_{F,t}^n/n_F; \xi_t^*]}{u_F(Y_{H,t}^n, Y_{F,t}^n)} = 1 - \Phi^*. \quad (13)$$

### 3.3 New Keynesian aggregate supply relations

Now suppose that goods prices are adjusted at random intervals as in Calvo (1983). Let  $\alpha$  be the probability that each good price remains unchanged in each period. We assume that this probability is identical in the two countries.

Consider the price adjustment in the home country. Suppose that the price of good  $i \in N_H$  can be adjusted at date  $t$ . The supplier of that good chooses  $p_t(i)$  to maximize its expected discounted profits:

$$\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ \left[ (1 - \Omega)p_t(i) - \frac{w_T(i)}{A_T} \right] \frac{Y_{H,T}}{n_H} \left[ \frac{p_t(i)}{P_{H,T}} \right]^{-\theta} \right\}.$$

The first-order condition for profit maximization is written as

$$\mathbb{E}_t \sum_{T=t}^{\infty} \alpha^{T-t} Q_{t,T} \left\{ \frac{Y_{H,T}}{n_H} \left( \frac{p_t(i)}{P_{H,T}} \right)^{-\theta-1} \left[ \frac{v_y[y_T(i); \xi_T]}{u_H(Y_{H,T}, Y_{F,T})} - (1 - \Phi) \frac{p_t(i)}{P_{H,T}} \right] \right\} = 0. \quad (14)$$

It follows that all producers that change their prices at date  $t$  choose the same price.

Log-linearizing equation (14) and the corresponding equation for the foreign country lead to the “New Keynesian” aggregate-supply relations:

$$\pi_{H,t} = \gamma_H x_{H,t} + \gamma_{HF} n_F x_{F,t} + \beta \mathbb{E}_t \pi_{H,t+1}, \quad (15)$$

$$\pi_{F,t}^* = \gamma_{HF} n_H x_{H,t} + \gamma_F x_{F,t} + \beta \mathbb{E}_t \pi_{F,t+1}^*. \quad (16)$$

Here  $\pi_{H,t} \equiv \ln P_{H,t} - \ln P_{H,t-1}$  and  $\pi_{F,t}^* \equiv \ln P_{F,t}^* - \ln P_{F,t-1}^*$  are the inflation rates for goods produced in the home and foreign countries, respectively;  $x_{j,t} \equiv \ln Y_{j,t} - \ln Y_{j,t}^n$  is the

“output gap” in country  $j = H, F$ ; and the coefficients are given by

$$\begin{aligned}\gamma_H &\equiv \zeta[1 + \omega + (\sigma - 1)n_H] > 0, \\ \gamma_{HF} &\equiv \zeta(\sigma - 1), \\ \gamma_F &\equiv \zeta[1 + \omega + (\sigma - 1)n_F] > 0, \\ \zeta &\equiv \frac{1 - \alpha}{\alpha} \frac{1 - \alpha\beta}{1 + \omega\theta}.\end{aligned}\tag{17}$$

### 3.4 Welfare approximation

When the two countries coordinate their policies, our welfare criterion for evaluating these is the average lifetime utility of the representative agents in the two countries:

$$n_H U_0 + n_F U_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(Y_{H,t}, Y_{F,t}) - \int_{N_H} v[y_t(i); \xi_t] di - \int_{N_F} v^*[y_t^*(i); \xi_t^*] di \right\}. \tag{18}$$

Specifically, we shall work with a quadratic loss function derived from a second-order approximation of (18) following Woodford (2003) and Benigno and Woodford (2005), among others.

Consider a non-stochastic steady state with zero inflation, and assume for simplicity that  $\Phi = \Phi^* = 0$ . Then, as shown in the Appendix, a second-order approximation of the world welfare (18) around the zero-inflation steady state is given by

$$n_H U_0 + n_F U_0^* \approx -\nu_0 E_0 \sum_{t=0}^{\infty} \beta^t L_t + \nu_1,$$

where  $\nu_0$  and  $\nu_1$  are constants independent of policy, and  $L_t$  is a quadratic measure of the world-welfare loss given by

$$L_t \equiv \frac{1}{2} x_t' \Lambda x_t + \frac{n_H}{2} \pi_{H,t}^2 + \frac{n_F}{2} \pi_{F,t}^{*2}. \tag{19}$$

Here,  $x_t \equiv (x_{H,t}, x_{F,t})'$ , and

$$\Lambda \equiv \frac{1}{\theta} \begin{bmatrix} \gamma_H n_H & \gamma_{HF} n_H n_F \\ \gamma_{HF} n_H n_F & \gamma_F n_F \end{bmatrix}. \tag{20}$$

Thus, up to a second-order approximation, our welfare criterion is given by the expected loss function:

$$\mathcal{L}_\tau \equiv E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} L_t, \tag{21}$$

where  $\tau$  is the period in which policies are evaluated<sup>4</sup>.

Our welfare measure clearly shows that what must be stabilized are the PPI inflation rates,  $\pi_{H,t}$  and  $\pi_{F,t}^*$ , rather than the consumer price index (CPI) inflation rates,  $\pi_t$  and  $\pi_t^*$  defined as  $\pi_t \equiv \ln P_t - \ln P_{t-1}$  and  $\pi_t^* \equiv \ln P_t^* - \ln P_{t-1}^*$ . An equivalent result is shown by Clarida, Gali, and Gertler (2001) as well as by Benigno and Benigno (2003), in different contexts.

### 3.5 Real interest rates in open economies

In an economy where a single good is produced and consumed, it is straightforward to define the real interest rate. If different commodities are produced and consumed, it becomes less obvious how to define the real interest rate. In the standard closed-economy New Keynesian model, a variety of differentiated commodities are produced and consumed, but there is still no ambiguity in the definition of the real interest rate, because every household consumes the same basket of goods, which also coincides with the basket of goods produced in the economy. There the CPI-based real interest rate coincides with the PPI-based real interest rate.

Since the real interest rate is defined unambiguously in the closed economy, so is the “natural rate of interest”: it is defined as the real interest rate in the flexible-price equilibrium. With sticky prices, to a first-order approximation, the inflation rate is completely eliminated if the nominal interest rate, the real interest rate, and the natural rate of interest are all equalized. As discussed by Eggertsson and Woodford (2003), however, if the natural interest rate becomes negative, the zero lower bound condition for the nominal interest rate binds in the optimal policy problem, and it is no longer possible to completely stabilize the inflation rate.

The corresponding notions in the open economy framework are less clear, however,

---

<sup>4</sup>As indicated by the quadratic world-welfare loss (19) à la Eggertsson and Woodford (2003), our model abstracts from the other possible avenue through which deflation impairs the economy than price dispersions. Those include debt-deflation which may be important during the great recession. If the welfare loss associated with debt-deflation is also considered, the zero lower bound policy could last longer than that proposed in the current model.

even in our specification where representative individuals in the two countries consume exactly the same basket of goods in equilibrium and both have access to the complete set of state-contingent claims at the same prices. The reason is that the two countries produce different baskets of goods. It follows that each country has a distinct PPI-based real interest rate, which is also different from the CPI-based real interest rate. Thus, the question of which real interest rate should be relevant to the monetary authority is now non-trivial. Naturally, the answer depends on its chosen objective. Here, the objective of the monetary authority is to minimize the expected world-welfare loss function (21), and thus to stabilize the PPI inflation rates,  $\pi_{H,t}$  and  $\pi_{F,t}^*$ . It follows that the relevant real interest rates are the PPI-based rates.

Let  $R_{H,t}$  be the gross real interest rate associated with the home-produced composite good  $Y_H$ :

$$R_{H,t} \equiv \left\{ E_t \left[ \frac{\beta u_H(Y_{H,t+1}, Y_{F,t+1})}{u_H(Y_{H,t}, Y_{F,t})} \right] \right\}^{-1}.$$

Similarly, let  $R_{F,t}$  be the gross real interest rate associated with the foreign-produced composite good  $Y_F$ :

$$R_{F,t} \equiv \left\{ E_t \left[ \frac{\beta u_F(Y_{H,t+1}, Y_{F,t+1})}{u_F(Y_{H,t}, Y_{F,t})} \right] \right\}^{-1}.$$

Given that the PPI inflation rates are the ones to be stabilized, the relevant natural rates of interest are also PPI-based: these are defined as the PPI-based real interest rates that obtain in equilibrium when prices in both countries are assumed to be flexible:

$$R_{H,t}^n \equiv \left\{ E_t \left[ \frac{\beta u_H(Y_{H,t+1}^n, Y_{F,t+1}^n)}{u_H(Y_{H,t}^n, Y_{F,t}^n)} \right] \right\}^{-1}, \quad (22)$$

$$R_{F,t}^n \equiv \left\{ E_t \left[ \frac{\beta u_F(Y_{H,t+1}^n, Y_{F,t+1}^n)}{u_F(Y_{H,t}^n, Y_{F,t}^n)} \right] \right\}^{-1}. \quad (23)$$

Here the natural rates of output,  $Y_{H,t}^n$  and  $Y_{F,t}^n$ , are defined by the productivity shocks,  $\xi_t$  and  $\xi_t^*$ , as shown in equations (13). It follows that the natural rates of interest,  $R_{H,t}^n$  and  $R_{F,t}^n$ , are also completely determined by the exogenous productivity shocks.<sup>5</sup>

---

<sup>5</sup>Notice that we could also define the CPI-based natural rate of interest, which turns out to be unique and the same across the two countries. However, it is not relevant to the determination of monetary policy in our model and so is omitted here.

Let  $r_{H,t} \equiv \ln R_{H,t}$  and  $r_{F,t} \equiv \ln R_{F,t}$ . Then a first-order approximation of the above equations yields

$$r_{H,t} = E_t \left\{ [1 + (\sigma - 1)n_H](x_{H,t+1} - x_{H,t}) + (\sigma - 1)n_F(x_{F,t+1} - x_{F,t}) \right\} + r_{H,t}^n, \quad (24)$$

$$r_{F,t} = E_t \left\{ [1 + (\sigma - 1)n_F](x_{F,t+1} - x_{F,t}) + (\sigma - 1)n_H(x_{H,t+1} - x_{H,t}) \right\} + r_{F,t}^n, \quad (25)$$

where  $r_{H,t}^n \equiv \ln R_{H,t}^n$  and  $r_{F,t}^n \equiv \ln R_{F,t}^n$ .

Following convention, the dynamic IS relations are obtained by log-linearizing the Euler equations and taking the conditional expectation. Here, since the PPIs are the inflation rates that the monetary authorities should watch, the relevant Euler equations are (6)-(7), and hence the corresponding IS relations become:

$$i_{H,t} = E_t \left\{ [1 + (\sigma - 1)n_H](x_{H,t+1} - x_{H,t}) + (\sigma - 1)n_F(x_{F,t+1} - x_{F,t}) + \pi_{H,t+1} \right\} + r_{H,t}^n, \quad (26)$$

$$i_{F,t} = E_t \left\{ [1 + (\sigma - 1)n_F](x_{F,t+1} - x_{F,t}) + (\sigma - 1)n_H(x_{H,t+1} - x_{H,t}) + \pi_{F,t+1}^* \right\} + r_{F,t}^n, \quad (27)$$

where  $i_{H,t}$  and  $i_{F,t}$  are the logs of the gross nominal interest rates. Note that equations (24) and (25) coincide with equations (26) and (27), respectively, because  $r_{H,t} = i_{H,t} - E_t \pi_{H,t+1}$  and  $r_{F,t} = i_{F,t} - E_t \pi_{F,t+1}^*$ . The zero bounds for the nominal interest rates are

$$i_{H,t} \geq 0, \quad (28)$$

$$i_{F,t} \geq 0. \quad (29)$$

A competitive equilibrium attains the first best outcome (up to a first-order approximation) if

$$\pi_{H,t} = \pi_{F,t}^* = x_{H,t} = x_{F,t} = 0,$$

at all dates and under all contingencies. Given equations (26) and (27), the nominal interest rates in such an equilibrium are equal to the PPI-based natural rates of interest:

$$i_{j,t} = r_{j,t}^n, \quad j = H, F.$$

In this sense, our definition of the natural rates of interest, equations (22) and (23), is a natural extension of the one used by Eggertsson and Woodford (2003) for the closed economy.

The fact that the natural rates of interest relevant to welfare are those based on the PPI inflation rate as in equations (22)-(23) explains why it is possible for the two countries to fall separately into liquidity traps even though the international asset market is complete. Here, representative agents in the two countries trade the complete set of state contingent claims at the same prices. For any given basket of goods, therefore, both the corresponding real interest rate and the corresponding natural rate of interest are identical for individuals in different countries. Thus the situation where just one of the countries is caught in a liquidity trap does not arise because agents in different countries face different real interest rates; rather, it arises because the two monetary authorities are watching different real interest rates and hence different natural rates of interest. Optimal monetary policy requires the monetary authority in each country to control the real interest rate based on the PPI; it does so by choosing a policy interest rate that takes into account the natural rate of interest defined in terms of that country's PPI, as shown in equations (24)-(25).

## 4 Optimal Monetary Policy in the face of a Global Liquidity Trap

### 4.1 Optimal Monetary Policy in the face of a Global Liquidity Trap

In this section we analyze the equilibrium when policy is optimal and coordinated with the optimal policy coordination.<sup>6</sup> Specifically, suppose that, at some date  $\tau$ , the two monetary authorities coordinate with each other and choose their policies with perfect commitment

---

<sup>6</sup>Existing studies on optimal monetary policy in open economies, such as Clarida, Gali, and Gertler (2001) and Benigno and Benigno (2003), investigate the Nash equilibrium along with the cooperative equilibrium. In this paper, we focus on the latter equilibrium because the examination of the Nash equilibrium gives a substantially different welfare measure.

In the Nash equilibrium, the optimal policy interest rate in the home country is chosen so as to maximize the utility of representative agents in the home country  $H$  given by equation (1). Thus, an optimizer in the home country no longer considers the welfare of representative agents in the foreign country.

in order to achieve an equilibrium that maximizes world welfare. The equilibrium in this case is obtained by solving the Ramsey problem, that is, by minimizing the world-welfare loss function (21) subject to the constraints (15), (16), (26), (27), (28), and (29). This equilibrium is referred to as the Ramsey equilibrium.

Consider first the case in which the zero bound conditions for the nominal interest rates, equations (28) and (29), never bind. Then the Ramsey equilibrium can be implemented by the following targeting rules:

$$\pi_{H,t} + \frac{1}{\theta} (x_{H,t} - x_{H,t-1}) = 0, \quad (30)$$

$$\pi_{F,t}^* + \frac{1}{\theta} (x_{F,t} - x_{F,t-1}) = 0. \quad (31)$$

These rules are inward looking in the sense that the monetary authority in each country only needs to look at the inflation rate and the output gap in its own country. Thus, as long as the zero bound conditions for the nominal interest rates do not bind, world welfare is maximized by a purely inward-looking policy.<sup>7</sup> This point has been previously made, for instance, by Clarida, Gali, and Gertler (2001). Note that this inward-looking feature of the optimal monetary policy does not depend on the value of  $\sigma$ .

However, the optimal monetary policy can no longer be described by inward-looking

---

Benigno and Benigno (2003) show that a quadratic measure of welfare includes linear terms for consumption and output. They conclude that the elimination of these linear terms requires the assumption of perfect price stability through the game theoretic strategies of the two central banks. In the face of a liquidity trap, however, perfect price stability cannot be attained. The linear term in the welfare measure makes the optimal monetary policy analysis impossible.

Alternatively, Clarida, Gali, and Gertler (2001) propose treating the foreign variables as given constants. This allows the following form of loss function to be derived:

$$L_t \equiv \sum_{t=0}^{\infty} \beta^t \frac{1}{2} (\gamma_H x_{H,t}^2 + \theta \pi_{H,t}^2).$$

While this quadratic measure of the welfare loss contains only home variables, comparison between the Nash equilibrium and the cooperative equilibrium involves a new difficulty because the sum of the quadratic measures of welfare loss in the two countries does not coincide with equation (19).

<sup>7</sup>Our assumption of producer currency pricing is also crucial for this result. As shown by Devereux and Engel (2003), under local currency pricing, the optimal monetary policy aims at stabilizing the nominal exchange rate and therefore takes foreign variables into consideration. Investigation of the optimal monetary policy under local currency pricing is left for our future research.

rules if the zero lower bound conditions bind with a positive probability. Even with the producer currency pricing, foreign variables must be included in the domestic targeting rule. The degree of influence from foreign variables is determined by  $\sigma$ . Denoting the Lagrange multipliers associated with inequalities (28) and (29) by  $\phi_{H,t}$  and  $\phi_{F,t}$ , the first order conditions under a commitment policy yield the following targeting rules:

$$\pi_{H,t} + \frac{1}{\theta} (x_{H,t} - x_{H,t-1}) = z_{H,t}, \quad (32)$$

$$\pi_{F,t}^* + \frac{1}{\theta} (x_{F,t} - x_{F,t-1}) = z_{F,t}, \quad (33)$$

where  $z_{H,t}$  and  $z_{F,t}$  are defined by

$$\begin{bmatrix} z_{H,t} \\ z_{F,t} \end{bmatrix} = Z(L) \begin{bmatrix} \phi_{H,t} \\ \phi_{F,t} \end{bmatrix}. \quad (34)$$

Here  $L$  is the lag operator and  $Z(L)$  is given by

$$Z(L) \equiv - \begin{bmatrix} \frac{\sigma + \omega + (\sigma - 1)\omega n_H}{\zeta(1 + \omega)(\omega + \sigma)} & \frac{(\sigma - 1)\omega n_F}{\zeta(1 + \omega)(\omega + \sigma)} \\ \frac{(\sigma - 1)\omega n_H}{\zeta(1 + \omega)(\omega + \sigma)} & \frac{\sigma + \omega + (\sigma - 1)\omega n_F}{\zeta(1 + \omega)(\omega + \sigma)} \end{bmatrix} (1 - L)(1 - \beta^{-1}L) + \begin{bmatrix} \beta^{-1} & 0 \\ 0 & \beta^{-1} \end{bmatrix} L. \quad (35)$$

Comparing the targeting rules (30)-(31) and (32)-(33), we see that when the zero bound binds, the effect is summarized by the term  $z_t = (z_{H,t}, z_{F,t})$ . Suppose that country  $j \in \{H, F\}$  is in a liquidity trap in some period  $\hat{t}$ , so that  $\phi_{j,\hat{t}} > 0$ . Then it affects  $z_t$  for three periods:  $t = \hat{t}, \hat{t} + 1, \hat{t} + 2$ , as shown by equation (35). If  $\phi_{H,t} = \phi_{F,t} = 0$  for all  $t$ , the optimal targeting rules (32)-(33) reduce to the inward-looking rules (30)-(31).

To understand better the effect of a liquidity trap on the optimal policy, Figure 2 plots how  $z_H$  and  $z_F$  respond to a one-time increase in  $\phi_H$  for different values of  $\sigma$  in equation (34). Specifically it shows how  $z_{H,t}$  and  $z_{F,t}$  vary when  $\phi_{H,t} = 0$  for all  $t \neq 1$  and  $\phi_{H,1} = 1$  with  $\phi_{F,t} = 0$  for all  $t$ .<sup>8</sup> Let us look at the top panel, which shows how the optimal targeting rule for the home country is affected when the home country falls into a liquidity trap in period 1. In the period that the zero bound binds, the monetary authority has to allow for deflation and a negative output gap, so that the targeting rule shifts downward:  $z_{H,1} < 0$ . However, such a downward shift in the targeting rule is alleviated by promising an upward

---

<sup>8</sup>The parameter values used to plot the figure are summarized in Table 1, which is discussed in Section 4.2.

shift in the targeting rule in the future,  $z_{H,2} > 0$ . In other words, a country caught in a liquidity trap can reduce the damage it sustains if the monetary authority commits itself to generating some inflation and positive output gaps in the future. This feature of the optimal monetary policy is the *history dependence* that is emphasized in previous studies on the closed economy, such as Eggertsson and Woodford (2003) and Jung, Teranishi, and Watanabe (2005).

The possibility of a global liquidity trap adds an additional feature to the optimal policy: *international dependence*. Mathematically, such interdependence can be seen by the fact that the country-specific Lagrange multipliers on the zero bound constraints,  $\phi_{H,t}$  and  $\phi_{F,t}$ , each affect both  $z_{H,t}$  and  $z_{F,t}$  as shown in equation (34) provided that  $\sigma \neq 1$ .<sup>9</sup> For instance, if the home country is in a liquidity trap in period  $t$ , then  $\phi_{H,t} > 0$ ; this will affect not only the home country's targeting rule (32), but also the foreign country's rule (33) through its influence on  $z_{H,t}$  and  $z_{F,t}$ . The optimal rate of inflation for each country is affected by whether or not the other country is caught in a liquidity trap. Economic efficiency is no longer attained simply by 'keeping one's house in order.'

The lower panel of Figure 2 shows how a liquidity trap in the home country affects the optimal targeting rule for the foreign country. The direction of the effect depends on whether  $\sigma$  is greater or less than unity. This follows from the fact that the source of the international dependence in our model is the dependence of the marginal utility from consuming the composite good produced in one country on the consumption of the composite good produced in the other country. When  $\sigma > 1$ , however, home goods and foreign goods are Edgeworth substitutes, i.e.,  $u_{HF} = u_{FH} < 0$ . The marginal utility of the consumption of the composite good produced in one country is affected in the same direction by the consumption of the composite good produced in either country, because  $u_{HH} < 0$  and  $u_{FF} < 0$ . Thus, in this case, a shift of the optimal targeting rule in one country is transmitted into a shift of the optimal targeting rule in the other country in the

---

<sup>9</sup>It is clear from equation (34) that if  $\sigma = 1$ , then  $\phi_j$  only affects  $z_j$  for each  $j = H, F$ . Thus, the targeting rules (32)-(33) do not exhibit the form of international dependence discussed here. In what follows, whenever we emphasize the international dependence of the optimal monetary policy, we are implicitly assuming that  $\sigma \neq 1$ .

same direction. This can be seen in the figure that  $\phi_{H,t}$  affects  $z_{F,t}$  and  $z_{H,t}$  in the same direction when  $\sigma = 2$ . To the contrary, when  $\sigma < 1$ , home goods and foreign goods are Edgeworth complements:  $u_{HF} = u_{FH} > 0$ . Thus, the marginal utility of the consumption of goods produced in each country is affected in the opposite directions by the consumption of goods produced in the two countries. As a result, the optimal targeting rule in the two countries shift in the *opposite* directions. This is consistent with the figure in the case of  $\sigma = 0.5$ .

A further insight into how monetary policy should be conducted in a global liquidity trap is obtained by looking at the dynamic IS curves (26)-(27) with the zero bound conditions (28)-(29). First, suppose that the natural rate of interest associated with the home good is negative in period  $t_0$ ,  $r_{H,t_0}^n < 0$ , so that the home country is in a liquidity trap:

$$0 = i_{H,t_0} = r_{H,t_0} + E_{t_0}\pi_{H,t_0+1},$$

where  $r_{H,t_0}$  denotes the real interest rate associated with the home good as defined in equation (24).

The optimal policy attempts to relax the degree to which the zero constraint binds. There are several ways to do this. One way is for the monetary authority in the home country to commit to future stimulation of the home economy once the natural rate returns to a positive level. Such a commitment makes  $E_{t_0}\pi_{H,t_0+1} > 0$  and  $E_{t_0}(x_{H,t_0+1} - x_{H,t_0}) > 0$ . Both of these would offset at least partially the depressing effect of the negative shock to the home natural rate. Additionally, if the foreign monetary authority also commits to achieve  $(\sigma - 1)E_{t_0}(x_{F,t_0+1} - x_{F,t_0}) > 0$ , then the zero constraint for the home country would be relaxed further. Thus, if  $\sigma > 1$  (respectively, if  $\sigma < 1$ ), a future expansion (contraction) of the foreign economy helps alleviate the severity of the current liquidity trap for the home economy. In this way, policy commitment by each of the two monetary authorities acts to reduce the welfare loss associated with the home country's liquidity trap.

Next suppose that the natural rate in the home country returns to a positive level in period  $t_1 > t_0$ . The IS curve for the foreign country is

$$i_{F,t_1} = r_{F,t_1} + E_{t_1}\pi_{F,t_1+1}^*,$$

where  $r_{F,t_1}$  denotes the real interest rate associated with the foreign good defined in (25). Given the home monetary authority's policy commitment, the home economy experiences a temporary boom in period  $t_1$ ,  $x_{H,t_1} > 0$ , which implies that  $E_{t_1}(x_{H,t_1+1} - x_{H,t_1}) < 0$ . From the perspective of the foreign monetary authority, if  $\sigma > 1$  ( $\sigma < 1$ ) this constitutes a negative (positive) shock to the real interest rate  $r_{F,t_1}$ . Thus, for  $\sigma > 1$  (for  $\sigma < 1$ ), the foreign monetary authority tends to lower (raise)  $i_{F,t}$  when the home natural rate,  $r_{H,t}$ , becomes positive. Notice also that such a response by the foreign monetary authority tends to raise (lower)  $x_{F,t_1}$  when  $\sigma > 1$  ( $\sigma < 1$ ); this is consistent with the foreign monetary authority's commitment to generate  $(\sigma - 1)E_{t_0}(x_{F,t_0+1} - x_{F,t_0}) > 0$  during periods when the home country is in the liquidity trap.

## 4.2 Numerical example

In order to further analyze the properties of the optimal policy, let us consider a numerical example, which extends the closed-economy experiment of Eggertsson and Woodford (2003) to our open-economy environment. The parameters assumed here are summarized in Table 1. Suppose that in the initial period  $t = 0$ , the world economy is in the steady state where the natural rate is  $\bar{r}^n = \frac{1-\beta}{\beta}$ , and the inflation rates and the output gaps are all zero:  $\bar{\pi}_H = \bar{\pi}_F^* = \bar{x}_H = \bar{x}_F = 0$ . Then, in period  $t = 1$ , the natural rates of interest in both countries drop unexpectedly to a negative level  $\underline{r}^n < 0$ . These negative natural rate shocks are temporary, and we assume that the natural rates evolve according to the following stochastic process: (i)  $r_{H,1}^n = r_{F,1}^n = \underline{r}^n$ ; (ii) if  $r_{H,t}^n = \underline{r}^n$ , then

$$r_{H,t+1}^n = \begin{cases} \bar{r}^n, & \text{with probability } p_t, \\ \underline{r}^n, & \text{with probability } 1 - p_t, \end{cases}$$

where  $p_t = p$  for  $1 \leq t \leq \bar{S} - 1$  and  $p_t = 1$  for  $t \geq \bar{S}$ ; (iii) if  $r_{F,t}^n = \underline{r}^n$ , then

$$r_{F,t+1}^n = \begin{cases} \bar{r}^n, & \text{with probability } q_t, \\ \underline{r}^n, & \text{with probability } 1 - q_t, \end{cases}$$

where  $q_t = q$  for  $1 \leq t \leq \bar{S}$  and  $q_t = 1$  for  $t > \bar{S}$ ; (iv) if  $r_{j,t}^n = \bar{r}^n$ , then  $r_{j,t+1}^n = \bar{r}^n$  with probability one, for  $j = H, F$  and for all  $t > 1$ . Here,  $\bar{S}$  is a large positive integer that

determines the maximal number of periods for which a country's natural rate may remain negative.

Let  $T_H$  and  $T_F$  be the stopping times defined respectively as the last periods in which  $r_{H,t}^n = \underline{r}^n$  and  $r_{F,t}^n = \underline{r}^n$ . The probability that  $(T_H, T_F) = (\tau_H, \tau_F)$  is  $(1-p)^{\tau_H-1}p(1-q)^{\tau_F-1}q$  for each  $(\tau_H, \tau_F) \in \{1, \dots, \bar{S}\}^2$ . For a given monetary policy, the equilibrium is described by a set of stochastic processes  $\{i_{H,t}, i_{F,t}, \pi_{H,t}, \pi_{F,t}^*, x_{H,t}, x_{F,t}\}_{t=1}^\infty$ , each of which is adapted to the filtration generated by the stopping times  $(T_H, T_F)$ . The optimal monetary policy chooses this set of stochastic processes so as to solve the Ramsey problem described in the previous subsection. The details of the numerical algorithm are given in the Appendix. In what follows, we examine equilibrium paths under the optimal policy associated with particular realizations of the stopping times  $(T_H, T_F)$ .

Let us begin with the symmetric case:  $T_H = T_F$ , that is, the case in which the natural rates of both economies return to the normal level  $\bar{r}$  in the same period. Figure 3 plots the paths of the nominal interest rates  $i_{H,t}$  and  $i_{F,t}$ , the inflation rates  $\pi_{H,t}$  and  $\pi_{F,t}^*$ , and the output gaps  $x_{H,t}$  and  $x_{F,t}$  for the case of  $T_H = T_F = 15$  (that is, both  $r_{H,t}^n$  and  $r_{F,t}^n$  become positive again when  $t = 16$ ). It is clear that the optimal policy exhibits the kind of history dependence discussed in the previous subsection. The nominal interest rate in each country remains set to zero for two more periods ( $t = 16, 17$ ) after its natural rate becomes positive. Correspondingly, the inflation rate and the output gap in each country become positive in period 16. As discussed in the previous subsection, such a commitment alleviates the contractionary effects from negative natural rates  $\underline{r}$  in earlier periods. With the symmetric realization of the shocks, however, it is difficult to tell the extent to which the optimal policy shows international dependence. It is more easily seen for cases when the realizations of the shocks are asymmetric.

Figure 4 depicts the case where  $T_H = 15$  and  $T_F = 10$  (that is, where  $r_{H,t}^n$  and  $r_{F,t}^n$  return to  $\bar{r}$  when  $t = 16$  and  $t = 11$ , respectively). Again, the history dependence is evident: the nominal interest rate in each country remains set to zero for a while even after its natural rate returns to normal; and in each economy both inflation rate and output gap are positive in the period its natural rate shifts from  $\underline{r}^n$  to  $\bar{r}^n$ . Furthermore,

the international dependence of the optimal policy can also be clearly seen. For instance, look at what happens to the foreign country's nominal interest rate  $i_{F,t}$  after the home country's natural rate returns to  $\bar{r}^n$  (*i.e.*,  $t = 16, 17$ ). The home country's output gap increases temporarily in period 16, as a result of which its expected growth rate from  $t$  to  $t + 1$  is negative for  $t = 16, 17$ . Given that our example has  $\sigma = 2$ , the negative growth of the home output gap works as a negative shock on the real interest rate  $r_{F,t}^n$  defined in equation (25). This is why the foreign nominal interest rate  $i_{F,t}$  declines for the periods  $t = 16, 17$ . Analogously, the negative expected growth rate implied by the foreign output gap in the period when the foreign natural rate returns to  $\bar{r}^n$  ( $t = 11$ ) acts as a negative shock on the real interest rate  $r_{H,t}$  defined in equation (24). In that period, however, the home country is still caught in a liquidity trap and the home nominal interest rate cannot be lowered further. Instead, the effect of this negative shock on  $r_{H,t}$  is mostly seen in the shape of a decline in the home output gap in period 11. Yet another form of the international dependence appears in the term  $E_t(x_{F,t+1} - x_{F,t})$  in equation (24) for periods  $t \leq T_H$ . When  $r_{F,t}^n$  returns to  $\bar{r}^n$  in period 11, the foreign output gap rises at first, and then declines for a few periods ( $t = 12, 13$  in Figure 3). After this, the foreign output gap starts to increase gradually (for  $t = 14, 15, 16$  in the figure). Although quantitatively small, this behavior of the foreign output gap for  $t = 14, 15, 16$  is enough to yield  $E_t(x_{F,t+1} - x_{F,t}) > 0$  during those periods, which helps to alleviate the severity of the liquidity trap that the home country is caught in.

Variation in the Ramsey equilibrium path across different realizations of the shocks is illustrated in Figure 5. There, the paths of the nominal interest rates under the optimal policy are plotted for the cases where  $T_F = 10$  but  $T_H$  varies from 12 to 17. These paths are interpreted in the same way as in the previous figure: The optimal policy is seen to be characterized primarily by international dependence and history dependence.

The next two figures demonstrate how the Ramsey equilibrium depends on the probabilities of the natural rates returning to normal,  $p$  and  $q$ . In Figure 6, we continue to assume that  $p = q$  as in the previous figures, but allow the value to vary. Specifically, we plot the paths of the nominal interest rates under the optimal policy when  $T_H = 15$

and  $T_F = 10$  and it is assumed that  $p = q \in \{0.15, 0.2, 0.25, 0.3\}$ . It can be seen that, as probabilities  $p$  and  $q$  get smaller, the history dependence effect becomes more marked: that is, the optimal monetary policy requires commitment to a lower interest rate for longer periods. The international dependence effect is also magnified by a smaller value of  $p$  and  $q$  (notice the larger drop in  $i_{F,t}$  after  $t = 16$ ). This is because a larger degree of history dependence amplifies the boom in a country when its natural rate returns to normal, and this in turn increases the impact on the real interest rate in the other country.

In Figure 7, we consider the case of asymmetric probabilities:  $p \neq q$ . We fix  $q = 0.25$  and let  $p$  vary from 0.2 to 0.275. To focus on the asymmetry of the probabilities, we look at the equilibrium path for a symmetric realization of the shocks,  $T_H = T_F = 10$ . The figure demonstrates that a lower value of  $p$  leads to a larger degree of history dependence in the home monetary policy and also a larger degree of international dependence from the perspective of the foreign monetary policy.

## 5 Simple Monetary-Policy Rules

In this section we examine the extent to which the optimal monetary policy can be approximated by a “simple” interest-rate rule. In the case of a closed economy with no possibility of falling into a liquidity trap, Schmitt-Grohé and Uribe (2007) show that the optimal policy is replicated fairly well by the class of interest-rate rules that respond only to the inflation rate. In the liquidity trap case, Eggertsson and Woodford (2003) argue that a simple price-level targeting policy performs well for the closed economy. Our question is whether a similarly simple such monetary policy rule can be identified for our model of a global liquidity trap. For this purpose, we restrict attention to classes of simple interest-rate rules where nominal interest rates respond to some combination of inflation rates, price levels, output gaps, and nominal exchange rates. We will see that a simple interest-rate rule that includes both the foreign price level and the output gap in addition to those of the home country can improve welfare. Among the various rules we consider, this is the one that best captures the key features of the optimal monetary policy analyzed in the previous section.

We start with the case where the nominal interest rate in each country is set to respond only to domestic variables. Specifically, consider the following two classes of interest rate rule: the interest rate rule with inflation targets:

$$\begin{aligned}\tilde{i}_{H,t} &= \phi_\pi (\pi_{H,t} - \bar{\pi}_{H,t}) + \phi_x x_{H,t} + \bar{r}, \\ \tilde{i}_{F,t} &= \phi_\pi (\pi_{F,t}^* - \bar{\pi}_{F,t}^*) + \phi_x x_{F,t} + \bar{r};\end{aligned}\tag{36}$$

and the interest rate rule with price-level targets:

$$\begin{aligned}\tilde{i}_{H,t} &= \phi_p (\ln P_{H,t} - \ln \bar{P}_{H,t}) + \phi_x x_{H,t} + \bar{r}, \\ \tilde{i}_{F,t} &= \phi_p (\ln P_{F,t}^* - \ln \bar{P}_{F,t}^*) + \phi_x x_{F,t} + \bar{r}.\end{aligned}\tag{37}$$

In what follows we assume that the target inflation rates in the interest-rate rules (36) are zero:  $\bar{\pi}_{H,t} = \bar{\pi}_{F,t}^* = 0$ ; and that the target price levels in the interest-rate rules (37) are the date-0 price levels:  $\bar{P}_{H,t} = P_{H,0}$  and  $\bar{P}_{F,t}^* = P_{F,0}^*$ . Due to the zero bound on nominal interest rates, the actual rates set by the monetary authorities are

$$i_{H,t} = \max\{\tilde{i}_{H,t}, 0\}, \quad \text{and} \quad i_{F,t} = \max\{\tilde{i}_{F,t}, 0\}.$$

Given that the natural rates,  $r_{H,t}^n$  and  $r_{F,t}^n$ , follow the stochastic process described in the previous section, we compare the expected world-welfare loss (21) evaluated in period 1 under alternative policy rules. For the interest-rate rules (36)-(37), we restrict the policy parameters so that  $1.1 \leq \phi_\pi \leq 5$ ,  $0 \leq \phi_x \leq 5$ , and  $1.1 \leq \phi_p \leq 5$ .<sup>10</sup> Furthermore, this parameter space is discretized with a grid size of 0.5, when searching for the optimal parameter configuration.

Table 2 shows the optimal configuration of parameters for each class of interest-rate rules and the associated world-welfare losses (21). They are normalized by the world-welfare loss for the optimal monetary policy. In the table, the label “ITR” denotes the interest-rate rule with inflation targets and “PLTR” denotes the interest-rate rule with price-level targets.

The second row of Table 2 shows that the best inflation-targeting rule puts a zero weight on the output gap. This is similar to what Schmitt-Grohé and Uribe (2007) find in

---

<sup>10</sup>This restriction acts to guarantee determinacy.

a closed-economy model without a liquidity trap. The fourth row indicates that a price-level targeting rule ought also to place some weight on the output gap. For comparison, the first and third rows of Table 2 provide the world-welfare losses under some conventional parameter configurations: specifically,  $\phi_\pi = 1.5$  and  $\phi_x = 0.5$  for the interest-rate rule with inflation targets, and  $\phi_p = 1.5$  and  $\phi_x = 0.5$  for the interest-rate rule with price-level targets.

In terms of welfare, the interest-rate rule with price-level targets performs far better than the inflation-targeting rule, as is shown in the fourth row of Table 2. The reason is that the inflation-targeting rule does not provide any history dependence, a key element in mitigating the severity of a liquidity trap. In contrast, with price-level targets, the nominal interest rate in each country is gradually adjusted to its steady-state level after the natural rate regains its steady-state value. This enables it to generate history dependence. This property of the policy with price-level targets is in line with the findings of Eggertsson and Woodford (2003) for the closed economy model with a liquidity trap.

Now let us look at the case where the home country adopts an interest-rate rule with inflation targets but the foreign country adopts an interest-rate rule with price-level targets.<sup>11</sup> The fifth row of Table 2 shows the substantial deterioration in world welfare in this case compared to when both countries adopt an interest-rate rule with price-level targets. The clear implication is that the two monetary authorities should jointly commit to an interest-rate rule with price-level targets when faced with a global liquidity trap. A single country's commitment to a history dependent policy is not enough.

We next examine if including foreign variables in the domestic policy rule improves welfare. For this purpose, we augment the interest-rate rule with price-level targets with foreign variables as follows:

$$\begin{aligned}\tilde{i}_{H,t} &= \phi_p (\ln P_{H,t} - \ln \bar{P}_H) + \phi_{pa} (\ln P_{F,t}^* - \ln \bar{P}_F^*) + \phi_x x_{H,t} + \phi_{xa} x_{F,t} + \bar{r}, \\ \tilde{i}_{F,t} &= \phi_p (\ln P_{F,t}^* - \ln \bar{P}_F^*) + \phi_{pa} (\ln P_{H,t} - \ln \bar{P}_H) + \phi_x x_{F,t} + \phi_{xa} x_{H,t} + \bar{r},\end{aligned}\tag{38}$$

---

<sup>11</sup>This may be viewed as a situation where one country commits to its policy but in the other policy remains discretionary. For the definition of discretionary policy in a liquidity trap, see Jung, Teranishi, and Watanabe (2005).

where we restrict the policy parameters so that  $0 \leq \phi_{xa} \leq 5$  and  $0 \leq \phi_{pa} \leq 5$ . Following these rules, a country lowers its policy rate when the other country experiences a downturn, *i.e.*, a negative output gap and a price level lower than the target level. Thus, these rules capture international dependence. The sixth row of Table 2 shows how augmenting the policy rule in this way improves welfare.

We can interpret the augmented rules (38) in terms of the nominal exchange rate. Note that it follows from the household's first-order conditions (10) that the nominal exchange rate  $\mathcal{E}_t$  satisfies

$$\mathcal{E}_t = \frac{P_{H,t}}{P_{F,t}^*} \frac{n_F}{n_H} \frac{Y_{H,t}}{Y_{F,t}}.$$

Define the log deviation of the nominal exchange rate,  $\mathcal{E}_t^n$ , as

$$\mathcal{E}_t^n = \frac{\bar{P}_H}{\bar{P}_F^*} \frac{n_F}{n_H} \frac{Y_{H,t}^n}{Y_{F,t}^n},$$

and let  $\epsilon_t$  denote the exchange rate gap, *i.e.*,  $\epsilon_t \equiv \ln \mathcal{E}_t - \ln \mathcal{E}_t^n$ . It follows that

$$\epsilon_t = (\ln P_{H,t} - \ln \bar{P}_H) - (\ln P_{F,t}^* - \ln \bar{P}_F^*) + x_{H,t} - x_{F,t}.$$

Given this, we can rewrite equations (38) as

$$\begin{aligned} \tilde{i}_{H,t} &= (\phi_p - \phi_{ex}) (\ln P_{H,t} - \ln \bar{P}_H) + (\phi_{pa} + \phi_{ex}) (\ln P_{F,t}^* - \ln \bar{P}_F^*) \\ &\quad + (\phi_x - \phi_{ex}) x_{H,t} + (\phi_{xa} + \phi_{ex}) x_{F,t} + \phi_{ex} \epsilon_t + \bar{r}, \\ \tilde{i}_{F,t} &= (\phi_p - \phi_{ex}) (\ln P_{F,t}^* - \ln \bar{P}_F^*) + (\phi_{pa} + \phi_{ex}) (\ln P_{H,t} - \ln \bar{P}_H) \\ &\quad + (\phi_x - \phi_{ex}) x_{F,t} + (\phi_{xa} + \phi_{ex}) x_{H,t} - \phi_{ex} \epsilon_t + \bar{r}, \end{aligned}$$

where  $\phi_{ex}$  is a positive parameter. Thus, the welfare gain from augmenting the rule can be interpreted as the benefit of letting the policy rate respond to the nominal exchange rate. This helps the policy rule to capture the features of desirable policy in a global liquidity trap, namely, history dependence and international dependence.<sup>12</sup>

---

<sup>12</sup>Here, since both of the two countries are under the liquidity trap, the exchange rate peg to another country's currency for escaping from a liquidity trap, so called foolproof way in Svensson (2001), is not relevant policy. Consequently, two central banks create inflation expectation by the commitment to the zero interest rate. Alternatively, if one country is out of a liquidity trap, then foolproof way could work as shown in Coenen and Wieland (2003).

## 6 Concluding Remarks

In this paper we consider a two-country New Open Economy Macroeconomics model, and analyze the optimal monetary policy when monetary authorities cooperate in the face of a global liquidity trap — that is, a situation where both countries are caught simultaneously in liquidity traps. Compared to the closed economy case, the most notable feature of the optimal policy in the global liquidity trap is its international dependence. Whether or not a country’s nominal interest rate is hitting the zero bound affects the target inflation rate in the other country. The direction of the effect depends on whether goods produced in the two countries are Edgeworth complements or substitutes. We also compare several classes of simple interest-rate rules. Our finding is that targeting the price level yields higher welfare than targeting the inflation rate, and that it is desirable to let the policy rate of each country respond not only to its own price level and output gap, but also to those in the other country.

The model considered in this paper is of course very stylized, and the robustness of our findings needs to be tested under alternative assumptions. For instance, our current analysis is restricted to the case where the monetary authorities in the two countries coordinate their monetary policy choices with each other so as to maximize world welfare. An alternative assumption would be that the monetary authorities set their respective policies in a non-cooperative way. Another extension of potential interest would be to consider how the results would be affected if we adopted local currency pricing, rather than producer currency pricing as at present. In addition, while the current paper focuses on the interest rate policy, incorporating other possible policy measures, such as quantitative easing and credit easing, is also important. These extensions are left for future research.

## References

- Adam, Klaus, and Roberto M. Billi (2006). “Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates.” *Journal of Money, Credit and Banking*, 38(7), 1877–1905.

- Adam, Klaus, and Roberto M. Billi (2007). “Discretionary monetary policy and the zero lower bound on nominal interest rates.” *Journal of Monetary Economics*, 54(3), 728–752.
- Benigno, Gianluca, and Pierpaolo Benigno (2003). “Price Stability in Open Economies.” *Review of Economic Studies*, 70(4), 743–764.
- Benigno, Pierpaolo, and Michael Woodford (2005). “Inflation Stabilization and Welfare: The Case of a Distorted Steady State.” *Journal of the European Economic Association*, 3(6), 1185–1236.
- Clarida, Richard, Jordi Gali, and Mark Gertler (2001). “Optimal Monetary Policy in Open versus Closed Economies: An Intergated Approach.” *American Economic Review*, 91, 248–252.
- Coenen, Gunter, and Volker Wieland (2003). “The Zero-Interest-Rate Bound and the Role of the Exchange Rate for Monetary Policy in Japan.” *Journal of Monetary Economics*, 50(5), 1071–1101.
- Devereux, Michael B., and Charles Engel (2003). “Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility.” *Review of Economic Studies*, 70(4), 765–783.
- Eggertsson, Gauti B., and Michael Woodford (2003). “The Zero Bound on Interest Rates and Optimal Monetary Policy.” *Brookings Papers on Economic Activity*, 34(1), 139–235.
- Jung, Taehun, Yuki Teranishi, and Tsutomu Watanabe (2005). “Optimal Monetary Policy at the Zero-Interest-Rate Bound.” *Journal of Money, Credit and Banking*, 37(5), 813–35.
- Kato, Ryo, and Shin-Ichi Nishiyama (2005). “Optimal monetary policy when interest rates are bounded at zero.” *Journal of Economic Dynamics and Control*, 29(1-2), 97–133.
- Nakajima, Tomoyuki (2008). “Liquidity Trap and Optimal Monetary Policy in Open Economies.” *Journal of the Japanese and International Economies*, 22(1), 1–33.
- Nakov, Anton (2008). “Optimal and Simple Monetary Policy Rules with Zero Floor on the Nominal Interest Rate.” *International Journal of Central Banking*, 4(2), 73–127.

- Reifschneider, David, and John C. Williams (2000). “Three Lessons for Monetary Policy in a Low-Inflation Era.” *Journal of Money, Credit and Banking*, pp. 936–978.
- Schmitt-Grohé, Stephanie, and Martín Uribe (2007). “Optimal Simple and Implementable Monetary and Fiscal Rules.” *Journal of Monetary Economics*, 54, 1702–1725.
- Svensson, Lars-E-O (2001). “The Zero Bound in an Open Economy: A Foolproof Way of Escaping from a Liquidity Trap.” *Monetary and Economic Studies*, 19(S1), 277–312.
- Woodford, Michael (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton: Princeton University Press.

Table 1: Parameter values

Parameters	Values	Explanation
$\beta$	0.99	Subjective discount factor
$\sigma$	2	Intertemporal elasticity of substitution
$\kappa$	0.024	Elasticity of inflation with respect to output
$\alpha$	0.66	Probability of price change
$\rho$	1	Elasticity of substitution between domestic and foreign goods
$\theta$	7.88	Elasticity of substitution among differentiated goods
$\omega$	0.47	Frish elasticity
$n_H$	0.5	Country size
$\Phi, \Phi^*$	0	Steady-state distortions
$\underline{r}^n$	-0.02/4	Negative natural rate shock
$\overline{S}$	50	Maximal length of periods with $r_t^n = \underline{r}^n$

Table 2: Relative losses under different rules

Rule	Parameters	Relative loss
Conventional ITR	$\phi_\pi = 1.5, \phi_x = 0.5$	232.8
Best rule in ITR	$\phi_\pi = 5, \phi_x = 0$	218.01
Conventional PLTR	$\phi_p = 1.5, \phi_x = 0.5$	5.76
Best rule in PLTR	$\phi_p = 5, \phi_x = 0.5$	2.93
Best rule in PLTR in the home country and best rule in ITR in the foreign country	$\phi_p = 5, \phi_x = 5$ and $\phi_\pi = 1.1, \phi_x = 5$	91.78
Best rule in PLTR with foreign output gap and foreign price level	$\phi_p = 5, \phi_x = 0.5, \phi_{xa} = 0.5$ and $\phi_{pa} = 5$	2.69

Note: The relative loss is given by dividing each loss by one under the optimal monetary policy. ITR denotes the interest-rate rule with inflation targets. PLTR denotes the interest-rate rule with price-level targets.

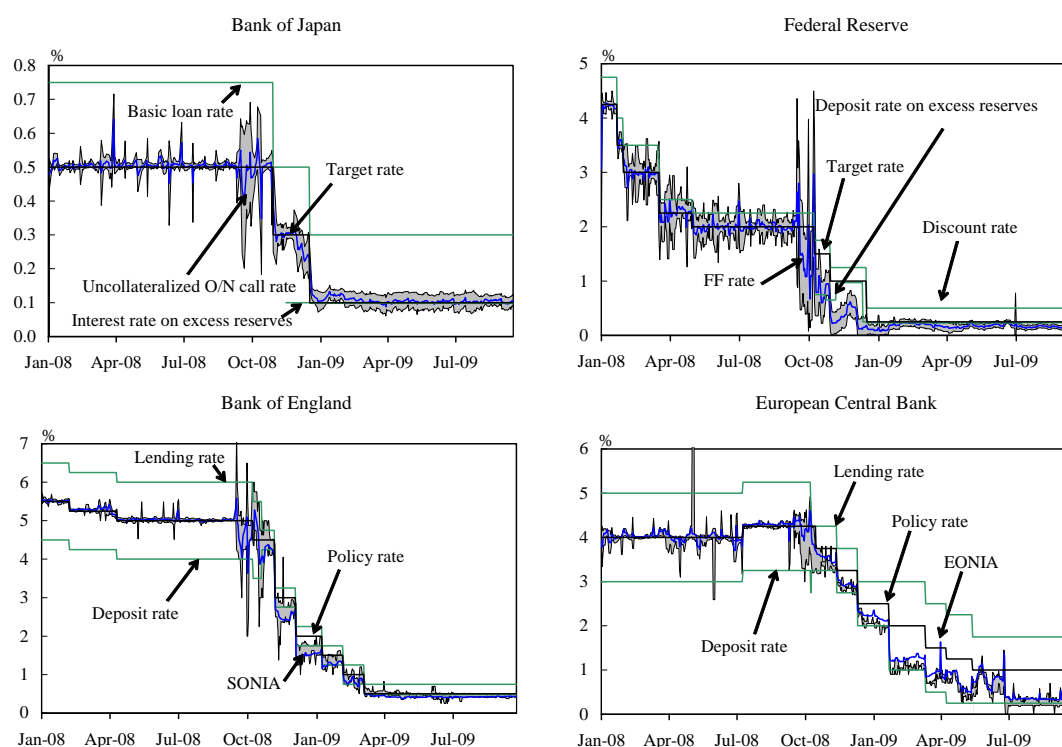


Figure 1: Policy interest rates in several advanced countries.

Note: Shadow indicates the range of the target rate plus/minus intraday one standard deviation for the Bank of Japan and the Federal Reserve, and the range of high-low euro deposit rates for the Bank of England and the European Central Bank. Sources: Bloomberg; Bank of Japan; Federal Reserve; Bank of England; European Central Bank.

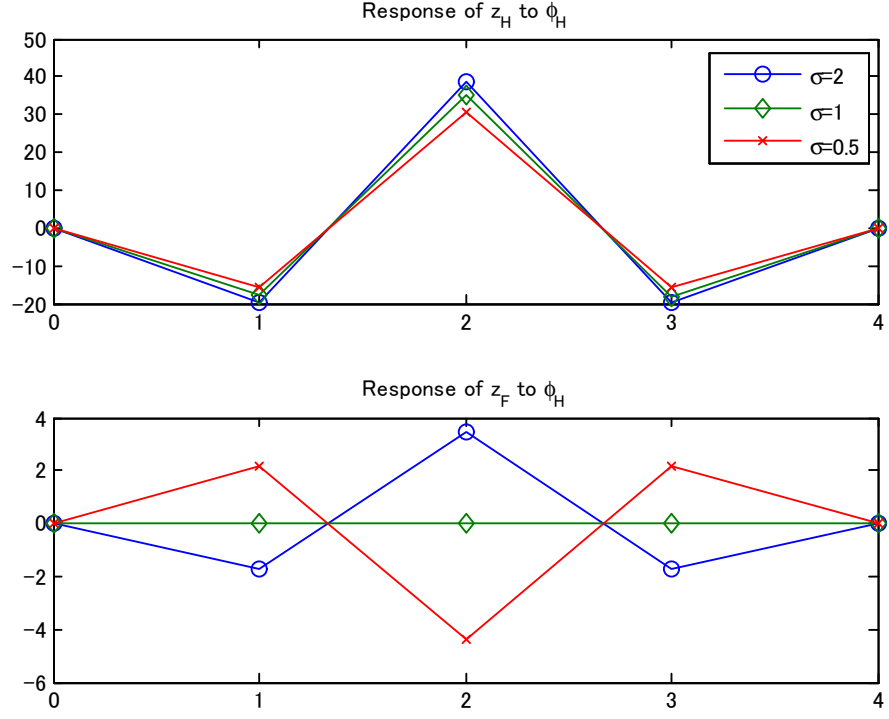


Figure 2: Response of  $z_H$  and  $z_F$  to a one-time increase in  $\phi_H$  for different values of  $\sigma$ . The upper panel shows the dynamics of  $z_H$  and the lower panel shows the dynamics of  $z_F$ .

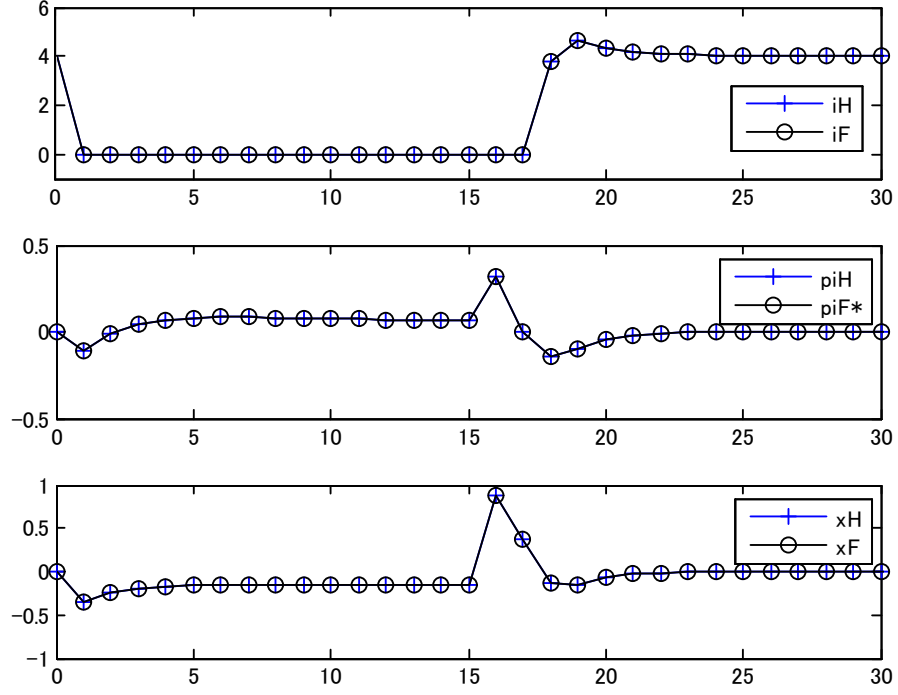


Figure 3: Case for  $T_H = T_F = 15$ .

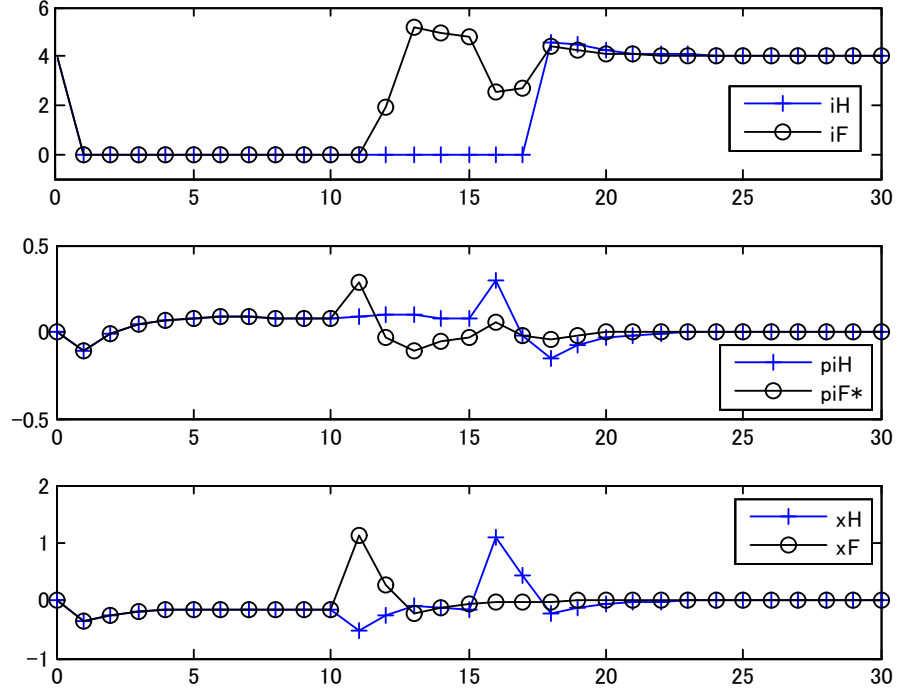


Figure 4: Case for  $T_F = 10$  and  $T_H = 15$ .

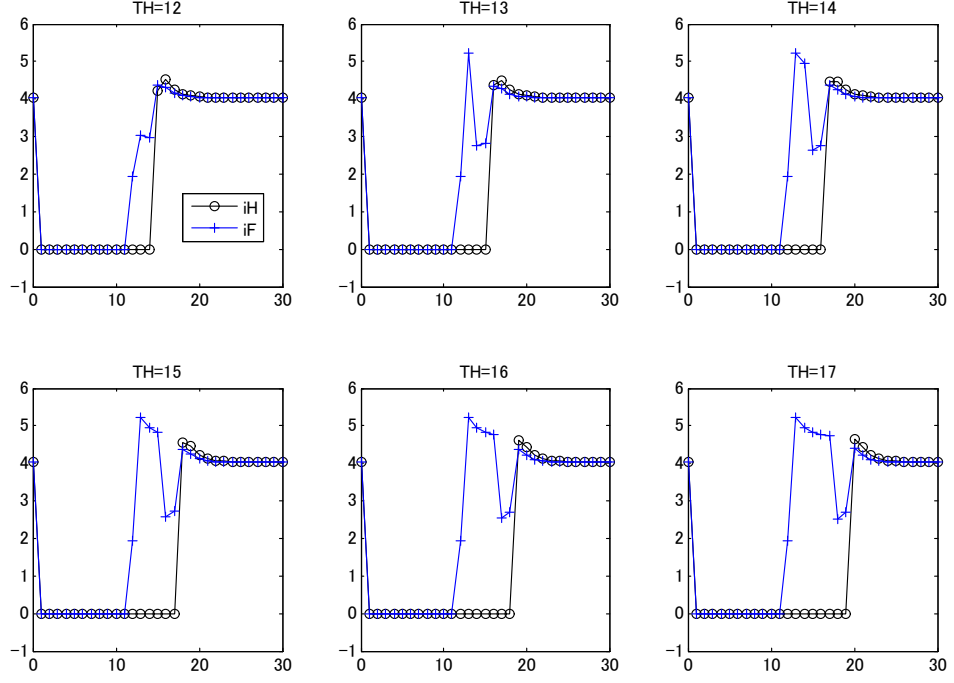


Figure 5: Case for from  $T_H = 12$  to  $T_H = 17$  and  $T_F = 10$ .

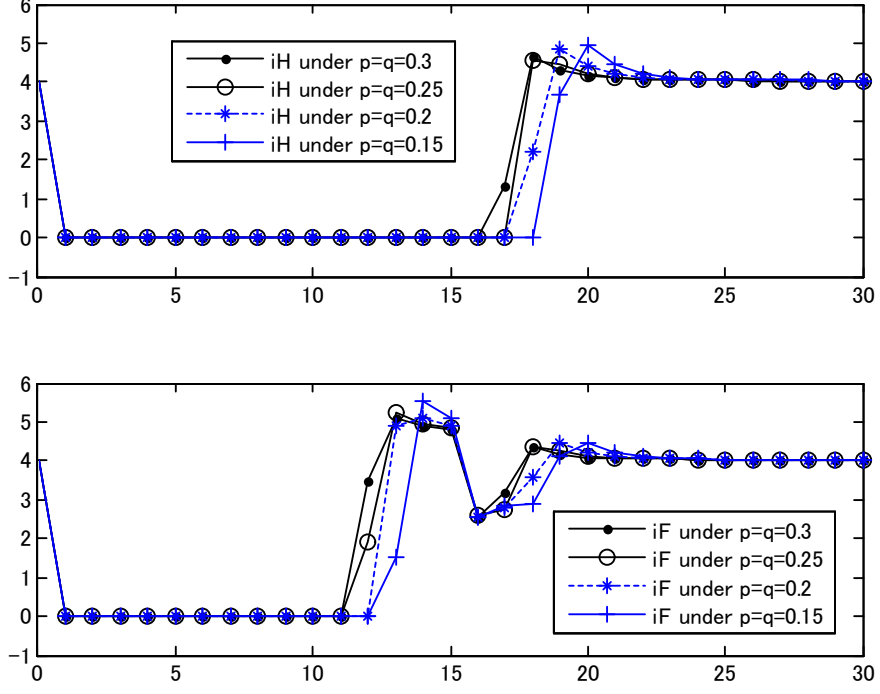


Figure 6: Case for  $T_F = 10$  and  $T_H = 15$  with different  $p = q$ .

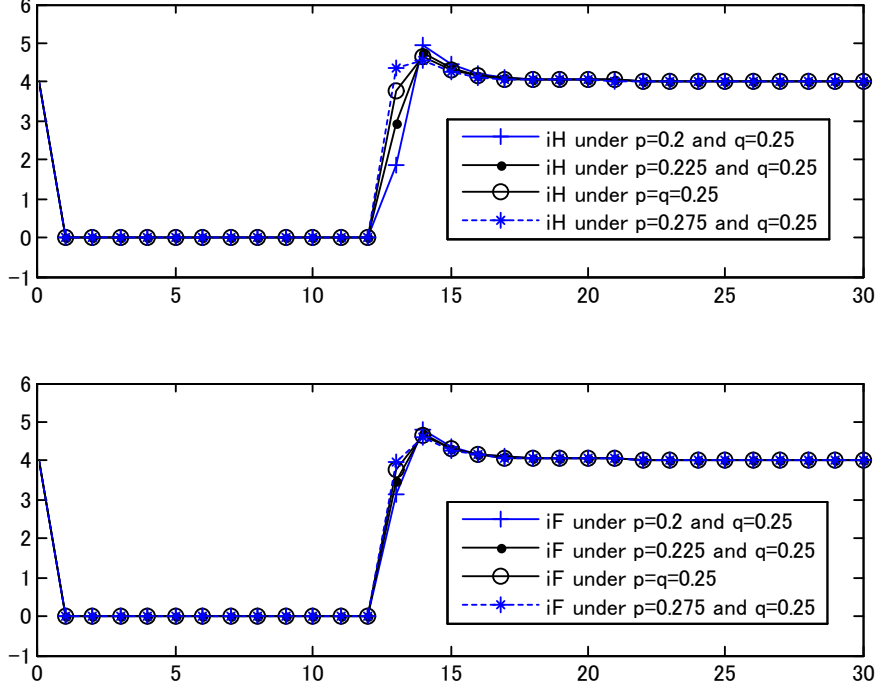


Figure 7: Case for  $T_H = T_F = 10$  with different probability  $p$  and  $q$ .

# Appendix

## A Welfare approximation

Here we follow Woodford (2003) and Benigno and Woodford (2005) to derive an approximate world welfare criterion. We assume that the monetary authorities in the two countries cooperate to maximize aggregate utility.

Given that preferences of the home and foreign household are given by equations (1) and (4), respectively, the level of average expected utility between the two countries is given by

$$n_H U_0 + n_F U_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(Y_{H,t}, Y_{F,t}) - \int_{N_H} v[y_t(i); \xi_t] di - \int_{N_F} v^*[y_t^*(i); \xi_t^*] di \right\}.$$

The cost minimization of the home household leads to the following derived demands:

$$C_{j,t} = n_H C_t \left( \frac{P_{j,t}}{P_t} \right)^{-1}, \quad j = H, F,$$

$$c_t(i) = \frac{1}{n_j} C_{j,t} \left( \frac{p_t(i)}{P_{j,t}} \right)^{-\theta}, \quad j = H, F.$$

The derived demands of the foreign household are similarly given. Using these conditions, we obtain

$$\int_{N_H} v[y_t(i); \xi_t] di = n_H v \left( \frac{Y_{H,t}}{n_H}; \xi_t \right) \Delta_{H,t},$$

$$\int_{N_F} v^*[y_t^*(i); \xi_t^*] di = n_F v^* \left( \frac{Y_{F,t}}{n_F}; \xi_t^* \right) \Delta_{F,t},$$

where  $\Delta_H$  and  $\Delta_F$  are the measures of price dispersion defined by

$$\Delta_{H,t} \equiv \frac{1}{n_H} \int_{N_H} \left( \frac{p_t(i)}{P_{H,t}} \right)^{-\theta(1+\omega)} di,$$

$$\Delta_{F,t} \equiv \frac{1}{n_F} \int_{N_F} \left( \frac{p_t^*(i)}{P_{F,t}^*} \right)^{-\theta(1+\omega)} di.$$

Then the world welfare measure is given by

$$n_H U_0 + n_F U_0^* = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u(Y_{H,t}, Y_{F,t}) - n_H v \left( \frac{Y_{H,t}}{n_H}; \xi_t \right) \Delta_{H,t} - n_F v^* \left( \frac{Y_{F,t}}{n_F}; \xi_t^* \right) \Delta_{F,t} \right\}. \quad (39)$$

Now, fix a non-stochastic steady state with zero inflation. In what follows, a bar over a variable denotes its steady state value, and a hat indicates the log-deviation from the steady-state value. Following Benigno and Woodford (2005), we take a second-order approximation of equation (39) around that steady state in terms of  $\Xi \equiv (\xi, \xi^*, \hat{\Delta}_{H,-1}^{1/2}, \hat{\Delta}_{F,-1}^{1/2}, \varphi)$ , where  $\varphi$  are parameters of policy rules normalized in such a way that  $\varphi = 0$  implies long-run output levels  $Y_{j,\infty} = \bar{Y}_j$ ,  $j = H, F$ , and for any small enough  $\varphi$ ,  $\ln Y_{j,\infty} - \ln \bar{Y}_j = \mathcal{O}(\|\varphi\|)$ ,  $j = H, F$ . Then,

$$\begin{aligned}
& u(Y_{H,t}, Y_{F,t}) - n_H v\left(\frac{Y_{H,t}}{n_H}; \xi_t\right) \Delta_{H,t} - n_F v\left(\frac{Y_{F,t}}{n_F}; \xi_t^*\right) \Delta_{F,t} \\
&= \frac{\bar{u}_H \bar{Y}_H}{n_H} \left\{ n_H \Phi \hat{Y}_{H,t} + n_F \Phi^* \hat{Y}_{F,t} + \frac{n_H}{2} \left[ (1 - \sigma) n_H - (1 + \omega)(1 - \Phi) \right] \hat{Y}_{H,t}^2 \right. \\
&\quad + (1 - \sigma) n_H n_F \hat{Y}_{H,t} \hat{Y}_{F,t} + \frac{n_F}{2} \left[ (1 - \sigma) n_F - (1 + \omega)(1 - \Phi^*) \right] \hat{Y}_{F,t}^2 \\
&\quad \left. - n_H (1 - \Phi) \hat{Y}_{H,t} \xi_t - n_F (1 - \Phi^*) \hat{Y}_{F,t} \xi_t^* - \frac{n_H (1 - \Phi)}{1 + \omega} \hat{\Delta}_{H,t} - \frac{n_F (1 - \Phi^*)}{1 + \omega} \hat{\Delta}_{F,t} \right\} \\
&\quad + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3),
\end{aligned} \tag{40}$$

where t.i.p. denotes the terms independent of policy. Also, it is shown that

$$\sum_{t=0}^{\infty} \beta^t \frac{n_H}{1 + \omega} \hat{\Delta}_{H,t} = \frac{\alpha \theta (1 + \omega \theta)}{(1 - \alpha)(1 - \alpha \beta)} \sum_{t=0}^{\infty} \beta^t \frac{n_H}{2} \pi_{H,t}^2 + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3), \tag{41}$$

$$\sum_{t=0}^{\infty} \beta^t \frac{n_F}{1 + \omega} \hat{\Delta}_{F,t} = \frac{\alpha \theta (1 + \omega \theta)}{(1 - \alpha)(1 - \alpha \beta)} \sum_{t=0}^{\infty} \beta^t \frac{n_F}{2} \pi_{F,t}^{*2} + \text{t.i.p.} + \mathcal{O}(\|\Xi\|^3). \tag{42}$$

For simplicity, we follow Woodford (2003) and assume that  $\Phi$  and  $\Phi^*$  are small to delete the linear terms in equation (40),  $n_H \Phi \hat{Y}_{H,t} + n_F \Phi^* \hat{Y}_{F,t}$ . Let  $Y_j^e$ ,  $j = H, F$ , denote the efficient levels of output in the absence of shocks, that is,

$$\frac{v_y(Y_H^e/n_H; 0)}{u_H(Y_H^e, Y_F^e)} = 1, \quad \frac{v_y(Y_F^e/n_F; 0)}{u_F(Y_H^e, Y_F^e)} = 1.$$

Then, let  $x_j^e \equiv \ln Y_j^e - \ln \bar{Y}_j$ ,  $j = H, F$ , denote the efficient levels of the output gaps, where  $\bar{Y}_j$ ,  $j = H, F$ , are the steady-state levels of output. When  $\Phi$  and  $\Phi^*$  are small,

$$\begin{aligned}
& [1 + \omega + (\sigma - 1)n_H] x_H^e + (\sigma - 1)n_F x_F^e = \Phi + \mathcal{O}(\|\Phi\|^2), \\
& (\sigma - 1)n_H x_H^e + [1 + \omega + (\sigma - 1)n_F] x_F^e = \Phi^* + \mathcal{O}(\|\Phi^*\|^2).
\end{aligned}$$

Then, using equations (40)-(42), a second-order approximation of the world welfare measure equation (39) is given by

$$n_H U_0 + n_F U_0^* = -\frac{\bar{u}_H \bar{Y}_H \theta}{n_H \zeta} \sum_{t=0}^{\infty} \beta^t L_t + \text{t.i.p.} + O(\|\Phi, \Phi^*, \Xi\|^3),$$

where  $\zeta$  is as defined in equation (17), and the world loss function  $L_t$  is given by

$$L_t \equiv \frac{1}{2}(x_t - x^e)' \Lambda (x_t - x^e) + \frac{n_H}{2} \pi_{H,t}^2 + \frac{n_F}{2} \pi_{F,t}^{*2}.$$

where  $x^e \equiv (x_H^e, x_F^e)' = (0, 0)'$ .

## B Numerical Algorithm for Stochastic Simulation

Suppose that the natural rates of the two countries follows the stochastic process as described in the main text, and consider the Ramsey problem in period  $\tau = 1$  of minimizing the world welfare function (21) subject to the constraints (15), (16), (26), (27), (28), and (29). Let  $\psi_t \equiv (\psi_{H,t}, \psi_{F,t})$  be the multipliers for constraints (15) and (16), and  $\phi_t \equiv (\phi_{H,t}, \phi_{F,t})$  be the multiplier associated with constraints (26), (27), (28), and (29). Then the Lagrangian for the Ramsey problem can be formed as:

$$\begin{aligned} \mathcal{L} = E_1 \sum_{t=1}^{\infty} \beta^t & \left\{ \frac{1}{2} x_t' \Lambda x_t + \frac{n_H}{2} \pi_{H,t}^2 + \frac{n_F}{2} \pi_{F,t}^{*2} \right. \\ & + \psi_{H,t} \left( n_H \pi_{H,t} - \gamma_H n_H x_{H,t} - \gamma_{HF} n_H n_F x_{F,t} - \beta n_H \pi_{H,t+1} \right) \\ & + \psi_{F,t} \left( n_F \pi_{F,t}^* - \gamma_{HF} n_H n_F x_{H,t} - \gamma_F n_F x_{F,t} - \beta n_F \pi_{F,t+1}^* \right) \\ & + \phi_{H,t} \left( n_H [1 + (\sigma - 1) n_H] (x_{H,t} - x_{H,t+1}) \right. \\ & \quad \left. + (\sigma - 1) n_H n_F (x_{F,t} - x_{F,t+1}) - n_H \pi_{H,t+1} - n_H r_{H,t}^n \right) \\ & + \phi_{F,t} \left( (\sigma - 1) n_H n_F (x_{H,t} - x_{H,t+1}) \right. \\ & \quad \left. + n_F [1 + (\sigma - 1) n_F] (x_{F,t} - x_{F,t+1}) - n_F \pi_{F,t+1}^* - n_F r_{F,t}^n \right) \Big\}, \end{aligned}$$

where  $\Lambda$  is defined in equation (20). The first-order conditions are given by

$$\Lambda x_t - \theta \Lambda \psi_t + A(\phi_t - \beta^{-1} \phi_{t-1}) = 0, \quad (43)$$

$$\pi_t + \psi_t - \psi_{t-1} - \beta^{-1} \phi_{t-1} = 0, \quad (44)$$

$$\pi_t = \theta N^{-1} \Lambda x_t + \beta E_t \pi_{t+1}, \quad (45)$$

$$i_t = E_t \left[ N^{-1} A(x_{t+1} - x_t) + \pi_{t+1} + r_t^n \right], \quad (46)$$

and

$$i_t \geq 0, \quad \phi_t \geq 0, \quad \phi_{H,t} i_{H,t} = \phi_{F,t} i_{F,t} = 0, \quad (47)$$

where  $x_t = (x_{H,t}, x_{F,t})$ ,  $\pi_t = (\pi_{H,t}, \pi_{F,t}^*)$ ,  $i_t = (i_{H,t}, i_{F,t})$ ,  $r_t^n = (r_{H,t}^n, r_{F,t}^n)$ , and

$$A = \begin{bmatrix} n_H(1 + (\sigma - 1)n_H) & (\sigma - 1)n_H n_F \\ (\sigma - 1)n_H n_F & n_F(1 + (\sigma - 1)n_F) \end{bmatrix}$$

$$N = \begin{bmatrix} n_H & 0 \\ 0 & n_F \end{bmatrix}$$

Depending on the signs of  $r_{H,t}^n$  and  $r_{F,t}^n$ , and also depending on whether or not the zero bound condition binds for  $i_{H,t}$  and  $i_{F,t}$ , we distinguish nine phases in the equilibrium dynamics:<sup>13</sup>

	$r_F^n < 0, i_F = 0$	$r_F^n > 0, i_F = 0$	$r_F^n > 0, i_F > 0$
$r_H^n < 0, i_H = 0$	phase (1,1)	phase (1,2)	phase (1,3)
$r_H^n > 0, i_H = 0$	phase (2,1)	phase (2,2)	phase (2,3)
$r_H^n > 0, i_H > 0$	phase (3,1)	phase (3,2)	phase (3,3)

Remember that  $(T_H, T_F) \in \{1, \dots, S\}^2$  are the stopping times describing the last periods in which  $r_{H,t}^n < 0$  and  $r_{F,t}^n < 0$ , respectively. Let  $k_{1,2}(T_F)$  and  $k_{2,1}(T_H)$  be the numbers

---

<sup>13</sup>In principle, it is possible to have other phases, for instance, the one where  $r_H^n < 0$  and  $i_H > 0$ . With our parameter configuration, however, we have confirmed in our numerical solution that these nine phases are the only possible ones that occur in the Ramsey equilibrium.

of periods in which phases (1,2) and (2,1) occur given realized values of  $T_F$  and  $T_H$ , respectively. Similarly, let  $k_{2,2}(T_H, T_F)$ ,  $k_{2,3}(T_H, T_F)$ , and  $k_{3,2}(T_H, T_F)$  denote the number of periods in which phases (2,2), (2,3), and (3,2) occur given realized values of  $T_H$  and  $T_F$ , respectively. Here,  $k_{1,2}(T_F)$ ,  $k_{2,1}(T_H)$ ,  $k_{2,2}(T_H, T_F)$ ,  $k_{2,3}(T_H, T_F)$ , and  $k_{3,2}(T_H, T_F)$  are all non-negative integers. Our numerical algorithm is to find a collection of functions  $\{k_{1,2}(T_F), k_{2,1}(T_H), k_{2,2}(T_H, T_F), k_{2,3}(T_H, T_F), k_{3,2}(T_H, T_F)\}$  and  $\{\pi_t(T_H, T_F), x_t(T_H, T_F), \psi_t(T_H, T_F), \phi_t(T_H, T_F), i_t(T_H, T_F)\}$  such that the conditions for the Ramsey equilibrium, (43)-(47) are all satisfied.

## C Simulation under Deterministic Shock

For simulations under deterministic shocks, following Jung, Teranishi, and Watanabe (2005), we assume that both private-agents and monetary authorities completely foresee the sequence of natural interest rates  $\{r_{H,t}^n, r_{F,t}^n\}_{t=1}^{\bar{S}+1}$  at period  $t = 1$ , where  $\bar{S} + 1$  is the time when economy is in the steady state.

Figure A1 displays the time paths of nominal interest rates  $i_{H,t}$  and  $i_{F,t}$ , inflation rates  $\pi_{H,t}$  and  $\pi_{F,t}^*$ , and output gaps  $x_{H,t}$  and  $x_{F,t}$  in the two countries from the top when adverse shocks in the two countries last until  $T_H = T_F = 10$ . Figure A2 displays the time paths of nominal interest rates, inflation rates, and output gaps in the two countries when a shock to natural rate of interest lasts longer in the domestic than in the foreign country at  $T_H = 15$  and  $T_F = 10$ . We can see that the case with deterministic shocks are similar to the case with stochastic shocks.

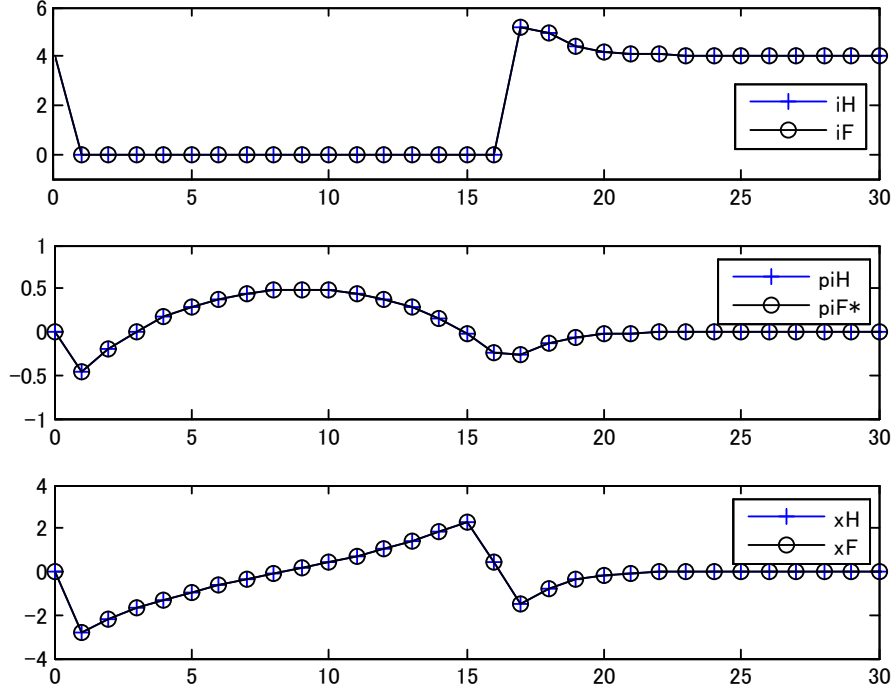


Figure A1: Case for  $T_H = T_F = 15$  under sthe deterministic shock.

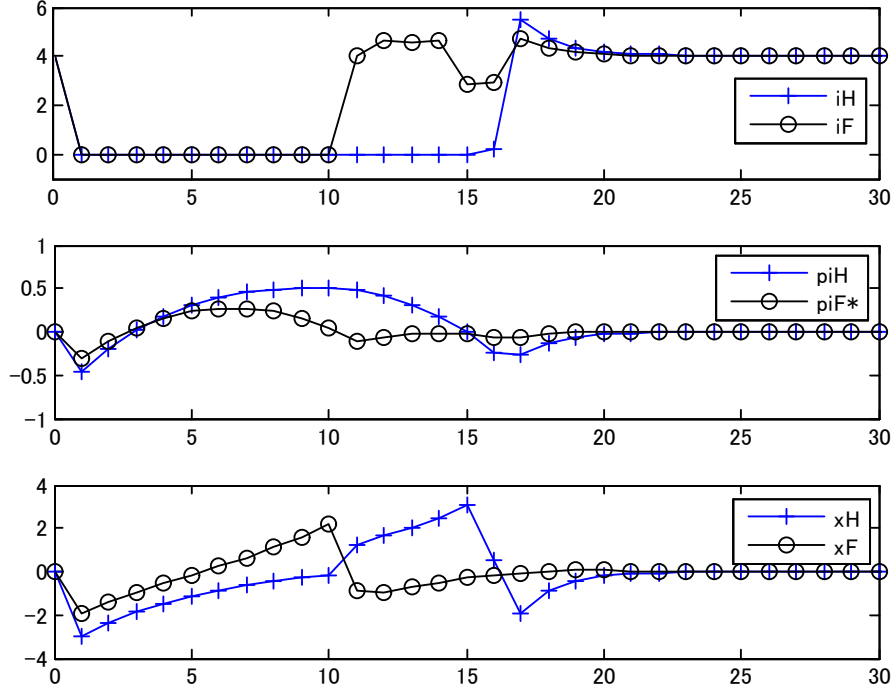


Figure A2: Case for  $T_F = 10$  and  $T_H = 15$  under the deterministic shock.